The Social Dynamics of Performance
Technical Appendix

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A Proof of Propositions 1 and 2

In this appendix, I derive the dynamics of Network A and Network B. The derivations of the density μ_t^A for Network A contained in equation (4) of Proposition 1 proceed along the same lines as those of Duffie and Manso (2007) and the references thereof, except for one aspect: agents meet in Network A with an intensity ηq_t^A that is network-specific. Since agents are restricted to meet with others currently positioned within the same network, types are drawn from a density that needs to be normalized by the size of the network such that it integrates up to one, i.e. $\frac{\mu_t^A}{q_t^A}$. Hence, for $n \in A$, the dynamics of μ_t^A evolve as

$$\frac{\mathrm{d}\mu_t^A(n)}{\mathrm{d}t} = \eta q_t^A \left(\sum_{k=1}^{n-1} \mu_t^A(n-k) \frac{\mu_t^A(k)}{q_t^A} - \mu_t^A \right)$$

which yields (4). The mass q_t^A is then obtained as follows: from (4), I can write

$$\mathrm{d}e^{\eta \int_0^t q_s^A \mathrm{d}s} \mu_t^A(n) = \eta e^{\eta \int_0^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_t^A(n-k)\mu_t^A(k)\mathrm{d}t.$$

Integrating, I obtain that

$$\mu_t^A(n) = e^{-\eta \int_0^t q_s^A \mathrm{d}s} \mu_0^A(n) + \eta \int_0^t e^{-\eta \int_u^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_u^A(n-k)\mu_u^A(k)\mathrm{d}u.$$

Hence, if $n = 1$, this implies that $\mu_t^A(1) = e^{-\eta \int_0^t q_s^A \mathrm{d}s}$ and

$$\mu_t^A(n) = \eta \int_0^t e^{-\eta \int_u^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_u^A(n-k)\mu_u^A(k)\mathrm{d}u$$

for $n \in A \backslash \{1\}$. Accordingly, the mass q_t^A of Network A satisfies

$$\begin{aligned} q_t^A &= \sum_{n=1}^{K-1} \mu_t^A(n) \\ &= e^{-\eta \int_0^t q_s^A \mathrm{d}s} \left(1 + \eta \int_0^t e^{\eta \int_0^u q_s^A \mathrm{d}s} \sum_{n=2}^{K-1} \sum_{k=1}^{n-1} \mu_u^A(n-k)\mu_u^A(k)\mathrm{d}u \right). \end{aligned}$$

Differentiating this expression, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} q_t^A = -\eta (q_t^A)^2 + \eta \sum_{n=2}^{K-1} \sum_{m=1}^{n-1} \mu_t^A(n-m)\mu_t^A(m), \;\; q_0^A = \omega_0.$$

Equation (5) in Proposition 2 is derived in two steps: *i)* I first take care of the set of agents who migrate to Network B. Taking discrete intervals of time Δ, the migration of agents to Network B satisfies

$$\mu^B_{t+\Delta} = (1 - \eta q^B_t \Delta)\mu^B_t + \mu^A_{t+\Delta} - \mu^A_t.$$

Rearranging and taking the limit on both sides, I get

$$\lim_{\Delta \to 0} \frac{\mu^B_{t+\Delta} - \mu^B_t}{\Delta} = -\eta q^B_t \mu^B_t + \lim_{\Delta \to 0} \frac{\mu^A_{t+\Delta} - \mu^A_t}{\Delta}.$$

Migrating agents have between K and $2(K-1)$ signals and their meetings further need to incorporate the restriction that both networks are disjoint: an agent who currently holds $K+1$ signals could not previously have K signals for, otherwise, she would already be in Network B. Likewise, an agent who holds $2(K-1)$ could only previously possess $K-1$ signals. Hence, the types k and $n-k$ whose meeting results into $n \in [K, 2(K-1)]$ have to be such that $1 \leq k \leq K-1$ and $1 \leq n-k \leq K-1$, which is equivalent to $1 \vee (n-(K-1)) \leq k \leq (K-1) \wedge (n-1)$ or $n-(K-1) \leq k \leq K-1$ given that $K \geq 2$. Therefore, migrating agents with $n \in [K, 2(K-1)]$ signals enter Network B at a rate $\frac{\mathrm{d}\mu^B_t(n)}{\mathrm{d}t} = -\eta q^B_t \mu^B_t(n) + \eta \sum_{k=n-(K-1)}^{K-1} \mu^A_t(n-k)\mu^A_t(k)$. First, there is no emigration term related to μ^A_t because agents achieving type $n \in [K, 2(K-1)]$ are not part of Network A. Their emigration takes now place with respect to Network B. Second, the convolution is truncated in such a way that the previous discussion holds.

Besides migrating agents, *ii)* I need to take care of the meetings among incumbents to Network B who necessarily achieve a type higher than $2(K-1)$. The derivation thereof proceeds along the same lines as that for Network A and may, therefore, be directly adapted: the types k and $n-k$ whose meeting results into $n \in [2K, N-1]$ have to be such that $K \leq k \leq N-1$ and $K \leq n-k \leq N-1$, which is equivalent to $K \vee (n-(N-1)) \leq k \leq (n-K) \wedge (N-1)$ or $K \leq k \leq n-K$ given that $N \geq 2K$. Hence, the convolution of incumbents is given by $\sum_{k=K}^{n-K} \mu^B_t(n-k)\mu^B_t(k)$ and the resulting density in (5) follows.

From (5), the mass q^B_t of agents located in Network B satisfies

$$q^B_t = \sum_{n=K}^{N-1} \mu^B_t(n) = \eta \int_0^t e^{-\eta \int_u^t q^B_s \mathrm{d}s} \sum_{n=K}^{N-1} \begin{pmatrix} \mathbf{1}_{\{n \in [K, 2(K-1)]\}} \sum_{k=n-(K-1)}^{K-1} \mu^A_u(n-k)\mu^A_u(k) \\ +\mathbf{1}_{\{[n \in [2K, N-1]]\}} \sum_{k=K}^{n-K} \mu^B_u(n-k)\mu^B_u(k) \end{pmatrix} \mathrm{d}u.$$

Differentiating this expression, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} q^B_t = -\eta (q^B_t)^2 + \eta \sum_{n=K}^{N-1} \begin{pmatrix} \mathbf{1}_{\{n \in [K, 2(K-1)]\}} \sum_{m=n-(K-1)}^{K-1} \mu^A_t(n-m)\mu^A_t(m) \\ +\mathbf{1}_{\{n \in [2K, N-1]\}} \sum_{m=K}^{n-K} \mu^B_t(n-m)\mu^B_t(m) \end{pmatrix}, \quad q^B_0 = 0.$$

Finally, the mass of agents perfectly informed is determined by the sum of the agents holding $n \in C$ signals for the set of integers $C = \{N, ..., 2(N-1)\} \subset \mathbb{N}^*$. Denoting by $\mu_t^C(\mathbb{C}) = \iota(\{j : j_t \in \mathbb{C}\})$ the density for each classes of number $n \geq N$ of signals and where $\mathbb{C} \subseteq C$, I can write

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_t^C(n) = \eta \sum_{k=K\vee(n-(N-1))}^{(N-1)\wedge(n-K)} \mu_t^B(n-k)\mu_t^B(k), \quad \mu_0^C(n) = \delta_0.$$

Migrating agents pile up in each class which have become irrelevant given that Network C indifferently grants access to perfect information. Hence, the mass q_t^C of agents located in Network C satisfies

$$\begin{aligned} q_t^C &= \sum_{n=N}^{2(N-1)} \mu_t^C(n) = \eta \int_0^t \sum_{n=N}^{2(N-1)} \sum_{k=K\vee(n-(N-1))}^{(N-1)\wedge(n-K)} \mu_s^B(n-k)\mu_s^B(k)\mathrm{d}s \\ &= 1 - q_t^A - q_t^B. \end{aligned}$$

■

B Proof of Proposition 3

In this appendix, I derive the Bayesian updating procedure in the presence of social interactions. In a first step, I derive the filter pertaining to the information accrued from the tape and, then, derive the filter related to the information processed through private discussions. When filtering from the price, agents apply a Kalman filter to the vector of unobservable variables $X_t \equiv (\Pi, \Theta_t)^\top$ using the information Y_t^c generated by the tape. The vector X_t has dynamics

$$\mathrm{d}X_t = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix} X_t \mathrm{d}t + \begin{bmatrix} 0 \\ \sigma_\Theta \end{bmatrix} \mathrm{d}B_t^\Theta, \; X_0 = \begin{bmatrix} \Pi \\ \Theta_0 \end{bmatrix}.$$

From (6), the price may be written as

$$P_t = \xi_t + (1 - \lambda_{1,t})\widehat{\Pi}_t^c$$

where $\xi_t \equiv \lambda_{1,t}\Pi + \lambda_{2,t}\Theta_t$. Furthermore, notice that observing the price is informationally equivalent to observing ξ_t, i.e. $\sigma(P_s : 0 \leq s \leq t) \Leftrightarrow \sigma(\xi_s : 0 \leq s \leq t)$ and thus $Y_t^c = \xi_t$. The dynamics of Y_t^c are obtained by applying Ito's lemma

$$\mathrm{d}Y_t^c = \left(\lambda_{1,t}'\Pi + (\lambda_{2,t}' - a_\Theta\lambda_{2,t})\Theta_t\right)\mathrm{d}t + \lambda_{2,t}\sigma_\Theta \mathrm{d}B_t^\Theta.$$

The Kalman filter under $\mathcal{F}^j$ for $j = c, l$ is obtained through the theorem below.

Theorem 1. *Denote the unobservable vector by X_t and the observable vector by Y_t with dynamics*

$$\begin{aligned} dX_t &= (a_0 + a_1 X_t)dt + b\,dB_t \\ dY_t &= (A_0 + A_1 X_t)dt + B\,dB_t \end{aligned}$$

where $dB_t = dB_t^{\Theta}$. The conditional mean $\widehat{X}_t$ with respect to the information set $\mathcal{F}_t^Y = \sigma(Y_s : 0 \le s \le t)$ has dynamics

$$d\widehat{X}_t = (a_0 + a_1\widehat{X}_t)dt + (O_t A_1^\top + bB^\top)(BB^\top)^{-\frac{1}{2}}d\widehat{B}_t$$

where $O_t = E\left[(X - \widehat{X}_t)(X - \widehat{X}_t)^\top \middle| \mathcal{F}_t^Y\right]$ is the positive semi-definite conditional variance-covariance matrix of X_t given by the solution to the Ricatti equation

$$\dot{O}_t = a_1 O_t + O_t a_1^\top + bb^\top - (O_t A_1^\top + bB^\top)(BB^\top)^{-1}(A_1 O_t + B^\top b)$$

and where the filter innovation $\widehat{B}_t$ satisfying

$$d\widehat{B}_t = (BB^\top)^{-\frac{1}{2}}(dY_t - (A_0 + A_1\widehat{X}_t)dt)$$

is a Brownian motion with respect to the filtration $\mathcal{F}_t^Y$.

Proof. See Lipster and Shiryaev (2001), Theorem 12.7. Q.E.D.

To apply Theorem 1, I need an expression for the variance-covariance matrix O of the filter: notice that because $\xi_t \in \mathcal{F}_t^c \subseteq \mathcal{F}_t^l$, it follows that $\xi_t = \lambda_{1,t}\Pi + \lambda_{2,t}\Theta_t \equiv \lambda_{1,t}\widehat{\Pi}_t^c + \lambda_{2,t}\widehat{\Theta}_t^c \equiv \lambda_{1,t}\widehat{\Pi}_t^l + \lambda_{2,t}\widehat{\Theta}_t^l$. This means, in turn, that

$$E\left[(\Theta_t - \widehat{\Theta}_t^j)^2 \middle| \mathcal{F}_t^j\right] = E\left[\left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 (\Pi - \widehat{\Pi}_t^j)^2 \middle| \mathcal{F}_t^j\right] = \left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 o_t^j, \quad j = c, l$$

and

$$E\left[(\Theta_t - \widehat{\Theta}_t^j)(\Pi - \widehat{\Pi}_t^j) \middle| \mathcal{F}_t^j\right] = E\left[-\frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^j)^2 \middle| \mathcal{F}_t^j\right] = -\frac{\lambda_{1,t}}{\lambda_{2,t}} o_t^j, \quad j = c, l.$$

Accordingly, O^j for $j = c, l$ may be written as

$$O_t^j = o_t^j \begin{bmatrix} 1 & -\frac{\lambda_{1,t}}{\lambda_{2,t}} \\ -\frac{\lambda_{1,t}}{\lambda_{2,t}} & \left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 \end{bmatrix}.$$

Further observing that

$$a_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; a_1 = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ \sigma_\Theta \end{bmatrix}, \; A_0 = 0, \; A_1 = \begin{bmatrix} \lambda'_{1,t} \\ \lambda'_{2,t} - a_\Theta \lambda_{2,t} \end{bmatrix}^\top$$

and $B = \lambda_{2,t}\sigma_\Theta$ and working out the expression for the conditional mean $\widehat{X}_t$ in Theorem 1, I get the following dynamics under common information

$$\begin{aligned} \mathrm{d}\begin{bmatrix} \widehat{\Pi}^c_t \\ \widehat{\Theta}^c_t \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix} \begin{bmatrix} \widehat{\Pi}^c_t \\ \widehat{\Theta}^c_t \end{bmatrix} \mathrm{d}t \\ &\quad + \frac{1}{\lambda^2_{2,t}\sigma_\Theta} \begin{bmatrix} o^c_t(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t})) \\ \lambda^2_{2,t}\sigma^2_\Theta + o^c_t\left(\frac{\lambda^2_{1,t}}{\lambda_{2,t}}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}) - \lambda_{1,t}\lambda'_{1,t}\right) \end{bmatrix} \mathrm{d}\widehat{B}^c_t \end{aligned}$$

where $\widehat{B}^c_t$ is a one-dimensional Brownian motion with respect to $\mathcal{F}^c_t$ with

$$\mathrm{d}\widehat{B}^c_t = \frac{1}{\lambda_{2,t}\sigma_\Theta}\left(\mathrm{d}\xi_t - (\lambda'_{1,t}\widehat{\Pi}^c_t + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})\widehat{\Theta}^c_t)\mathrm{d}t\right). \tag{21}$$

Using the definition of k_t in (11), I get the equation (9) for the common filter. Furthermore, substituting the expression obtained above for O^c_t into the Ricatti equation of Theorem 1 and working out the equation, I obtain the following ordinary differential equation

$$\frac{\mathrm{d}o^c_t}{\mathrm{d}t} = -\frac{(o^c_t)^2}{\lambda^4_{2,t}\sigma^2_\Theta}(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}))^2 = -k^2_t(o^c_t)^2.$$

This yields the equation for the filtered variance under common information appearing in (12).

Similarly, when agents l do not meet anyone and, thus, collect information from only watching the tape, their forecasts evolve as

$$\mathrm{d}\begin{bmatrix} \widehat{\Pi}^l_t \\ \widehat{\Theta}^l_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix} \begin{bmatrix} \widehat{\Pi}^l_{t_-} \\ \widehat{\Theta}^l_{t_-} \end{bmatrix} \mathrm{d}t + \begin{bmatrix} o^l_t\left(n^l_{t_-}\right)k_t \\ \left(\sigma_\Theta - o^l_t\left(n^l_{t_-}\right)\frac{\lambda_{1,t}}{\lambda_{2,t}}k_t\right) \end{bmatrix} \mathrm{d}\widehat{B}^l_t \tag{22}$$

where

$$\mathrm{d}\widehat{B}^l_t = \frac{1}{\lambda_{2,t}\sigma_\Theta}(\mathrm{d}\xi_t - (\lambda'_{1,t}\widehat{\Pi}^l_{t_-} + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})\widehat{\Theta}^l_{t_-})\mathrm{d}t) \tag{23}$$

is a one-dimensional Brownian motion with respect to $\mathcal{F}^l_{t_-}$. The variance of their filter evolves as $\mathrm{d}o^l_t\left(n^l_t\right) = -k^2_t\left(o^l_t\left(n^l_{t_-}\right)\right)^2\mathrm{d}t$.

I now turn to show how both the beliefs about Π and $(\Theta_t)_{t\geq 0}$ are updated when an agent is met: whenever an agent l meets another agent holding m_t signals, she gets a sequence $\{S^l_k\}_{k=1}^{m_t}$ of incremental signals. Conveniently, by Gaussian theory, $\bar{S}^l_{m,t} \equiv \frac{1}{m_t}\sum_{k=1}^{m_t} S^l_k$ is a sufficient statistic for the latter. Notice that $\bar{S}^l_{m,t}$ is conditionally distributed as $\bar{S}^l_{m,t} \sim \mathcal{N}(\Pi, \frac{\sigma_S^2}{m_t})$. Then, by Bayes' rule, the conditional density $p^l_t(\Pi|\mathcal{F}^l_t)$ may be written in the following recursive way

$$p^l_t(\Pi|\mathcal{F}^l_t) = \frac{p^l_{t_-}(\Pi|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-\Pi)^2}{2\sigma_S^2/m_t}}}{\int_{\mathbb{R}} p^l_{t_-}(x|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-x)^2}{2\sigma_S^2/m_t}}\,\mathrm{d}x}.$$

Hence, at time 0, given the prior $\Pi \sim \mathcal{N}(0, \sigma_\Pi^2)$ and immediately after investor l receives her initial private signal (in which case, $n = 1$)

$$p^l_0(\Pi|\mathcal{F}^l_0) = \frac{e^{-\frac{1}{2}\left(\frac{\Pi}{\sigma_\Pi}\right)^2} e^{-\frac{1}{2}\left(\frac{S^l-\Pi}{\sigma_S}\right)^2}}{\int_{\mathbb{R}} e^{-\frac{1}{2}\left(\frac{x}{\sigma_\Pi}\right)^2} e^{-\frac{1}{2}\left(\frac{S^l-x}{\sigma_S}\right)^2}\,\mathrm{d}x} = \sqrt{\frac{\frac{1}{\sigma_S^2}+\frac{1}{\sigma_\Pi^2}}{2\pi}}\,e^{-\frac{((\Pi)\sigma_S^2+(\Pi-S^l)\sigma_\Pi^2)^2}{2\sigma_S^2\sigma_\Pi^2(\sigma_S^2+\sigma_\Pi^2)}}$$

which may be used as an initial condition for the above recursion. I can then compute the integral in the denominator of the expression above to obtain

$$\int_{\mathbb{R}} p^l_{t_-}(x|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-x)^2}{2\sigma_S^2/m_t}}\,\mathrm{d}x = \sqrt{\frac{\widehat{o}^l_t\left(n^l_t\right)}{o^l_t\left(n^l_{t_-}\right)}}\,e^{-\left(\frac{\left(\bar{S}^l_{m,t}\right)^2}{2\sigma_S^2/m_t}+\frac{\left(\widehat{\Pi}^l_{t_-}\right)^2}{2o^l_t\left(n^l_{t_-}\right)}\right)+\left(\frac{\widehat{\Pi}^l_{t_-}}{o^l_t\left(n^l_{t_-}\right)}+\frac{\bar{S}^l_{m,t}}{\sigma_S^2/m_t}\right)^2\frac{\widehat{o}^l_t\left(n^l_t\right)}{2}}$$

where $\widehat{o}^l_t\left(n^l_t\right) \equiv \left(\frac{1}{o^l_t\left(n^l_{t_-}\right)}+\frac{m_t}{\sigma_S^2}\right)^{-1}$. Substituting this expression back, I obtain

$$p^l_t(\Pi|\mathcal{F}^l_t) = \frac{1}{\sqrt{2\pi\widehat{o}^l_t\left(n^l_t\right)}}e^{-\frac{\left(\left(\frac{\widehat{\Pi}^l_{t_-}}{o^l_t\left(n^l_{t_-}\right)}+\frac{\bar{S}^l_{m,t}}{\sigma_S^2/m_t}\right)\widehat{o}^l_t\left(n^l_t\right)-\Pi\right)^2}{2\widehat{o}^l_t\left(n^l_t\right)}} = \frac{1}{\sqrt{2\pi o^l_t\left(n^l_t\right)}}e^{-\frac{1}{2}\frac{(\widehat{\Pi}^l_t-\Pi)^2}{o^l_t\left(n^l_t\right)}} \tag{24}$$

where the second equality follows from that the conditional distribution $p^l_t(\Pi|\mathcal{F}^l_t)$ is Gaussian for any t. Comparing the two expressions in (24) yields the updating rule for the variance

$$\frac{1}{o^l_t\left(n^l_t\right)} = \frac{1}{o^l_t\left(n^l_{t_-}\right)}+\frac{m_t}{\sigma_S^2} \tag{25}$$

and the mean

$$\frac{\widehat{\Pi}_t^l}{o_t^l\left(n_t^l\right)} = \frac{\widehat{\Pi}_{t_-}^l}{o_t^l\left(n_{t_-}^l\right)} + \frac{\bar{S}_{m,t}^l}{\sigma_S^2/m_t} = \frac{\widehat{\Pi}_{t_-}^l}{o_t^l\left(n_t^l\right)} + \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}. \tag{26}$$

I still need to take care of the updating rule for the noisy supply $(\Theta_t)_{t\geq 0}$: observing that information ξ_t accruing from the price is continuous, I can write $\widehat{\Theta}_t^l - \widehat{\Theta}_{t_-}^l = -\frac{\lambda_{1,t}}{\lambda_{2,t}}(\widehat{\Pi}_t^l - \widehat{\Pi}_{t_-}^l)$ and, in consideration of (26), the updating rule immediately follows

$$\frac{\widehat{\Theta}_t^l}{o_t^l\left(n_t^l\right)} = \frac{\widehat{\Theta}_{t_-}^l}{o_t^l\left(n_t^l\right)} - \frac{\lambda_{1,t}}{\lambda_{2,t}} \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}. \tag{27}$$

Alternatively, the conditional p.d.f. $p_t^l(\Theta_t|\mathcal{F}_t^l)$ for the noisy supply satisfies the recursion

$$p_t^l(\Theta_t|\mathcal{F}_t^l) = \frac{p_{t_-}^l(\Theta_t|\mathcal{F}_{t_-}^l) e^{-\frac{(\bar{S}_{m,t}^l - (\widehat{\Pi}_t^l + \frac{\lambda_{2,t}}{\lambda_{1,t}}(\widehat{\Theta}_t^j - \Theta_t)))^2}{2\sigma_S^2/m_t}}}{\int_{\mathbb{R}} p_{t_-}^l(x|\mathcal{F}_{t_-}^l) e^{-\frac{(\bar{S}_{m,t}^l - (\widehat{\Pi}_t^l + \frac{\lambda_{2,t}}{\lambda_{1,t}}(\widehat{\Theta}_t^j - x)))^2}{2\sigma_S^2/m_t}} \mathrm{d}x}$$

with initial condition

$$p_0^l(\Theta_0|\mathcal{F}_0^l) = \sqrt{\frac{\frac{\lambda_{2,0}^2}{\lambda_{1,0}^2\sigma_S^2} + \frac{2a_\Theta}{\sigma_\Theta^2}}{2\pi}} e^{-\frac{1}{2}\frac{(2a_\Theta\Theta_0\lambda_{1,0}^2\sigma_S^2 + \lambda_{2,0}(S^l\lambda_{1,0} + \Theta_0\lambda_{2,0})\sigma_\Theta^2)^2}{2a_\Theta\lambda_{1,0}^4\sigma_S^4\sigma_\Theta^2 + \lambda_{1,0}^2\lambda_{2,0}^2\sigma_S^2\sigma_\Theta^4}}.$$

In consideration of the updating rules in (25), (26) and (27), when an agent is met at time t, the filtered fundamental and the filtered noisy supply experience a jump whose size is respectively given by

$$\Delta\widehat{\Pi}_t^l \equiv \widehat{\Pi}_t^l - \widehat{\Pi}_{t_-}^l = \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t} o_t^l\left(n_t^l\right) \text{ and } \Delta\widehat{\Theta}_t^l \equiv \widehat{\Theta}_t^l - \widehat{\Theta}_{t_-}^l = -\frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t} o_t^l\left(n_t^l\right). \tag{28}$$

Likewise, the variance of posteriors experiences a jump of size

$$\Delta o_t^l \equiv o_t^l\left(n_t^l\right) - o_t^l\left(n_{t_-}^l\right) = -\frac{1}{\sigma_S^2} o_t^l\left(n_t^l\right) o_t^l\left(n_{t_-}^l\right) m_t.$$

Moreover, the respective filters of Π, a constant, and $(e^{a_\Theta t}\Theta_t)_{t\geq 0}$, a martingale, need to be martingales. Hence, I need to compensate their jump size adequately so as to enforce their martingality. To do so, I have to pin down the distribution of the jump size in posteriors or, equivalently, the distribution of $Z_{m,t}^l \equiv \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t} o_t^l\left(n_t^l\right)$. Let $\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)$ denote the required density. Moreover, notice that the probability of meeting someone in the time interval $[t, t+\mathrm{d}t)$ is $\mathbb{P}[N^l_{t+\mathrm{d}t} - N^l_t = 1] = \eta_t \mathrm{d}t$ and the probability of getting m incremental signals given $n^l_{t\_}$ currently held and conditional on meeting someone in $[t, t+\mathrm{d}t)$ is

$$\mathbb{P}[m_t = m | N^l_{t+\mathrm{d}t} - N^l_t = 1, n^l_{t\_}] = \frac{\mu^A_t(m)}{q^A_t}\mathbf{1}_{\{n^l_{t\_} \in A; m \in A\}} + \frac{\mu^B_t(m)}{q^B_t}\mathbf{1}_{\{n^l_{t\_} \in B; m \in B\}}.$$

Hence, the probability of getting m signals in $[t, t+\mathrm{d}t)$ given $n^l_{t\_}$ is

$$\mathbb{P}[\{m_t = m\} \cap \{N^l_{t+\mathrm{d}t} - N^l_t = 1\} | n^l_{t\_}] = \eta \left(\mu^A_t(m)\mathbf{1}_{\{n^l_{t\_} \in A; m \in A\}} + \mu^B_t(m)\mathbf{1}_{\{n^l_{t\_} \in B; m \in B\}}\right) \mathrm{d}t$$

and the distribution $\nu^l_t(\cdot)$ satisfies

$$\begin{aligned} \nu^l_t(m; \mathrm{d}t; \mathrm{d}Z) &= \mathbb{P}[\{m_t = m\} \cap \{N^l_{t+\mathrm{d}t} - N^l_t = 1\} | n^l_{t\_}] \times \mathbb{P}[Z^l_{m,t} \in \mathrm{d}Z | \mathcal{F}^l_{t\_}, m_t] \quad (29) \\ &= \eta \left(\mu^A_t(m)\mathbf{1}_{\{n^l_{t\_} \in A; m \in A\}} + \mu^B_t(m)\mathbf{1}_{\{n^l_{t\_} \in B; m \in B\}}\right) \mathrm{d}t \\ &\times \mathcal{N}\left(0, \frac{m_t \sigma^2_S (o^c_t)^2}{(\sigma^2_S + n^l_{t\_} o^c_t)(\sigma^2_S + n^l_t o^c_t)}\right) \mathrm{d}Z \end{aligned}$$

where the last expression follows from that

$$E\left[Z^l_{m,t} | \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t o^l_t(n^l_t)}{\sigma^2_S} E\left[\Pi + \frac{1}{m_t}\sum_{j=1}^{m_t} \epsilon^j - \widehat{\Pi}^l_{t\_} \middle| \mathcal{F}^l_{t\_}, m_t\right] = 0$$

and

$$V\left[Z^l_{m,t} | \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t^2 (o^l_t(n^l_t))^2}{\sigma^4_S} V\left[\Pi + \frac{1}{m_t}\sum_{j=1}^{m_t} \epsilon^j \middle| \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t^2 (o^l_t(n^l_t))^2}{\sigma^4_S}(o^l_t(n^l_{t\_}) + \frac{\sigma^2_S}{m_t}).$$

Since the expected jump size is null, the compensation is null and the filtered dynamics in (10) and the variance dynamics

$$\mathrm{d}o^l_t(n^l_t) = -k^2_t (o^l_t(n^l_{t\_}))^2 \mathrm{d}t - \frac{1}{\sigma^2_S} o^l_t(n^l_t) o^l_t(n^l_{t\_}) m_t \mathrm{d}N^l_t$$

immediately follow by putting together (22) and the jump sizes in (28).

Finally, letting $K^c_t \equiv \frac{1}{o^c_t}$, I can write $\mathrm{d}K^l_t\left(n^l_t\right) = \mathrm{d}K^c_t + \frac{m_t}{\sigma^2_S}\mathrm{d}N^l_t$ and obtain the explicit relation

$$K^l_t\left(n^l_t\right) = K^l_0 + K^c_t + \int_0^t \frac{m_s}{\sigma^2_S}\mathrm{d}N^l_s = K^c_t + \frac{n^l_t}{\sigma^2_S}$$

where K^c_t satisfies $\mathrm{d}K^c_t = k^2_t \mathrm{d}t$. This yields the expression for $o^l_t\left(n^l_t\right)$ in (12). ■

C Proof of Proposition 4

In this appendix, I turn to agents i and l's optimization problem. Following the notations of He and Wang (1995), I denote by Q the excess return on the price. The latter satisfies

$$\mathrm{d}Q_t = \mathrm{d}P_t - rP_t\mathrm{d}t + \mathbf{1}_{\{t=T\}}\Delta P_T. \tag{30}$$

Accordingly, agent l who currently holds $n^l_{t\_}$ signals chooses a portfolio strategy $\theta^l_{t\_} \equiv \theta^l(\Psi^l_{t\_}, n^l_{t\_}, t)$ with Ψ^l to be shortly described in order to maximize

$$\sup_{\theta^l} E\left[-e^{-\gamma W^l_T}\middle|\, \mathcal{F}^l_{t\_}\right] \text{ s.t. } \quad \mathrm{d}W^l_t = rW^l_t\mathrm{d}t + \theta^l_{t\_}\mathrm{d}Q_t \tag{31}$$

while agent i chooses a portfolio strategy $\theta^i_t \equiv \theta^i(\Psi_t, t)$ in order to maximize

$$\sup_{\theta^i} E\left[-e^{-\gamma W^i_T}\middle|\, \mathcal{F}^i_t\right] \text{ s.t. } \quad \mathrm{d}W^i_t = rW^i_t\mathrm{d}t + \theta^i_t\mathrm{d}Q_t. \tag{32}$$

I first solve the problems in (31) and (32) over $[0, T)$ and then solve the ones prevailing at the horizon date which, in turn, will provide boundary conditions. In doing so, I need to determine the state variables relevant to (31) and (32): due to the CARA form of utility, W will act as a trivial state variable. Besides t whose dependence is triggered by $T < \infty$, the other relevant state variables, Ψ, appear to be the ones driving Q.

I proceed first with (31) and, then, obtain the solution to (32) as the special case when $n^l \to \infty$: notice that, because $r = 0$, $Q_t = E[P_t|\, \mathcal{F}^l_t] = \lambda_{1,t}\widehat{\Pi}^l_t + \lambda_{2,t}\widehat{\Theta}^l_t + (1-\lambda_{1,t})\widehat{\Pi}^c_t$. The dynamics of $\widehat{\Pi}^l_t$ and $\widehat{\Theta}^l_t$ being obtained from (10), I just need to derive the dynamics of $\widehat{\Pi}^c_t$ with respect to $\mathcal{F}^l_{t\_}$: substitutions between the two Brownians in (21) and (23) lead to

$$\mathrm{d}\widehat{B}^c_t = \mathrm{d}\widehat{B}^l_t + \frac{1}{\lambda_{2,t}\sigma_\Theta}(\lambda'_{1,t}(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t) + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})(\widehat{\Theta}^l_{t\_} - \widehat{\Theta}^c_t))\mathrm{d}t.$$

Using the equivalence relations between $\widehat{\Theta}^c_t$ and $\widehat{\Theta}^l_t$, this change of measure is written as

$$\begin{aligned}\mathrm{d}\widehat{B}^c_t &= \mathrm{d}\widehat{B}^l_t + \frac{1}{\lambda^2_{2,t}\sigma_\Theta}(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}))(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t)\mathrm{d}t \\ &= \mathrm{d}\widehat{B}^l_t + k_t(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t)\mathrm{d}t\end{aligned} \tag{33}$$

with the associated Radon-Nikodym derivative

$$\left.\frac{\mathrm{d}\widehat{\mathbb{P}}^c}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}^l_t} = e^{-\frac{1}{2}\int_0^t (k_s(\widehat{\Pi}^l_{s\_} - \widehat{\Pi}^c_s))^2\mathrm{d}s + \int_0^t k_s(\widehat{\Pi}^l_{s\_} - \widehat{\Pi}^c_s)\mathrm{d}\widehat{B}^c_s}. \tag{34}$$

I assume that (34) is a martingale (and not only a local martingale) and Girsanov's theorem applies. Then $\widehat{\Pi}_t^c$ satisfies the following dynamics

$$\begin{aligned}
\mathrm{d}\widehat{\Pi}_t^c &= \frac{o_t^c}{\lambda_{2,t}^4\sigma_\Theta^2}(\lambda_{1,t}'\lambda_{2,t} - \lambda_{1,t}(\lambda_{2,t}' - a_\Theta\lambda_{2,t}))^2(\widehat{\Pi}_{t_}^l - \widehat{\Pi}_t^c)\mathrm{d}t \\
&\quad + \frac{o_t^c}{\lambda_{2,t}^2\sigma_\Theta}(\lambda_{1,t}'\lambda_{2,t} - \lambda_{1,t}(\lambda_{2,t}' - a_\Theta\lambda_{2,t}))\mathrm{d}\widehat{B}_t^l = o_t^c k_t^2(\widehat{\Pi}_{t_}^l - \widehat{\Pi}_t^c)\mathrm{d}t + o_t^c k_t \mathrm{d}\widehat{B}_t^l
\end{aligned}$$

with respect to $\mathcal{F}_{t\_}^l$. An application of Ito's lemma then shows

$$\begin{aligned}
\mathrm{d}Q_t &= ((\lambda_{1,t}' + (1-\lambda_{1,t})o_t^c k_t^2)(\widehat{\Pi}_{t_}^l - \widehat{\Pi}_t^c) + (\lambda_{2,t}' - a_\Theta\lambda_{2,t})\widehat{\Theta}_{t_}^l)\mathrm{d}t \\
&\quad + (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t})o_t^c k_t)\mathrm{d}\widehat{B}_t^l.
\end{aligned} \tag{35}$$

Inspection of (35) reveals that $\Psi_t^l := (1, \widehat{\Theta}_t^l, \Delta_t^l)^\top$ where the first element is introduced to capture linear dependencies along with some constant and where $\Delta_t^l \equiv \widehat{\Pi}_t^l - \widehat{\Pi}_t^c$ denotes the difference between investors l's estimate of the stock value and the estimate solely based on market information. By Ito's lemma, the latter satisfies

$$\mathrm{d}\Delta_t^l = -o_t^c k_t^2 \Delta_{t\_}^l \mathrm{d}t + \left(o_t^l\left(n_{t\_}^l\right) - o_t^c\right)k_t \mathrm{d}\widehat{B}_t^l + \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t\_}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t \mathrm{d}N_t^l.$$

Moreover, using the expression for $\widehat{\Theta}_t^l$ in (10) in Proposition 3, I get

$$\mathrm{d}\widehat{\Theta}_t^l = -a_\Theta\widehat{\Theta}_{t\_}^l\mathrm{d}t + \left(\sigma_\Theta - o_t^l\left(n_{t\_}^l\right)\frac{\lambda_{1,t}}{\lambda_{2,t}}k_t\right)\mathrm{d}\widehat{B}_t^l - \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t\_}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t \mathrm{d}N_t^l.$$

As a result, the excess return Q and the filtered state variables $\Psi_t^l \equiv E[\Psi_t|\mathcal{F}_t^l]$ follow the coupled process

$$\begin{aligned}
\mathrm{d}Q_t &= A_{Q,t}\Psi_{t_}^l\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^l + C_{Q,t}\left(\Psi_{t_}^l, n_t^l\right)\mathrm{d}N_t^l \\
\mathrm{d}\Psi_t^l &= A_{\Psi,t}\Psi_{t_}^l\mathrm{d}t + B_{\Psi,t}^l\left(n_{t_}^l\right)\mathrm{d}\widehat{B}_t^l + C_{\Psi,t}^l\left(\Psi_{t_}^l, n_t^l\right)\mathrm{d}N_t^l
\end{aligned}$$

over $[0, T)$ where

$$A_{\Psi,t} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a_\Theta & 0 \\ 0 & 0 & -k_t^2 o_t^c \end{pmatrix}, \quad B_{\Psi,t}^l\left(n_t^l\right) = \begin{pmatrix} 0 \\ \sigma_\Theta - \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{k_t\sigma_S^2}{K_t^c\sigma_S^2 + n_t^l} \\ \left(\frac{\sigma_S^2}{\sigma_S^2 K_t^c + n_t^l} - \frac{1}{K_t^c}\right)k_t \end{pmatrix},$$

and

$$C_{\Psi,t}^l\left(\Psi_t^l, n_t^l\right) = \begin{pmatrix} 0 & -\frac{\lambda_{1,t}}{\lambda_{2,t}} & 1 \end{pmatrix}^\top \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t\_}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t$$

along with

$$A_{Q,t} = [\ 0 \quad \lambda_{2,t}' - a_\Theta \lambda_{2,t} \quad \lambda_{1,t}' + (1-\lambda_{1,t}) o_t^c k_t^2\ ], \quad B_{Q,t} = \lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t}) o_t^c k_t \tag{36}$$

and $C_{Q,t} = 0$. As expected, the excess return does not jump, expect, perhaps, at the horizon date T when the stock pays out.

Finally, notice that Ψ_t^l does not constitute a sufficient statistic for (31) because, unlike He and Wang (1995), the variance o^l is not a sole function of time. Instead, o^l jumps at random times and one needs to include it as an additional state variable. In that respect, the relation between o_t^c and $o_t^l\left(n_t^l\right)$ described in Proposition 3 implies that one may choose to keep track either of n_t^l or o_t^l. Accordingly, I let the state variables for (31) be (W^l, Ψ^l, n^l, t) and let the associated value function J^l be of the form $J^l(W^l, \Psi^l, n^l, t)$. Using these, I can write the Hamilton-Jacobi-Bellman (HJB) equation associated with (31). By the standard martingale argument, it satisfies

$$\begin{aligned}
0 = \sup_{\theta_{t-}^l} & \left\{ J_W^l A_{Q,t} \Psi_{t-}^l \theta_{t-}^l + \frac{1}{2} J_{WW}^l B_{Q,t}^2 (\theta_{t-}^l)^2 + B_{Q,t} (B_{\Psi,t}^l(n_{t-}^l))^\top J_{W\Psi}^l \theta_{t-}^l \right\} \\
& + J_t^l + (J_\Psi^l)^\top A_{\Psi,t} \Psi_{t-}^l + \frac{1}{2} \mathrm{tr}(J_{\Psi\Psi}^l B_{\Psi,t}^l(n_{t-}^l)(B_{\Psi,t}^l(n_{t-}^l))^\top) \\
& + E^{\nu_t^l}[J^l(W_t^l, \Psi_{t-}^l + C_{\Psi,t}^l\left(\Psi_{t-}^l, n_t^l\right), n_{t-}^l + m_t, t) - J^l(W_t^l, \Psi_{t-}^l, n_{t-}^l, t)]
\end{aligned}$$

with terminal boundary condition $J^l(W^l, \Psi^l, n^l, T) = -e^{-\gamma W_T^l}$ and where $\mathrm{tr}(\cdot)$ denotes the trace operator. The first-order condition reads

$$J_W^l A_{Q,t} \Psi_{t-}^l + J_{WW}^l B_{Q,t}^2 \theta_{t-}^l + B_{Q,t} (B_{\Psi,t}^l(n_{t-}^l))^\top J_{W\Psi}^l = 0 \tag{37}$$

and the second-order condition for optimality is $J_{WW}^l < 0$. Substituting the first-order condition into the HJB equation, I obtain the following PDE

$$\begin{aligned}
& J_t^l + (J_\Psi^l)^\top A_{\Psi,t} \Psi_{t-}^l + \frac{1}{2} \mathrm{tr}(J_{\Psi\Psi}^l B_{\Psi,t}^l(n_{t-}^l)(B_{\Psi,t}^l(n_{t-}^l))^\top) \\
& + E^{\nu_t^l}[J^l(W_t^l, \Psi_{t-}^l + C_{\Psi,t}^l\left(\Psi_{t-}^l, n_t^l\right), n_{t-}^l + m_t, t) - J^l(W_t^l, \Psi_{t-}^l, n_{t-}^l, t)] \\
& - \frac{1}{2} \frac{(J_W^l A_{Q,t} \Psi_{t-}^l + B_{Q,t} (B_{\Psi,t}^l(n_{t-}^l))^\top J_{W\Psi}^l)^2}{J_{WW}^l B_{Q,t}^2} = 0.
\end{aligned} \tag{38}$$

To solve (38), I make the following observation: if my setting were absent of social relations, Ψ^l would follow a bi-dimensional OU process and the setting would be affine quadratic. As shown in Cheng and Scaillet (2007), I could then conjecture that

$$J^l(W^l, \Psi^l, n^l, t) = -\exp\left(-\gamma W^l - \frac{1}{2} (\Psi^l)^\top M_t^l(n^l) \Psi^l\right) \tag{39}$$

where $M_t^l(n^l)$ is a $(3\times 3)-$symmetric matrix of coefficients to be determined and that satisfies boundary conditions pinned down by the problem to be solved at the horizon date. Yet, due to social interactions, learning is not purely Brownian and the combination of the jump in posteriors along with the quadratic form of (39) make the setup not LQJD (Linear-Quadratic-Jump-Diffusion) but QJD. Hence, unlike the setting of Cheng and Scaillet (2007) or Piazzesi (2005), for instance, the quadratic state variables jump along with the affine ones and the jump size in (38) is of the form

$$E^{\nu_t^l}[J^l(W^l,\Psi_{t-}^l + C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right), n_{t-}^l + m_t, t) - J^l(W_t^l,\Psi_{t-}^l,n_{t-}^l,t)] = \tag{40}$$
$$J^l(W^l,\Psi_{t-}^l,n_{t-}^l,t)\times$$
$$E^{\nu_t^l}\left[\exp\left(-\frac{1}{2}\begin{pmatrix}\left(C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right)\right)^\top M_t^l(n_t^l)C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right)\\ +(\Psi_{t-}^l)^\top(M_t^l(n_t^l)-M_t^l(n_{t-}^l))\Psi_{t-}^l\\ +(\Psi_{t-}^l)^\top M_t^l(n_t^l)C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right)\\ +\left(C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right)\right)^\top M_t^l(n_t^l)\Psi_{t-}^l\end{pmatrix}\right)-1\right].$$

In this context, Chen, Filipovic, and Poor (2004) have shown that the exponential quadratic ansatz in (39) for the value function fails to hold. Instead, for (39) to hold and in the spirit of the approximation considered in Duffie, Gârleanu, and Pedersen (2007) and Vayanos and Weill (2008), I linearize the jump size in (40) by means of a first-order Taylor approximation of the latter around zero:

$$\begin{aligned}
&E^{\nu_t^l}[J^l(W^l,\Psi_{t-}^l + C_{\Psi,t}^l\left(\Psi_{t-}^l,n_t^l\right), n_{t-}^l + m_t, t) - J^l(W_t^l,\Psi_{t-}^l,n_{t-}^l,t)]\\
\approx& -\frac{1}{2}\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(C_{\Psi,t}^l\left(\Psi_{t-}^l,n_{t-}^l+m\right)\right)^\top M_t^l(n_{t-}^l+m)C_{\Psi,t}^l\left(\Psi_{t-}^l,n_{t-}^l+m\right)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\\
&-\frac{1}{2}\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(C_{\Psi,t}^l(\Psi_{t-}^l,n_{t-}^l+m)\right)^\top M_t^l(n_{t-}^l+m)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\Psi_{t-}^l\\
&-\frac{1}{2}(\Psi_{t-}^l)^\top\left(\int_{\mathbb{N}^*\times\mathbb{R}}M_t^l(n_{t-}^l+m)C_{\Psi,t}^l(\Psi_{t-}^l,n_{t-}^l+m)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\\
&-\frac{1}{2}(\Psi_{t-}^l)^\top\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(M_t^l(n_{t-}^l+m)-M_t^l(n_{t-}^l)\right)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\Psi_{t-}^l\\
\equiv& -\frac{1}{2}\left(\phi_{0,t-}^l(M^l)+\left(\phi_{1,t-}^l(M^l)\right)^\top\Psi_{t-}^l+(\Psi_{t-}^l)^\top\phi_{1,t-}^l(M^l)+(\Psi_{t-}^l)^\top\phi_{2,t-}^l(M^l)\Psi_{t-}^l\right).
\end{aligned}$$

This allows to make the setting LQJD and to preserve its tractablity. An upper bound on the error induced by the linearization is provided in Appendix D. Further substituting the ansatz in (38), I obtain the following matrix differential equation

$$\begin{aligned}\dot{M}_t^l(n_{t\_}^l) &= -\eta\phi_{2,t\_}^l(M^l) - \mathrm{diag}(\phi_{1,t\_}^l(M^l))I_{(1:3,1)}^{(3)} - I_{(1,1:3)}^{(3)}\mathrm{diag}(\phi_{1,t\_}^l(M^l)) \\ &M_t^l(n_{t\_}^l)\left(\frac{B_{\Psi,t}^l(n_{t\_}^l)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right) + \left(\frac{B_{\Psi,t}^l(n_{t\_}^l)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)^\top M_t^l(n_{t\_}^l) \\ &- \frac{A_{Q,t}^\top A_{Q,t}}{(B_{Q,t})^2} - \left(\mathrm{tr}\left(M_t^l(n_{t\_}^l)B_{\Psi,t}^l(n_{t\_}^l)(B_{\Psi,t}^l(n_{t\_}^l))^\top\right) + \eta\phi_{0,t\_}^l(M^l)\right)I_{11}^{(3)}\end{aligned} \tag{41}$$

where $I_{i,j}^{(N)}$ is a $N \times N-$index matrix with its elements being zero except elements (i,j) being 1.

Since, as per (29), the expected jump size is null, it is immediate that

$$\phi_{1,t}^l(M^l) = [\ 0 \quad 0 \quad 0\ ]^\top \quad \forall l \in \mathcal{I}_2. \tag{42}$$

To determine the coefficients $\phi_{0,t}^l$ and $\phi_{2,t}^l$, I denote by $c_{\Psi,t} \equiv \left[\ 0 \quad -\frac{\lambda_{1,t}}{\lambda_{2,t}} \quad 1\ \right]^\top$ the jump scale in such a way that $C_\Psi^l = c_\Psi Z^l$. I then integrate over the different realizations of the incremental number m_t of signals: clearly, since agents only meet others within their own network, ϕ's need to be network-specific. This yields

$$\phi_{0,t\_}^l(M^l) = \begin{cases} \sum_{m\in A}\mu_t^A(m)c_{\Psi,t}^\top M_t^l(n_{t\_}^l+m)c_{\Psi,t}\frac{m\sigma_S^2}{(\sigma_S^2K_t^c+n_{t\_}^l)(\sigma_S^2K_t^c+n_{t\_}^l+m)} & \text{if } n_{t\_}^l \in A \\ \sum_{m\in B}\mu_t^B(m)c_{\Psi,t}^\top \\ \times\left(\begin{array}{c}\mathbf{1}_{\{n_{t\_}^l+m\in A\cup B\}}\left(M_t^l(n_{t\_}^l+m)c_{\Psi,t}\frac{m\sigma_S^2}{(\sigma_S^2K_t^c+n_{t\_}^l)(\sigma_S^2K_t^c+n_{t\_}^l+m)}\right) \\ +\mathbf{1}_{\{n_{t\_}^l+m\notin A\cup B\}}\left(M_t^i c_{\Psi,t}\frac{\sigma_S^2}{\sigma_S^2K_t^c+n_{t\_}^l}\right)\end{array}\right) & \text{if } n_{t\_}^l \in B\end{cases}$$

and

$$\phi_{2,t\_}^l(M^l) = \begin{cases} \sum_{m\in A}\mu_t^A(m)\left(M_t^l(n_{t\_}^l+m) - M_t^l(n_{t\_}^l)\right) & \text{if } n_{t\_}^l \in A \\ \sum_{m\in B}\mu_t^B(m) \\ \left(\begin{array}{c}\mathbf{1}_{\{n_{t\_}^l+m\in A\cup B\}}\left(M_t^l(n_{t\_}^l+m) - M_t^l(n_{t\_}^l)\right) \\ +\mathbf{1}_{\{n_{t\_}^l+m\notin A\cup B\}}\left(M_t^i - M_t^l(n_{t\_}^l)\right)\end{array}\right) & \text{if } n_{t\_}^l \in B\end{cases}$$

The solution to agents i's problem is directly obtained as a particular case when $n^l \to \infty$: the HJB equation for (32) is

$$\begin{aligned}0 = \sup_{\theta_t^i}&\left\{J_W^i A_{Q,t}\Psi_t\theta_t^i + \frac{1}{2}J_{WW}^i B_{Q,t}^2(\theta_t^i)^2 + B_{Q,t}(B_{\Psi,t}^i)^\top J_{W\Psi}^i\theta_t^i\right\} \\ &+ J_t^i + (J_\Psi^i)^\top A_{\Psi,t}\Psi_t + \frac{1}{2}\mathrm{tr}(J_{\Psi\Psi}^i B_{\Psi,t}^i(B_{\Psi,t}^i)^\top)\end{aligned} \tag{43}$$

with $J^i(W^i,\Psi,T) = -e^{-\gamma W_T^i}$ and $\Psi_t = (1, \Pi - \widehat{\Pi}_t^c, \Theta_t)^\top$ and $B_{\Psi,t}^i = \lim_{n^l\to\infty} B_{\Psi,t}^l(n^l)$. After substitution of the first-order condition and using the conjecture $J^i(W^i,\Psi,t) = -e^{-\gamma W^i - \frac{1}{2}(\Psi)^\top M_t^i \Psi}$, which, in this case, is exact, I obtain

$$\begin{aligned}\dot{M}_t^i = M_t^i \left(\frac{B_{\Psi,t}^i A_{Q,t}}{B_{Q,t}} - A_{\Psi,t} \right) + \left(\frac{B_{\Psi,t}^i A_{Q,t}}{B_{Q,t}} - A_{\Psi,t} \right)^\top M_t^i \\ - \frac{A_{Q,t}^\top A_{Q,t}}{(B_{Q,t})^2} - \operatorname{tr}\left(M_t^i B_{\Psi,t}^i (B_{\Psi,t}^i)^\top \right) I_{11}^{(3)}.\end{aligned} \tag{44}$$

I can go one step further and simplify the matrix differential equations above: first, considering (41) and (42) and (44) along with the terminal condition $M^{j,(1,2)}(T) = M^{j,(1,3)}(T) = 0$ for $j = l, i$ which follows from (52) below and since the differential equations for $M^{j,(1,2)}$ and $M^{j,(1,3)}$ have no generator term, $M_t^l(n_t^l)$ is of the form

$$M_t^l(n_t^l) = \begin{bmatrix} M_t^{l,(1,1)}(n_t^l) & 0 & 0 \\ 0 & M_t^{l,(2,2)}(n_t^l) & M_t^{l,(2,3)}(n_t^l) \\ 0 & M_t^{l,(2,3)}(n_t^l) & M_t^{l,(3,3)}(n_t^l) \end{bmatrix} \tag{45}$$

where M_t^i follows from $n^l \to \infty$. (45) implies that (41) and (44) may be decoupled from $M_t^{l,(1,1)}(n_t^l)$ and $M_t^{i,(1,1)}$: I shall redefine $c_{\Psi,t}^\star \equiv \begin{bmatrix} -\frac{\lambda_{1,t}}{\lambda_{2,t}} & 1 \end{bmatrix}^\top$ and

$$\begin{aligned} A_{\Psi,t}^\star &= \begin{bmatrix} -a_\Theta & 0 \\ 0 & -o_t^c k_t^2 \end{bmatrix}, \quad B_{\Psi,t}^{\star,l}(n_t^l) = \begin{bmatrix} \sigma_\Theta - \frac{\lambda_{1,t}}{\lambda_{2,t}} \frac{k_t \sigma_S^2}{\sigma_S^2 K_t^c + n_t^l} \\ (\frac{\sigma_S^2}{\sigma_S^2 K_t^c + n_t^l} - \frac{1}{K_t^c}) k_t \end{bmatrix}, \\ A_{Q,t}^\star &= \begin{bmatrix} \lambda_{2,t}' - a_\Theta \lambda_{2,t} \\ \lambda_{1,t}' + (1-\lambda_{1,t}) \frac{k_t^2}{K_t^c} \end{bmatrix}^\top, \quad B_{\Psi,t}^{\star,i} = \begin{bmatrix} \sigma_\Theta \\ -o_t^c k_t \end{bmatrix} \end{aligned} \tag{46}$$

along with matrices $M^{\star,l}$ and $M^{\star,i}$ whose dependence on $\phi_2^{\star,l} \equiv \phi_2^l(M^{\star,l})$ is emphasized

$$M_t^{\star,l}(n_t^l, \phi_{2,t}^{\star,l}) = \begin{bmatrix} M_t^{l,(2,2)}(n_t^l, \phi_{2,t}^{\star,l}) & M_t^{l,(2,3)}(n_t^l, \phi_{2,t}^{\star,l}) \\ M_t^{l,(2,3)}(n_t^l, \phi_{2,t}^{\star,l}) & M_t^{l,(3,3)}(n_t^l, \phi_{2,t}^{\star,l}) \end{bmatrix} \text{ and } M_t^{\star,i} = \begin{bmatrix} M_t^{i,(2,2)} & M_t^{i,(2,3)} \\ M_t^{i,(2,3)} & M_t^{i,(3,3)} \end{bmatrix}.$$

Then, reorganizing (41) and (44) respectively yields

$$\begin{aligned}\dot{M}_t^{\star,l}(n_{t-}^l, \phi_{2,t-}^{\star,l}) = -\eta \phi_{2,t-}^{\star,l} + M_t^{\star,l}(n_{t-}^l, \phi_{2,t-}^{\star,l}) \left(\frac{B_{\Psi,t}^{\star,l}(n_{t-}^l) A_{Q,t}^\star}{B_{Q,t}} - A_{\Psi,t}^\star \right) \\ + \left(\frac{B_{\Psi,t}^{\star,l}(n_{t-}^l) A_{Q,t}^\star}{B_{Q,t}} - A_{\Psi,t}^\star \right)^\top M_t^{\star,l}(n_{t-}^l, \phi_{2,t-}^{\star,l}) - \frac{(A_{Q,t}^\star)^\top A_{Q,t}^\star}{(B_{Q,t})^2},\end{aligned} \tag{47}$$

$$\dot{M}_t^{\star,i} = M_t^{\star,i}\left(\frac{B_{\Psi,t}^{\star,i}A_{Q,t}^{\star}}{B_{Q,t}} - A_{\Psi,t}^{\star}\right) + \left(\frac{B_{\Psi,t}^{\star,i}A_{Q,t}^{\star}}{B_{Q,t}} - A_{\Psi,t}^{\star}\right)^{\top} M_t^{\star,i} - \frac{(A_{Q,t}^{\star})^{\top}A_{Q,t}^{\star}}{(B_{Q,t})^2}. \quad (48)$$

Substituting (39) into (37) and the ansatz for agents i into the first-order condition in (43) yields

$$\theta_t^l \equiv \theta^l(\widehat{\Theta}_{t_-}^l, \widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c, n_{t_-}^l, t) = \frac{A_{Q,t}^{\star} - B_{Q,t}(B_{\Psi,t}^{\star,l}(n_{t_-}^l))^{\top} M_t^{\star,l}(n_{t_-}^l, \phi_{2,t_-}^{\star,l})}{\gamma B_{Q,t}^2}\begin{bmatrix}\widehat{\Theta}_{t_-}^l \\ \widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c\end{bmatrix}$$

and

$$\theta_t^i \equiv \theta^i(\Theta_t, \Pi - \widehat{\Pi}_t^c, t) = \frac{A_{Q,t}^{\star} - B_{Q,t}(B_{\Psi,t}^{\star,i})^{\top} M_t^{\star,i}}{\gamma B_{Q,t}^2}\begin{bmatrix}\Theta_t \\ \Pi - \widehat{\Pi}_t^c\end{bmatrix},$$

delivering the respective optimal portfolio choices in (13) and (14) in Proposition 4.

Boundary conditions to (48) and (47) are provided by agents' optimization problem at the very last round of trading. Here, I need to take care of the jump in (30):

$$\begin{aligned} J^l(W_{T_-}^l, \Psi_{T_-}^l, n_T^l, T) &= \sup_{\theta_{T_-}^l} E\left[-e^{-\gamma W_{T_-}^l - \gamma\theta_{T_-}^l \Delta P_T}\middle| \mathcal{F}_{T_-}^l\right] \quad (49) \\ &= \sup_{\theta_{T_-}^l} -e^{-\gamma W_{T_-}^l - \gamma\theta_{T_-}^l E\left[\Delta P_T|\mathcal{F}_{T_-}^l\right] + \frac{1}{2}\gamma^2\left(\theta_{T_-}^l\right)^2 V\left[\Delta P_T|\mathcal{F}_{T_-}^l\right]} \end{aligned}$$

where I used the Laplace transform of a normal random variable. Solving for the first-order condition yields the optimal portfolio

$$\theta_{T_-}^l = \frac{E\left[\Delta P_T|\,\mathcal{F}_{T_-}^l\right]}{\gamma V\left[\Delta P_T|\,\mathcal{F}_{T_-}^l\right]}. \quad (50)$$

Using that $\Delta P_T = \Pi + \delta - P_{T_-} = (\delta + (1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T)$, I obtain, for agents l and i,

$$\theta_{T_-}^l = \frac{(1-\lambda_{1,T_-})(\widehat{\Pi}_{T_-}^l - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\widehat{\Theta}_{T_-}^l}{\gamma(o_{T_-}^l(n_T^l) + \sigma_\delta^2)}, \quad \theta_{T_-}^i = \frac{(1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T}{\gamma\sigma_\delta^2}. \quad (51)$$

Substituting (51) into the terminal value function in (49), I get

$$J^l(W_{T_-}^l, \Psi_{T_-}^l, n_T^l, T) = \exp\left(-\gamma W_{T_-}^l - \frac{1}{2}\frac{\left((1-\lambda_{1,T_-})(\widehat{\Pi}_{T_-}^l - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\widehat{\Theta}_{T_-}^l\right)^2}{o_{T_-}^l(n_T^l) + \sigma_\delta^2}\right) \quad (52)$$

for agents l. Similarly, I obtain $J^i(W_{T_-}^i, \Psi, T) = e^{-\gamma W_{T_-}^i - \frac{1}{2}\frac{\left((1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T\right)^2}{\sigma_\delta^2}}$ for agents i. But given the shape of the value function conjecture in (39), the boundary conditions are

$$M_{T_-}^{\star,l}(n_T^l) = \frac{1}{o_{T_-}^l(n_T^l) + \sigma_\delta^2} \begin{bmatrix} \lambda_{2,T_-}^2 & -\lambda_{2,T_-}(1-\lambda_{1,T_-}) \\ -\lambda_{2,T_-}(1-\lambda_{1,T_-}) & (1-\lambda_{1,T_-})^2 \end{bmatrix} \tag{53}$$

and

$$M_{T_-}^{\star,i} = \frac{1}{\sigma_\delta^2} \begin{bmatrix} \lambda_{2,T_-}^2 & -\lambda_{2,T_-}(1-\lambda_{1,T_-}) \\ -\lambda_{2,T_-}(1-\lambda_{1,T_-}) & (1-\lambda_{1,T_-})^2 \end{bmatrix}. \tag{54}$$

Notice that these boundary conditions are given in terms of the terminal price coefficients λ_{1,T_-} and λ_{2,T_-}. The boundary conditions for these will be derived in Appendix E. ■

D On the Approximation Error

In this appendix, I derive the dual counterpart to the primal problem in (31): I use the dual formulation to obtain a probabilistic characterization to the first-order Taylor approximation used in Proposition 4. Then, I use the dual approach along the lines of Haugh, Kogan, and Wang (2006) to provide an upper bound on the approximation error taking prices as given.

The primal problem in (31) may be alternatively written as

$$\sup_{\theta^l} E\left[-e^{-\gamma W_{T_-}^l + A} \middle| \mathcal{F}_{t_-}^l \right] \text{ s.t. } \mathrm{d}W_t = \theta_{t_-}\mathrm{d}P_t = \theta_{t_-}(A_{Q,t}\Psi_{t_-}^l \mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^l) \tag{55}$$

where A is a $\mathcal{F}_{T_-}$−measurable random variable given in (52). I denote by $(\theta_t^\star)_{t\geq 0}$ the optimal portfolio strategy associated with (55) and by $V_t^\star$ the value function associated with (55) and evaluated at the optimal strategy $(\theta_t^\star)_{t\geq 0}$. Also, I denote by $(\widetilde{\theta}_t)_{t\geq 0}$ the portfolio strategy in (13) which is associated with the value function $\widetilde{V}_t$ whose jump has been linearized. Clearly, because the policy $(\widetilde{\theta}_t)_{t\geq 0}$ is suboptimal, it is immediate that

$$\widetilde{V}_t \leq V_t^\star, \quad \forall t \in [0, T). \tag{56}$$

That is, the approximated value function $\widetilde{V}_t$ provides a lower bound to the optimal value function $V_t^\star$. To get a sense of what $V_t^\star$ and, thereby, the magnitude of the approximation error may be, it is necessary to include an upper bound on $V_t^\star$ as well. The latter is provided by duality theory: consider a fictitious market which is comprised of the original risky stock with equilibrium price P and dynamics

$$\mathrm{d}P_t = \sigma_t(\kappa_t \mathrm{d}t + \mathrm{d}\widehat{B}_t^l)$$

where $\kappa_t \equiv \frac{\mu_t}{\sigma_t}$ denotes its market price of risk. Furthermore, I introduce a market completion in the form of a fictitious asset with price S and dynamics

$$\mathrm{d}S_t = \mathrm{d}N_t^l - g_t \mathrm{d}t.$$

An agent facing such a market builds a portfolio strategy (θ, ψ) where θ and ψ denote the fractions invested in P and S, respectively. Her wealth therefore evolves according to

$$\mathrm{d}W_t = \theta_{t\_}\mathrm{d}P_t + \psi_{t\_}\mathrm{d}S_t = \theta_{t\_}\sigma_t(\kappa_t \mathrm{d}t + \mathrm{d}\widehat{B}_t^l) + \psi_{t\_}(\mathrm{d}N_t^l - g_t \mathrm{d}t). \tag{57}$$

The dual state variable $H \in \mathcal{K}$ to (57), or equivalently its Lagrange multiplier, is assumed to satisfy

$$\mathrm{d}H_t = \beta_t H_{t\_}\mathrm{d}\widehat{B}_t^l + \delta_t H_{t\_}\mathrm{d}M_t^l \tag{58}$$

where $M_t^l = N_t^l - \int_0^t \eta_s \mathrm{d}s$ is a $\widehat{\mathbb{P}}^l-$martingale and $\mathcal{K}$ is assumed to be as follows.

Assumption 1. *The space $\mathcal{K}$ is such that $(\beta, \delta) \in \mathbb{R} \times (-1, +\infty)$ are square integrable and predictable processes satisfying the conditions*

$$\int_0^t \beta_s^2 ds < \infty, \ \int_0^t \delta_s \eta_s ds < \infty \ \ \forall t \in [0, T) \text{ and such that } H_t > 0, \ E[H_t] = 1,$$
$$H_t W_t \text{ a local martingale } \forall t \in [0, T) \text{ and } E[H_T(\log(H_T) - A)] < \infty.$$

The dual state variable in (58) is assumed to take an exponential martingale form to enforce its positivity. Furthermore, an application of Ito's lemma yields

$$\begin{aligned}\mathrm{d}H_t W_t = {} & H_{t\_}\theta_{t\_}\sigma_t(\kappa_t + \beta_t)\mathrm{d}t + H_{t\_}(\theta_t \sigma_t + W_{t\_}\beta_t)\mathrm{d}\widehat{B}_t^l \\ & + (\psi_{t\_}(1 + \delta_t) + \delta_t W_{t\_})H_{t\_}\mathrm{d}N_t^l - (\psi_{t\_}g_t + W_{t\_}\delta_t \eta_t)H_{t\_}\mathrm{d}t.\end{aligned}$$

Since, from Assumption 1, $(H_t W_t)_{t \geq 0}$ is restricted to be a local martingale, it must be that $\beta_t = -\kappa_t$ and $(N_t^l)_{t \geq 0}$ should be appropriately compensated, i.e.

$$\psi_t g_t + W_t \delta_t \eta_t = \eta_t(\psi_t(1 + \delta_t) + \delta_t W_t)$$

and thus $\delta_t = \frac{g_t}{\eta_t} - 1$. Therefore, any candidate dual state variable H for the above fictitious market takes the following form

$$\begin{aligned}H_T &= e^{-\frac{1}{2}\int_0^T \kappa_s^2 \mathrm{d}s - \int_0^T \kappa_s \mathrm{d}\widehat{B}_t^l} e^{\int_0^T (\eta_s - g_s)\mathrm{d}s} \prod_{0 \leq s \leq t} \left(1 + \left(\frac{g_s}{\eta_s} - 1\right) \Delta N_s^l\right) \\ &= \mathcal{E}\left(-\kappa \widehat{B}^l + \left(\frac{g}{\eta} - 1\right) M^l\right)_T\end{aligned}$$

where $\mathcal{E}(\cdot)$ denotes the Doléans-Dade exponential. The remaining conditions in Assumption 1 are imposed so that H defines a proper change of measure and such that the dual problem is well-defined. The optimization problem faced by an agent in the fictitious market is then given by

$$\begin{aligned} V_t^{(g)} &\equiv \sup_{(\theta,\psi)} E\left[-e^{-\gamma W_{T_-}^{\theta,\psi}(g)+A}\right] \\ &\text{s.t. } \mathrm{d}W_t^{\theta,\psi}(g) = \theta_{t\_}\mathrm{d}P_t + \psi_{t\_}\mathrm{d}S_t^{(g)} \\ &\text{s.t. } \psi_t^\star(g) = 0 \;\; \forall t \in [0,T). \end{aligned} \tag{59}$$

That is, at the optimum, $g^\star$ is such that an agent finds it optimal not to invest in the fictitious asset. The portfolio strategies associated with the fictitious problem in (59) may be obtained as follows: by the martingale representation theorem, the discounted wealth may be written as

$$\begin{aligned} H_t W_t^{\theta,\psi} &= W_0 + \int_0^t \varphi_s \mathrm{d}\widehat{B}_s^l + \int_0^t \phi_s \mathrm{d}M_s^l \\ &= W_0 + \int_0^t H_s(\theta_s\sigma_s - W_s^{\theta,\psi}\kappa_s)\mathrm{d}\widehat{B}_s^l + \int_0^t H_s\left(\psi_s\frac{g_s}{\eta_s} + W_s^{\theta,\psi}\left(\frac{g_s}{\eta_s} - 1\right)\right)\mathrm{d}M_s^l \end{aligned}$$

where (φ,ϕ) are predictable processes. Accordingly, for any arbitrary dual candidate $H^{(g)}$, the portfolio strategy is

$$\theta_t^{(g)} = (\sigma_t)^{-1}(\varphi_t^{(g)}\left(H_t^{(g)}\right)^{-1} + \kappa_t W_t^{(g)}) \text{ and } \psi_t^{(g)} = \left(\frac{g_t}{\eta_t}\right)^{-1}\left(\phi_t^{(g)}\left(H_t^{(g)}\right)^{-1} + \left(1 - \frac{g_t}{\eta_t}\right)W_t^{(g)}\right). \tag{60}$$

Also, for any arbitrary candidate $H^{(g)}$, I can write the optimization problem in a static form à la Cox-Huang as

$$\sup_{W_T^{(g)}} E_0\left[-e^{-\gamma W_{T_-}^{(g)}+A}\right] \text{ s.t. } E_0\left[H_T^{(g)}W_T^{(g)}\right] \leq W_0. \tag{61}$$

Solving the associated Lagrangian, the value function $V^{(g)}$ for any $H^{(g)}$ may be written as

$$V_0^{(g)} = -e^{-\gamma W_0 - E_0[H_{T_-}^{(g)}(\log(H_{T_-}^{(g)})-A)]}. \tag{62}$$

Since $W_t^{(g)} \geq W_t^\star$, (62) constitutes an upper bound to $V_t^\star$, i.e. $V_t^\star \leq V_t^{(g)} \;\; \forall t \in [0,T)$, and is expected to be strictly equal to it at the optimum $V_t^\star = \inf_H V_t^{(g)}$. Hence, taking into account (56), $V_t^\star$ is bounded by

$$\widetilde{V}_t \;\; \forall(\widetilde{\theta}_t)_{t\geq 0} \text{ given} \leq V_t^\star \leq V_t^{(g)} \;\; \forall g > 0 \text{ given}$$

and, thus, the approximation error is bounded by

$$0 \leq \left| V_t^\star - \widetilde{V}_t \right| \leq \left| V_t^{(g)} - \widetilde{V}_t \right|$$

for given prices. The upper bound in (62) requires the computation of $E_0[H_T^{(g)}(\log(H_T^{(g)}) - A)]$: since $H \in \mathcal{K}$, H represents a change of measure and I can accordingly define the following Radon-Nikodym derivative

$$\left.\frac{\mathrm{d}\widetilde{\mathbb{P}}^l}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}_t^l} = H_t. \tag{63}$$

By Girsanov's theorem, this change of measure implies that $\widetilde{B}_t^l = \widehat{B}_t^l + \int_0^t \kappa_s \mathrm{d}s$ is a $\widetilde{\mathbb{P}}^l-$Brownian motion and

$$\widetilde{M}_t^l = M_t^l - \int_0^t (g_s - \eta_s)\mathrm{d}s$$

is a $\widetilde{\mathbb{P}}^l-$martingale and $(\widetilde{N}_t^l)_{t\geq 0}$ is a $\widetilde{\mathbb{P}}^l-$Poisson process with intensity g_t. Hence,

$$E_0[H_T^{(g)}(\log(H_T^{(g)}) - A)] = \widetilde{E}_0[\log(H_T^{(g)})] - \widetilde{E}_0[A].$$

Moreover, applying the change of measure in (63) and Ito's lemma shows that

$$\mathrm{d}\log(H_t^{(g)}) = \frac{1}{2}\kappa_t^2 \mathrm{d}t - \kappa_t \mathrm{d}\widetilde{B}_t^l - \eta_t \left(\frac{g_t}{\eta_t} - 1\right)\mathrm{d}t + \log\left(\frac{g_t}{\eta_t}\right)\mathrm{d}\widetilde{N}_t^l$$

such that

$$\widetilde{E}_0[\log(H_T^{(g)})] = \frac{1}{2}\widetilde{E}_0\left[\int_0^T \kappa_s^2 \mathrm{d}t\right] + \widetilde{E}_0\left[\int_0^T \left(\log\left(\frac{g_t}{\eta_t}\right) g_t - g_t + \eta_t\right)\mathrm{d}t\right].$$

To actually compute an upper bound, I further need to pick a particular g: one such choice includes $g = \eta$ as a natural candidate for reasons that will become clear shortly. With this candidate at hand, the dual state variable $H_t^{(\eta)} = \mathcal{E}(-\kappa\widehat{B}^l)_t$ does not jump and the upper bound reduces to

$$V_0^{(\eta)} = -e^{-\gamma W_0 - \widetilde{E}_0[\frac{1}{2}\int_0^T \kappa_s^2 \mathrm{d}s - A]} \equiv -e^{-\gamma W_0 - \frac{1}{2}f(\Psi^l, n, 0)}.$$

Recalling that the market price of risk is given by $\kappa_t = \frac{A_{Q,t}\Psi_t^l}{B_{Q,t}}$ and that the change of measure in (63) implies that $(\Psi^l)_{t\geq 0}$ still evolves as an OU process under $\widetilde{\mathbb{P}}^l$

$$\mathrm{d}\Psi_t^l = \left(A_{\Psi,t} - B_{\Psi,t}\frac{A_{Q,t}}{B_{Q,t}}\right)\Psi_{t\_}^l \mathrm{d}t + B_{\Psi,t}\mathrm{d}\widetilde{B}_t^l + C_{\Psi,t}\mathrm{d}N_t^l$$

where the particular choice of $g = \eta$ implies that $(N_t^l)_{t\geq 0}$ is the same Poisson process under both measures, I can write the following PDE for the function f

$$0 = \left(\frac{A_{Q,t}\Psi_t^l}{B_{Q,t}}\right)^2 + f_t + (f_\Psi)^\top \left(A_{\Psi,t} - B_{\Psi,t}\frac{A_{Q,t}}{B_{Q,t}}\right)\Psi_t^l + \frac{1}{2}\mathrm{tr}\left(J_{\Psi\Psi}B_{\Psi,t}B_{\Psi,t}^\top\right)$$
$$+ E^\nu\left[f(\Psi_{t_-}^l + C_{\Psi,t}, n+m, t) - f(\Psi_{t_-}^l, n, t)\right]$$

with boudary condition $f(\Psi^l, n, T) = A$. Given that $(\Psi^l)_{t\geq 0}$ follows a Gaussian process under $\widetilde{\mathbb{P}}^l$ and that f is quadratic, one may conjecture that $f(\Psi^l, n, t) \equiv (\Psi^l)^\top R_t(n)\Psi^l$ and substitution of this conjecture into the above PDE after separation of variables yields (41) and inspection of A, implies that $R_t(n) = M_t(n)$, or

$$V_t^{(\eta)} = -e^{-\gamma W_t - \frac{1}{2}(\Psi_t^l)^\top M_t(n)\Psi_t^l}$$

which is nothing but the conjecture in (39). One may then obtain the unconditional upper bound at time 0, by integrating out the vector Ψ_0^l. To that purpose, one should notice that the latter is distributed as $\Psi_0^l \sim \mathcal{N}(\mathbf{0}, \Sigma_0)$ with

$$\Sigma_0 = \begin{bmatrix} \frac{\sigma_\Theta^2}{2a_\Theta} - \left(\frac{\lambda_{1,0}}{\lambda_{2,0}}\right)^2 \frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} & \frac{\lambda_{1,0}}{\lambda_{2,0}}\frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} \\ \frac{\lambda_{1,0}}{\lambda_{2,0}}\frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} & \frac{\sigma_\Pi^2\sigma_S^2}{\sigma_\Pi^2+\sigma_S^2} \end{bmatrix}.$$

Therefore,

$$E^{\Psi_0^l}\left[V_0^{(\eta)}\right] = -e^{-\gamma W_0 - \frac{1}{2}M_0^{l,(1,1)}(1)} \int_{\mathbb{R}^2} e^{-\frac{1}{2}\left(\Psi_0^{(\star,l)}\right)^\top M_0^{\star,l}(1)\Psi_0^{(\star,l)}} \mathrm{d}\Phi\left(\Psi_0^{(\star,l)}\right)$$
$$= -e^{-\gamma W_0 - \frac{1}{2}M_0^{l,(1,1)}(1)} \left|I + \Sigma_0 M_0^{\star,l}(1)\right|^{-\frac{1}{2}}$$

where the second equality follows from completing the square in the bivariate normal distribution. Given this explicit form for the upper bound, a probabilistic interpretation to the linearization is obtained by inspection of the portfolio strategy implied by $g = \eta$: the static problem in (61) induces the first-order condition $U'(W_T) = \lambda_t \frac{H_T^{(\eta)}}{H_t^{(\eta)}}$ where λ_t denotes the Lagrange multiplier of the static constraint at time t. Furthermore, the envelope theorem implies that $\frac{\partial V_t^{(\eta)}}{\partial W_t} = \lambda_t$. Thus, $\frac{H_T^{(\eta)}}{H_t^{(\eta)}} = U'(W_T)\left(\frac{\partial V_t^{(\eta)}}{\partial W_t}\right)^{-1}$ and, as a result, $\frac{H_s^{(\eta)}}{H_t^{(\eta)}} = \frac{\partial V_s^{(\eta)}}{\partial W_s} / \frac{\partial V_t^{(\eta)}}{\partial W_t}$ $\forall s \geq t$, which finally implies that $\mathrm{d}\log(H_t^{(\eta)}) = \mathrm{d}\log\left(\frac{\partial V_t^{(\eta)}}{\partial W_t}\right)$. Matching the diffusion terms yields

$$\theta_t^{(\eta)} = -(\sigma_t)^{-1}\left(\frac{\partial^2 V_t}{\partial W_t^2}\right)^{-1}\left(\frac{\partial V_t}{\partial W_t}\kappa_t + \left(\frac{\partial^2 V_t}{\partial W_t \partial \Psi_t^l}\right)^\top B_{\Psi,t}\right),$$

which, after substitution of the explicit upper bound $V_t^{(\eta)}$, needless to say, yields the portfolio policy in (13). Further matching the jump terms yields

$$0 = \log\left(V_t^{(\eta)}(W_{t_-} + \psi_t, \Psi^l_{t_-} + C_{\Psi,t})\right) - \log\left(V_t^{(\eta)}(W_{t_-}, \Psi^l_{t_-})\right).$$

Substituting the explicit form and using the portfolio policy in (60) shows that

$$\psi_t^{(\eta)} \equiv \phi_t^{(\eta)}\left(H_t^{(\eta)}\right)^{-1} = -\frac{1}{2\gamma}\left(\left(\Psi^l_{t_-} + C_{\Psi,t}\right)^\top M_t(n+m)\left(\Psi^l_{t_-} + C_{\Psi,t}\right) - \left(\Psi^l_{t_-}\right)^\top M_t(n)\Psi^l_{t_-}\right).$$

Hence, the approximate policy $(\widetilde{\theta}_t)_{t\geq 0}$ boils down to pick the strategy $(\theta_t^{(\eta)})_{t\geq 0}$ and make it an optimal one by setting $\psi_t^{(\eta)} \approx 0$ since, as per the fictitious problem in (59), an optimal strategy must be so that $\psi_t = 0$. Alternatively, the Taylor approximation of the jump size tantamount to set the loading ϕ_t on the Poissonian risk in the martingale representation of the discounted wealth to zero.

I finally compute the value function $\widetilde{V}_t$ associated with the approximate strategy $(\theta_t^{(\eta)})_{t\geq 0}$ and $\psi_t = 0$. Since the approximate wealth satisfies

$$\widetilde{W}_T = W_0 + \int_0^T \theta_s^{(\eta)}\mathrm{d}P_s = W_T^{(\eta)} - \int_0^T \psi_s \mathrm{d}S_s^{(\eta)},$$

it remains to compute

$$\widetilde{V}_0 = E_0\left[-e^{-\gamma\left(W_0 + \int_0^T \theta_s^{(\eta)}\sigma_s(\kappa_s \mathrm{d}s + \mathrm{d}\widehat{B}^l_s)\right) + A}\right].$$

I introduce the following change of measure

$$\left.\frac{\mathrm{d}\bar{\mathbb{P}}^l}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}^l_t} = e^{-\gamma\int_0^T \theta_s^{(\eta)}\sigma_s \mathrm{d}\widehat{B}^l_s - \frac{1}{2}\gamma^2\int_0^T\left(\theta_s^{(\eta)}\sigma_s\right)^2\mathrm{d}s} \tag{64}$$

such that

$$\widetilde{V}_0 = -e^{-\gamma W_0}\bar{E}_0\left[e^{-\gamma\int_0^T \theta_s^{(\eta)}\sigma_s\left(\kappa_s - \frac{1}{2}\gamma\theta_s^{(\eta)}\sigma_s\right)\mathrm{d}s + A}\right] \equiv -e^{-\gamma W_0}h(\Psi^l, n, 0).$$

Recalling that $\theta_t^{(\eta)} = \frac{A_{Q,t} - B_{Q,t}B^\top_{\Psi,t}M^l_t(n)}{\gamma B_{Q,t}}\Psi^l_t$, I can write, after simplifications,

$$h(\Psi^l, n, 0) = \bar{E}_0\left[e^{-\frac{1}{2}\int_0^T\left(\Psi^l_t\right)^\top\frac{A^\top_{Q,t}A_{Q,t} - B^2_{Q,t}M^l_t(n)B_{\Psi,t}B^\top_{\Psi,t}M^l_t(n)}{B^2_{Q,t}}\Psi^l_t \mathrm{d}t + A}\right]. \tag{65}$$

The computation of the function $h(\cdot)$ is generally complicated precisely because it involves a quadratic jump. Therefore, for the purpose of computing the lower bound, I resort to Monte Carlo Simulations. I simulate the dynamics of Ψ^l under $\bar{\mathbb{P}}^l$ using a standard Euler scheme. Given the change of measure in (64), $\bar{B}_t^l = \widehat{B}^l + \gamma \int_0^t \theta_s^{(\lambda)} \sigma_s \mathrm{d}s$ is a $\bar{\mathbb{P}}^l-$Brownian motion by Girsanov's theorem and Ψ^l satisfies

$$\mathrm{d}\Psi_t^l = \left(A_{\Psi,t} - B_{\Psi,t} \frac{A_{Q,t} - B_{Q,t} B_{\Psi,t}^\top M_t^l(n)}{B_{Q,t}} \right) \Psi_{t\_}^l \mathrm{d}t + B_{\Psi,t} \mathrm{d}\bar{B}_t^l + C_{\Psi,t} \mathrm{d}N_t^l.$$

I further draw m out of the cross-sectional distribution of types by drawing a uniform random variable according to $\mathcal{U}_{[0,1]}$ and cumulate μ_t. I then compute the integral in (65) by means of the trapezoid rule.

E Proof of Proposition 5

In this appendix, I derive the equilibrium equations for the price coefficients λ_1 and λ_2. To that purpose, I first observe that, from (13), individual portfolios take the form

$$\theta^l(\Psi_t^l, n_t^l, t) = d_{\Theta,t}^l(n_t^l)\widehat{\Theta}_t^l + d_{\Delta,t}^l(n_t^l)\Delta_t^l. \tag{66}$$

Aggregating the latter over the population of agents first calls for the average beliefs $\int_{j\in I} \widehat{\Pi}_t^j \mathrm{d}\iota(j)$. I recall that, given that $\xi_t \in \mathcal{F}_t^c \subseteq \mathcal{F}_t^l$, I can write

$$\xi_t = \lambda_{1,t}\Pi + \lambda_{2,t}\Theta_t \equiv \lambda_{1,t}\widehat{\Pi}_t^c + \lambda_{2,t}\widehat{\Theta}_t^c \equiv \lambda_{1,t}\widehat{\Pi}_t^l + \lambda_{2,t}\widehat{\Theta}_t^l.$$

It then follows that $\widehat{\Theta}_t^c = \frac{\xi_t - \lambda_{1,t}\widehat{\Pi}_t^c}{\lambda_{2,t}}$ and $\widehat{\Theta}_t^l = \frac{\xi_t - \lambda_{1,t}\widehat{\Pi}_t^l}{\lambda_{2,t}}$: using these expressions along with equation (9) for the filter $\widehat{\Pi}_t^c$ of Proposition 3, an application of Ito's lemma delivers

$$\mathrm{d}\frac{\widehat{\Pi}_t^c}{o_t^c} = k_t \mathrm{d}\widehat{B}_t^c + k_t^2 \widehat{\Pi}_t^c \mathrm{d}t = k_t \mathrm{d}\widehat{B}_t^l + k_t^2 \widehat{\Pi}_{t\_}^l \mathrm{d}t \tag{67}$$

where the second equality follows from the change of measure in (33). Similarly, using (10), one may write

$$\mathrm{d}\frac{\widehat{\Pi}_t^l}{o_t^l} = k_t \mathrm{d}\widehat{B}_t^l + k_t^2 \widehat{\Pi}_{t\_}^l \mathrm{d}t + \bar{S}_{m,t}^l \frac{m_t}{\sigma_S^2} \mathrm{d}N_t^l \tag{68}$$

where the jump size directly follows from (26). Bunching (67) and (68) together, I can write

$$\mathrm{d}\left(\frac{\widehat{\Pi}_t^l}{o_t^l\left(n_t^l\right)} - \frac{\widehat{\Pi}_t^c}{o_t^c} \right) = \frac{\bar{S}_{m,t}^l}{\sigma_S^2} m_t \mathrm{d}N_t^l.$$

This equation may then be solved using the initial conditions $\widehat{\Pi}_0^c = 0$ and $o_0^c = \sigma_\Pi^2$ and applying the updating rule in (26) to $\widehat{\Pi}_0^l$. Doing so, I obtain the following lemma.

Lemma 1. *A manager l's expectations $\widehat{\Pi}^l$ satisfies*

$$\widehat{\Pi}^l_t = \frac{o^l_t\left(n^l_t\right)}{o^c_t}\widehat{\Pi}^c_t + \widehat{\Pi}^l_0\frac{o^l_t\left(n^l_t\right)}{o^l_0(1)} + \frac{o^l_t\left(n^l_t\right)}{\sigma^2_S}\int_0^t \bar{S}^l_{m,t} m_s dN^l_s = \frac{o^l_t\left(n^l_t\right)}{o^c_t}\widehat{\Pi}^c_t + \frac{o^l_t\left(n^l_t\right)}{\sigma^2_S}\bar{S}^l_{n,t}n^l_t \tag{69}$$

taking into account that $\mathrm{d}n^l_t = (m_t\mathbf{1}_{\{m_t+n^l_{t\_}\in A\cup B\}} + \infty\mathbf{1}_{\{m_t+n^l_{t\_}\notin A\cup B\}})\mathrm{d}N^l_t, n^l_0 = 1$ when integrating. Moreover, recalling that $o^l_t = \frac{o^c_t\sigma^2_S}{\sigma^2_S+n^l_t o^c_t}$ and that the law of large numbers implies $\int_{j_t\in I_2}\bar{S}^j_{n,t}\mathrm{d}\iota(j_t) = \omega_t\Pi$ for all n, I finally write the average beliefs as

$$\begin{aligned}\int_{j_t\in I}\widehat{\Pi}^j_t\mathrm{d}\iota(j_t) &= \int_{j_t\in I_2}\left(\frac{o^j_t\left(n^l_t\right)}{o^c_t}\widehat{\Pi}^c_t + \frac{o^j_t\left(n^l_t\right)}{\sigma^2_S}\bar{S}^j_{n,t}n^l_t\right)\mathrm{d}\iota(j_t) + (1-\omega_t)\Pi \qquad (70)\\ &= \sum_{n\in A\cup B}\mu_t(n)\left(\frac{\sigma^2_S}{\sigma^2_S+no^c_t}\widehat{\Pi}^c_t + \frac{o^c_t n}{\sigma^2_S+no^c_t}\Pi\right) + (1-\omega_t)\Pi\\ &\equiv \alpha_t\widehat{\Pi}^c_t + (1-\alpha_t)\Pi\end{aligned}$$

where, similar to He and Wang (1995), $\alpha_t \equiv \sum_{n\in A\cup B}\mu_t(n)\frac{\sigma^2_S}{\sigma^2_S+no^c_t}$ and where

$$\mu_t(n) = \begin{cases}\mu^A_t(n) & \text{if } n\in A\\ \mu^B_t(n) & \text{if } n\in B\end{cases}.$$

Equation (70) allows to characterize the impact of percolation on higher-order beliefs (HOB). Since beliefs of higher order collapse to a linear combination of the first-order expectations and denoting agent l's expectation of $\int_{j_t\in I}\widehat{\Pi}^j_t\mathrm{d}\iota(j_t)$ by $\widehat{\Pi}^{(l,2)}_t \equiv E[\int_{j_t\in I}\widehat{\Pi}^j_t\mathrm{d}\iota(j_t)|\mathcal{F}^l_t]$, one may readily observe that $\widehat{\Pi}^{(l,2)}_t = \alpha_t\widehat{\Pi}^c_t + (1-\alpha_t)\widehat{\Pi}^l_t$ in such a way that the second-order expectation writes

$$\widehat{\Pi}^{(2)}_t \equiv \int_{j_t\in I}\widehat{\Pi}^{(j,2)}_t\mathrm{d}\iota(j_t) = (1-(1-\alpha_t)^2)\widehat{\Pi}^c_t + (1-\alpha_t)^2\Pi.$$

Iterating, the $k-$th order expectation $\widehat{\Pi}^{(k)}_t$ writes

$$\widehat{\Pi}^{(k)}_t = \int_{j_t\in I}\widehat{\Pi}^{(j,k)}_t\mathrm{d}\iota(j_t) = (1-(1-\alpha_t)^k)\widehat{\Pi}^c_t + (1-\alpha_t)^k\Pi.$$

Observing that the weight α_t is increasing in the cross-sectional average number of signals, and that the latter is roughly increasing like $e^{\eta t}$, the following pattern occurs. Because information percolation produces some inertia in updating initially, HOB play a stronger role at the beginning of the economy as compared with a setup in which signals continuously accrue. Yet, after some time, percolation takes off and signals accrue at a rate beyond that of a continuous flow of signals. This ultimately causes the effect of HOB to decrease after some time has elapsed.

This also allows to show how the conjecture for linear prices in Definition 1 obtains. Following Hong and Wang (2000), the price may be written as

$$P_t = \int_{j_t \in I} E[F_t|\mathcal{F}_t^j]\mathrm{d}\iota(j_t) + \lambda_{2,t}\Theta_t$$

with $F_t = E\left[e^{-r(T-t)}(\Pi + \delta)\middle|\, \mathcal{F}_t\right] = \Pi + \delta$ denoting the expected value of the terminal dividend under full information $\mathcal{F}_t$ discounted at the riskfree rate and where the second equality follows from $r = 0$. The term $\lambda_{2,t}\Theta_t$ represents a discount for inventory risk. Notice that $\widehat{\Theta}_t^c$ does not appear in the price as ξ_t implies that

$$\lambda_{1,t}(\Pi - \widehat{\Pi}_t^c) = \lambda_{2,t}(\Theta_t - \widehat{\Theta}_t^c)$$

since $E[\xi_t|\mathcal{F}_t^c] = \lambda_{1,t}\widehat{\Pi}_t^c + \lambda_{2,t}\widehat{\Theta}_t^c$ and, given $\xi_t \subseteq \mathcal{F}_t^c$, $E[\xi_t|\mathcal{F}_t^c] = \xi_t$. Putting everything together and taking into account that all agents have null prior regarding δ, I can write

$$P_t = (1 - \alpha_t)\Pi + \alpha_t\widehat{\Pi}_t^c + \lambda_{2,t}\Theta_t$$

and the conjecture in (6) follows. This expression should be considered to hold on $[0, T)$ so as to allow a discontinuity in the stock price at the horizon when it attains the final payoff $\Pi + \delta$.

Using (70), I can further compute the average beliefs regarding the supply

$$\int_{j_t \in I} \widehat{\Theta}_t^j \mathrm{d}\iota(j_t) = \frac{\lambda_{1,t}}{\lambda_{2,t}}\Pi + \Theta_t - \frac{\lambda_{1,t}}{\lambda_{2,t}}\int_{j_t \in I} \widehat{\Pi}_t^j \mathrm{d}\iota(j_t) = \Theta_t + \alpha_t\frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^c).$$

Applying the law of large numbers, I then obtain the aggregate demand

$$\begin{aligned}\int_{j_t \in I} \theta_t^j \mathrm{d}\iota(j_t) = \sum_{n \in A\cup B} \mu_t(n) &\left(\begin{array}{c} \frac{A_Q^{(2)} - B_Q D_l^{(2)}(n)}{\gamma B_Q^2}\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\sigma_S^2}{\sigma_S^2 + o_t^c n}(\Pi - \widehat{\Pi}_t^c)\right) \\ + \frac{A_Q^{(3)} - B_Q D_l^{(3)}(n)}{\gamma B_Q^2}\frac{o_t^c n}{\sigma_S^2 + o_t^c n}(\Pi - \widehat{\Pi}_t^c)) \end{array}\right) \\ + (1 - \omega_t)&\left(\frac{A_Q^{(2)} - B_Q D_i^{(2)}}{\gamma B_Q^2}\Theta_t + \frac{A_Q^{(3)} - B_Q D_i^{(3)}}{\gamma B_Q^2}(\Pi - \widehat{\Pi}_t^c)\right)\end{aligned} \tag{71}$$

where $D_j = B_\Psi^j M^j(t)$ for $j = l, i$. Clearing the markets yields

$$\begin{aligned}A_Q^{\star,(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}A_Q^{\star,(1)} = &\left(\begin{array}{c} -\frac{\lambda_{1,t}}{\lambda_{2,t}}\gamma B_Q^2 + \sum_{n\in A\cup B}\mu_t(n)B_Q\left(D_l^{\star,(2)}(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}}D_l^{\star,(1)}(n)\right)\frac{o_t^c n}{\sigma_S^2 + o_t^c n} \\ +(1-\omega_t)B_Q\left(D_i^{\star,(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}D_i^{\star,(1)}\right) \end{array}\right) \\ &\times \frac{1}{\sum_{n\in A\cup B}\mu_t(n)\frac{o_t^c n}{\sigma_S^2 + o_t^c n} + 1 - \omega_t}\end{aligned} \tag{72}$$

and,

$$\sum_{n\in A\cup B}\mu_t(n)\frac{A_Q^{\star,(1)}-B_QD_l^{\star,(1)}(n)}{\gamma B_Q^2}+(1-\omega_t)\frac{A_Q^{\star,(1)}-B_QD_i^{\star,(1)}}{\gamma B_Q^2}=1. \tag{73}$$

These equilibrium equations are obtained from the market-clearing condition $\int_{j_t\in I}\theta_t^j \mathrm{d}\iota(j_t)=\Theta_t$ and $A_Q^\star$, $A_\Psi^\star$ and $B_\Psi^{\star,j}$ in (46) and (72) follows from (73). Both equations (72) and (73) determine the equilibrium behavior of $\lambda_{1,t}$ and $\lambda_{2,t}$ over $[0,T)$. Yet, they lack boundary conditions: at the very date before the economy ends, agents hold myopic portfolios of the kind in (50). Aggregating these, I obtain

$$\begin{aligned}\int_{j_T\in I}\theta_{T_-}^j\mathrm{d}\iota(j_T)&=\int_{j_T\in I_2}\frac{\widehat{\Pi}_{T_-}^j-P_{T_-}}{\gamma(o_{T_-}^j+\sigma_\delta^2)}\mathrm{d}\iota(j_T)+(1-\omega_T)\frac{\Pi-P_{T_-}}{\gamma\sigma_\delta^2}\\&=\sum_{n\in A\cup B}\mu_T(n)\frac{\frac{\sigma_S^2}{\sigma_S^2+o_{T_-}^c n}\widehat{\Pi}_{T_-}^c+\frac{o_{T_-}^c n}{\sigma_S^2+o_{T_-}^c n}\Pi-P_{T_-}}{\gamma(o_{T_-}^l(n)+\sigma_\delta^2)}+(1-\omega_T)\frac{\Pi-P_{T_-}}{\gamma\sigma_\delta^2}=\Theta_T.\end{aligned}$$

Using that

$$P_{T_-}=\lambda_{1,T_-}\Pi+(1-\lambda_{1,T_-})\widehat{\Pi}_{T_-}^c+\lambda_{2,T_-}\Theta_T$$

and simplifying yields

$$\begin{aligned}\lambda_{1,T_-}=&\left(\sum_{n\in A\cup B}\mu_T(n)\frac{(o_{T_-}^c n+\sigma_S^2)\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)^{-1}\\&\times\left(\sum_{n\in A\cup B}\mu_T(n)\frac{o_{T_-}^c n\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)\end{aligned} \tag{74}$$

and

$$\lambda_{2,T_-}=-\gamma\sigma_\delta^2\left(\sum_{n\in A\cup B}\mu_T(n)\frac{(o_{T_-}^c n+\sigma_S^2)\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)^{-1}. \tag{75}$$

This provides the two required boundary conditions associated with (72) and (73) and takes care of the final jump in prices. Indeed, knowing λ_{1,T_-} and λ_{2,T_-} and, thus, P_{T_-}, the jump size is then simply obtained as $\Delta P_T=\Pi+\delta-P_{T_-}$. Substituting (74) and (75) into (53) and (54) makes all the boundary conditions depending on $o_{T_-}^c$ which represents the only unknown terminal condition, hence the shooting method suggested in the main text in Subsection 3.2.

When solving the equilibrium, one needs to jointly solve the HJB equations in (47) and (48) in Proposition 4 along with the price equations (72) and (73) and the equation (12) for the common variance in Proposition 3. The resulting system of differential equations is not explicit, for the derivatives $\lambda'_{1,t}$ and $\lambda'_{2,t}$ appear through the coefficient k_t. The solving procedure is alleviated if one is able to transform the system of equations to be solved into an explicit one. As it turns out, a trick which works as a continuous-time dynamic equivalent of Admati (1985)'s lemma is available: considering the second equilibrium equation in (73) and observing that the derivatives of $\lambda_{1,t}$ and $\lambda_{2,t}$ are contained in $A_{Q,t}$, one may spell out the left-hand side of (72) and write

$$A_{Q,t}^{\star,(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}} A_{Q,t}^{\star,(1)} = \lambda'_{1,t} + (1-\lambda_{1,t}) k_t^2 o_t^c - \frac{\lambda_{1,t}}{\lambda_{2,t}} (\lambda'_{2,t} - a_\Theta \lambda_{2,t}) \\ = \lambda_{2,t} \sigma_\Theta k_t + (1-\lambda_{1,t}) k_t^2 o_t^c. \tag{76}$$

Substituting this expression in place of the left-hand side of (72) produces a quadratic equation in k_t. The latter has two real roots, one of which is of the form $k_t = \frac{\sigma_\Theta \lambda_{2,t}}{o_t^c(\lambda_{1,t}-1)}$. This root may be verified, by substitution into the equilibrium equations, to correspond to the fully-revealing equilibrium. That is, one in which the diffusion of the price P_t is constantly null over $[0, T)$ and the price thus reveals Π. As this equilibrium is trivial, I shall discard it and for obvious reasons only consider the second root. Substituting the second root allows to make the system of equilibrium equations explicit and to considerably facilitate the numerical implementation of the equilibrium. ■

F Serial Correlation, Trading Strategies, and Measures of Performance

In this appendix, I compute the different quantities plotted in the results sections.

F.1 Serial Correlation

Integrating the supply in (1) along with common expectations in (9), I can write

$$\Theta_t = \Theta_0 e^{-a_\Theta t} + \sigma_\Theta \int_0^t e^{a_\Theta (s-t)} \mathrm{d}B_s^\Theta, \tag{77}$$

$$\widehat{\Pi}_t^c = \Pi \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \equiv E[\widehat{\Pi}_t^c | \Pi] + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \tag{78}$$

where I used that $\widehat{\Pi}_0^c = 0$. Accordingly, P_t may be written as

$$P_t = \lambda_{1,t} \Pi + \lambda_{2,t} \left(\Theta_0 e^{-a_\Theta t} + \sigma_\Theta \int_0^t e^{a_\Theta (s-t)} \mathrm{d}B_s^\Theta \right) \\ + (1-\lambda_{1,t}) \left(\Pi \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \right).$$

As a result, the price difference $\Delta P_t := P_{t+\Delta} - P_t$ over $[t, t+\Delta]$ is given by

$$\begin{aligned}
\Delta P_t = &\left[\begin{array}{c} \lambda_{1,t+\Delta} - \lambda_{1,t} + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right)\int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t+\Delta})\int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]\Pi \\
&+ \int_0^t \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right)\mathrm{d}B_s^\Theta \\
&+ \int_t^{t+\Delta}\left(\sigma_\Theta \lambda_{2,t+\Delta}e^{a_\Theta(s-(t+\Delta))} + (1-\lambda_{1,t+\Delta})o_s^c k_s e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\right)\mathrm{d}B_s^\Theta \\
&+ (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{-a_\Theta t}\Theta_0.
\end{aligned}$$

Similarly, I can compute the price difference $\Delta P_{t-\Delta}$ over $[t-\Delta, t]$ and get

$$\begin{aligned}
&\mathrm{var}(\Delta P_{t-\Delta}) = (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})^2 e^{-2a_\Theta(t-\Delta)}\frac{\sigma_\Theta^2}{2a_\Theta} + \\
&\left[\begin{array}{c} \lambda_{1,t} - \lambda_{1,t-\Delta} + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right)\int_0^{t-\Delta} o_s^c k_s^2 e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t})\int_{t-\Delta}^{t} o_s^c k_s^2 e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]^2 \sigma_\Pi^2 \\
&+ \int_0^{t-\Delta} \left(\begin{array}{c} (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{a_\Theta(s-(t-\Delta))}\sigma_\Theta \\ + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right) o_s^c k_s e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u} \end{array}\right)^2 \mathrm{d}s \\
&+ \int_{t-\Delta}^{t}\left(\sigma_\Theta \lambda_{2,t}e^{a_\Theta(s-t)} + (1-\lambda_{1,t})o_s^c k_s e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\right)^2 \mathrm{d}s, \text{ along with} \\
&\mathrm{cov}(\Delta P_{t+\Delta}, \Delta P_{t-\Delta}) = (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{-a_\Theta t}(\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{-a_\Theta(t-\Delta)}\frac{\sigma_\Theta^2}{2a_\Theta} + \\
&\left[\begin{array}{c} \lambda_{1,t+\Delta} - \lambda_{1,t} + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right)\int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t+\Delta})\int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right] \\
&\times \left[\begin{array}{c} \lambda_{1,t} - \lambda_{1,t-\Delta} + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right)\int_0^{t-\Delta} o_s^c k_s^2 e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t})\int_{t-\Delta}^{t} o_s^c k_s^2 e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]\sigma_\Pi^2 \\
&+ \int_0^{t-\Delta} \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right) \\
&\times \left(\begin{array}{c} (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{a_\Theta(s-(t-\Delta))}\sigma_\Theta \\ + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right) o_s^c k_s e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u} \end{array}\right)\mathrm{d}s \\
&+ \int_{t-\Delta}^{t} \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right) \\
&\times \left(\sigma_\Theta \lambda_{2,t}e^{a_\Theta(s-t)} + (1-\lambda_{1,t})o_s^c k_s e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\right)\mathrm{d}s
\end{aligned}$$

where the variance of $\Delta P_{t-\Delta}$ and the covariance between $\Delta P_{t-\Delta}$ and ΔP_t follow from Ito isometry. The econometrician then computes the serial correlation of stock returns by projecting ΔP_t onto $\Delta P_{t-\Delta}$. Applying the projection theorem, it follows that

$$E[\Delta P_t|\Delta P_{t-\Delta}] = \beta_t(\Delta)\Delta P_{t-\Delta}$$

where, as in Wang (1993), $\beta_t(\Delta) = \frac{\text{cov}(\Delta P_t, \Delta P_{t-\Delta})}{\text{var}(\Delta P_{t-\Delta})}$. Hence, as in Banerjee, Kaniel, and Kremer (2009), returns exhibit momentum whenever $\beta_t(\Delta) > 0$ and reversal whenever $\beta_t(\Delta) < 0$. This is the measure often used in the empirical literature as in Jegadeesh and Titman (1993), for instance.

F.2 Trading Strategies

From (66) and using the relation in (69) along with $\widehat{\Theta}^l_t = \Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}^l_t)$, the portfolio strategy of an agent l holding n signals may be re-expressed as

$$\begin{aligned}\theta^l_t(n) &= d^l_{\Theta,t}(n)\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \alpha^l_t(n)\widehat{\Pi}^c_t - (1-\alpha^l_t(n))\bar{S}^l_{n,t})\right) \\ &\quad + d^l_{\Delta,t}(n)(1-\alpha^l_t(n))(\bar{S}^l_{n,t} - \widehat{\Pi}^c_t) \\ &= d^l_{\Theta,t}(n)\Theta_t + \varphi^l_t(n)(\Pi - \widehat{\Pi}^c_t) + (\varphi^l_t(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}}d^l_{\Theta,t}(n))\frac{1}{n}\sum_{k=1}^{n}\epsilon^k\end{aligned} \tag{79}$$

where

$$\varphi^l_t(n) \equiv \alpha^l_t(n)\frac{\lambda_{1,t}}{\lambda_{2,t}}d^l_{\Theta,t}(n) + (1-\alpha^l_t(n))d^l_{\Delta,t}(n)$$

and where $\alpha^l_t(n) \equiv \frac{\sigma^2_S}{\sigma^2_S + o^c_t n}$.

As in Brennan and Cao (1997) or He and Wang (1995), I further isolate the part of θ^l that is solely associated with private information. Since, from (71), agent l contributes a fraction $\mu_t(n)d^l_{\Theta,t}(n)$ to the per capita supply shock, I denote by $\widetilde{\theta}^l := \theta^l_t(n) - d^l_{\Theta,t}(n)\Theta_t$, the part of agent l's portfolio that is absent of market-making concerns. From (78), it then follows that

$$\begin{aligned}\widetilde{\theta}^l_t(n) &= \varphi^l_t(n)\left(1 - \int_0^t o^c_s k^2_s e^{-\int_s^t o^c_u k^2_u \mathrm{d}u}\mathrm{d}s\right)\Pi - \varphi^l_t(n)\int_0^t o^c_s k_s e^{-\int_s^t o^c_u k^2_u \mathrm{d}u}\mathrm{d}B^\Theta_s \\ &\quad + (\varphi^l_t(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}}d^l_{\Theta,t}(n))\frac{1}{n}\sum_{k=1}^{n}\epsilon^k.\end{aligned}$$

Further assuming, similar to Watanabe (2008), that agent l remains of type n over Δ and dropping the index n for convenience, the informational portfolio variation over Δ

is given by

$$\Delta\widetilde{\theta}_t^l = \begin{pmatrix} \varphi_{t+\Delta}^l - \varphi_t^l + \left(\varphi_{t+\Delta}^l e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - \varphi_t^l\right) \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s \\ -\varphi_{t+\Delta}^l \int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s \end{pmatrix} \Pi$$
$$- \left(\varphi_{t+\Delta}^l e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - \varphi_t^l\right) \int_0^t e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} o_s^c k_s \mathrm{d}B_s^\Theta - \varphi_{t+\Delta}^l \int_t^{t+\Delta} e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} o_s^c k_s \mathrm{d}B_s^\Theta$$
$$+ \left(\varphi_{t+\Delta}^l - \varphi_t^l - \left(d_{\Theta,t+\Delta}^l \frac{\lambda_{1,t+\Delta}}{\lambda_{2,t+\Delta}} - d_{\Theta,t}^l \frac{\lambda_{1,t}}{\lambda_{2,t}}\right)\right) \frac{1}{n} \sum_{k=1}^n \epsilon^k$$

The econometrician, who can only make sense of $\Delta\widetilde{\theta}^l$ with respect to what she observes, considers $E[\Delta\widetilde{\theta}_t^l|\Delta P_t]$. Applying the projection theorem, one writes

$$E[\Delta\widetilde{\theta}_t^l|\Delta P_t] = \rho_t(\Delta)\Delta P_t \tag{80}$$

with $\rho_t(\Delta) = \frac{\mathrm{cov}(\Delta\widetilde{\theta}_t^l, \Delta P_t)}{\mathrm{var}(\Delta P_t)}$ and where, by Ito isometry and $\{\epsilon^k\}_{k=1}^n \perp \Pi \perp (B_t^\Theta)_{t\geq 0}$,

$$\mathrm{cov}(\Delta\widetilde{\theta}_t^l, \Delta P_t) = \begin{pmatrix} \varphi_{t+\Delta}^l - \varphi_t^l + \left(\varphi_{t+\Delta}^l e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - \varphi_t^l\right) \int_0^t o_s^c k_s^2 e^{-\int_0^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s \\ -\varphi_{t+\Delta}^l \int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s \end{pmatrix}$$
$$\times \begin{pmatrix} \left((1-\lambda_{1,t+\Delta}) e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) \int_0^t e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} o_s^c k_s^2 \mathrm{d}s \\ +\lambda_{1,t+\Delta} - \lambda_{1,t} + (1-\lambda_{1,t+\Delta}) \int_t^{t+\Delta} e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} o_s^c k_s^2 \mathrm{d}s \end{pmatrix} \sigma_\Pi^2$$
$$- \int_0^t \begin{matrix} \left(\varphi_{t+\Delta}^l e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - \varphi_t^l\right) e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} o_s^c k_s \\ \times \begin{pmatrix} \sigma_\Theta(\lambda_{2,t+\Delta} e^{-a_\Theta \Delta} - \lambda_{2,t}) e^{a_\Theta (s-t)} \\ + \left((1-\lambda_{1,t+\Delta}) e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{pmatrix} \end{matrix} \mathrm{d}s$$
$$- \varphi_{t+\Delta}^l \int_t^{t+\Delta} \begin{matrix} e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} o_s^c k_s \\ \times \left(\sigma_\Theta \lambda_{2,t+\Delta} e^{-a_\Theta(s-(t+\Delta))} + (1-\lambda_{1,t+\Delta}) e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} o_s^c k_s\right) \end{matrix} \mathrm{d}s.$$

Hence, as in Brennan and Cao (1997), agent l follows the trend whenever $\rho_t(\Delta) > 0$ and pursues a contrarian strategy whenever $\rho_t(\Delta) < 0$. This is the measure of momentum strategies used in Grinblatt, Titman, and Wermers (1995), for instance. Notice that the measure of trading behavior in (80) is not equivalent to that of Wang (1993), for instantaneous covariances ignore the contribution of private discussions. To see this, observe that prices have continuous sample paths

$$\mathrm{d}P_t = E[\mathrm{d}P_t|\mathcal{F}_t^l] + (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t}) o_t^c k_t) \mathrm{d}\widehat{B}_t^l.$$

Moreover, defining $f_t^l(n) = \frac{A_{Q,t} - B_{Q,t}(B_{\Psi,t}^l(n))^\top M_t^l(n)}{\gamma B_{Q,t}^2}$ along with the jump size of this function

$$\Delta f_t^l(n) = -\frac{(B_{\Psi,t}^l(n+m_t))^\top M_t^l(n+m_t) - (B_{\Psi,t}^l(n))^\top M_t^l(n)}{\gamma B_{Q,t}},$$

an application of Ito-Tanaka's formula yields

$$\mathrm{d}\theta_t^l = E[\mathrm{d}\theta_t^l|\mathcal{F}_t^l] + f_t^l(n_{t\_}^l)B_{\Psi,t}^l(n_{t\_}^l)\mathrm{d}\widehat{B}_t^l + (\Delta f_t^l(n_{t\_}^l)\Psi_{t\_}^l + f_t^l(n_{t\_}^l + m_t)C_{\Psi,t}^l)\mathrm{d}N_t^l.$$

Hence, I get

$$E[\mathrm{d}\theta_t^l \mathrm{d}P_t|\mathcal{F}_t^l] = (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t})o_t^c k_t)\frac{A_{Q,t} - B_{Q,t}(B_{\Psi,t}^l(n_{t\_}^l))^\top M_t^l(n_{t\_}^l)}{\gamma B_{Q,t}^2} B_{\Psi,t}^l(n_{t\_}^l)\mathrm{d}t$$

where the contribution of private meetings gets flushed out.

F.3 Measures of Performance

In the sequel, I derive a fund's NAV and the structure of the regression in (17). In so doing, I consider the performance solely induced by the informational portfolio $\widetilde{\theta}^l$, for one should not give credit to a manager for making money on noise traders.

***i)* Net Asset Value.** One may want to make the following preliminary observation: were agents risk-neutral, their expected trading gains would be null, for prices P would be martingales and, thus, $E[\int_0^T \theta_t^l \mathrm{d}P_t] = 0$ as in Hirshleifer, Subrahmanyam, and Titman (1994). Due to the serial correlation in returns, this result is expected to be changed: using (79), expected gains up to time τ may be expressed as

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = E\left[\int_0^\tau \varphi_t^l(\Pi - \widehat{\Pi}_t^c)A_{Q,t}\Psi_t \mathrm{d}t\right] + E\left[\int_0^\tau \left(\varphi_t^l - \frac{\lambda_{1,t}}{\lambda_{2,t}}d_{\Theta,t}^l\right)\frac{1}{n}\sum_{k=1}^n \epsilon^k \mathrm{d}P_t\right]$$
$$+ E\left[\int_0^\tau \widetilde{\theta}_t^l B_{Q,t}\mathrm{d}B_t^\Theta\right] + \mathbf{1}_{\{\tau=T\}}E[\widetilde{\theta}_{T\_}^l \Delta P_T].$$

where I set $W_0^l = 0$: since agents have CARA utility, this assumption is immaterial. Since $\{\epsilon^k\}_{k=1}^n \perp (\Psi_t)_{t\geq 0}$ and that ϵ^k has zero mean for all k and that the third term inside the expectation is an Ito integral, it follows, under regularity conditions, that

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = E\left[\int_0^\tau \varphi_t^l(\Pi - \widehat{\Pi}_t^c)\left(A_{Q,t}^{(2)}\Theta_t + A_{Q,t}^{(3)}(\Pi - \widehat{\Pi}_t^c)\right)\mathrm{d}t\right] + \mathbf{1}_{\{\tau=T\}}E[\widetilde{\theta}_{T\_}^l \Delta P_T].$$

Moreover, using (51), one may write

$$\widetilde{\theta}_{T\_}^l = \frac{(1-\alpha_{T\_}^l)\frac{1}{n}\sum_{k=1}^n \epsilon^k + (1-\alpha_{T\_}^l - \lambda_{1,T\_})(\Pi - \widehat{\Pi}_{T\_}^c)}{\gamma(o_{T\_}^l + \sigma_\delta^2)}$$

and it follows, applying Fubini's theorem, that

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = \int_0^\tau \varphi_t^l E\left[(\Pi-\widehat{\Pi}_t^c)\left(A_{Q,t}^{(2)}\Theta_t + A_{Q,t}^{(3)}(\Pi-\widehat{\Pi}_t^c)\right)\right]\mathrm{d}t$$
$$+\mathbf{1}_{\{\tau=T\}}\frac{1-\alpha_{T_-}^l}{\gamma(o_{T_-}^l+\sigma_\delta^2)}E\left[(\Pi-\widehat{\Pi}_{T_-}^c)\left((1-\lambda_{1,T_-})(\Pi-\widehat{\Pi}_{T_-}^c)-\lambda_{2,T_-}\Theta_T\right)\right].$$

Substituting (77) and (78), and using Ito isometry along with $\Pi \perp \Theta_0 \perp (B_t^\Theta)_{t\geq 0}$, I get

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = -\sigma_\Theta \int_0^\tau \varphi_t^l A_{Q,t}^{(2)} \int_0^t o_s^c k_s e^{-a_\Theta(t-s)-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\mathrm{d}t$$
$$+\int_0^\tau \varphi_t^l A_{Q,t}^{(3)}\left(\left(1-\int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2\mathrm{d}u}\mathrm{d}s\right)^2\sigma_\Pi^2+\int_0^t (o_s^c k_s)^2 e^{-2\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)\mathrm{d}t$$
$$+\mathbf{1}_{\{\tau=T\}}\frac{1-\alpha_{T_-}^l-\lambda_{1,T_-}}{\gamma(o_{T_-}^l+\sigma_\delta^2)}\times$$
$$\left(\begin{array}{c}(1-\lambda_{1,T_-})\left(\begin{array}{c}\left(1-\int_{[0,T)} o_s^c k_s^2 e^{-\int_{[s,T)} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)^2\sigma_\Pi^2 \\ +\int_{[0,T)}(o_s^c k_s)^2 e^{-2\int_{[s,T)} o_u^c k_u^2\mathrm{d}u}\mathrm{d}s\end{array}\right) \\ +\lambda_{2,T_-}\sigma_\Theta \int_{[0,T)} o_s^c k_s e^{-a_\Theta(T-s)-\int_{[s,T)} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\end{array}\right).$$

***ii)* Performance Regression.** Denoting by $\widehat{\theta}^c = \frac{A_Q^{(1)}\widehat{\Theta}^c}{\gamma B_Q^2}$ the myopic market portfolio and using that $\xi \in \mathcal{F}^c \subset \mathcal{F}^l$, I can write a manager l's myopic portfolio $\widehat{\theta}^l$ as

$$\widehat{\theta}_{t_-}^l = \widehat{\theta}_t^c + \frac{A_{Q,t}^{(2)}-\frac{\lambda_{1,t}}{\lambda_{2,t}}A_{Q,t}^{(1)}}{\gamma B_{Q,t}^2}(\widehat{\Pi}_{t_-}^l-\widehat{\Pi}_t^c).$$

Using (36) and (76), it follows that $A_{Q,t}^{(2)}-\frac{\lambda_{1,t}}{\lambda_{2,t}}A_{Q,t}^{(1)} = B_{Q,t}k_t$ and I get

$$\widehat{\theta}_{t_-}^l = \widehat{\theta}_t^c + \frac{k_t}{\gamma B_{Q,t}}(\widehat{\Pi}_{t_-}^l-\widehat{\Pi}_t^c) = \widehat{\theta}_t^c + \frac{k_t}{\gamma B_{Q,t}}(1-\alpha_t^l)(\bar{S}_t^l-\widehat{\Pi}_t^c)$$

where the last equality follows from (69). Further observing that $\Pi-\widehat{\Pi}_t^c = \frac{1}{\lambda_{1,t}}(P_t - \widehat{\Pi}_t^c - \lambda_{2,t}\Theta_t)$, one obtains (16).

The difference between the returns generated by managers i and market returns follows from observing that

$$\mathrm{d}R_t^i := \widetilde{\theta}_t^i \mathrm{d}P_t = d_{\Delta,t}^i(\Pi-\widehat{\Pi}_t^c)(A_{Q,t}^{(1)}\Theta_t\mathrm{d}t + (A_{Q,t}^{(2)}-B_{Q,t}k_t)(\Pi-\widehat{\Pi}_t^c)\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^c)$$

and

$$\mathrm{d}R_t^c := \widetilde{\theta}_t^c \mathrm{d}P_t = d_{\Theta,t}^c\frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi-\widehat{\Pi}_t^c)\left(A_{Q,t}^{(1)}\left(\Theta_t+\frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi-\widehat{\Pi}_t^c)\right)\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^c\right).$$

G Description of the Calibration

This appendix provides further comments on the calibration of Subsection 3.3.2, which is used in the results sections.

I set a_Θ above 0.05, as estimated by Campbell and Kyle (1993), for this parameter value would make the supply counterfactually persistent. The chosen calibration matches that used in Huang and Wang (1997) and Hong and Wang (2000). The volatility of the supply σ_Θ is consistent with that estimated by Campbell and Kyle (1993) who show that this parameter would be related to risk aversion in equilibrium. Inspection of Section 3 in Campbell and Kyle (1993) shows that Θ is equivalent to X in their setting. This further implies that σ_Θ is of the form $\frac{1-\Phi_2\eta\sigma_M\sigma_N}{\psi r\sigma_M^2}\text{diff}(\mathrm{d}N_t)$. After correcting a mistake in the first and third equations of B.12 and B.7, respectively, and substituting the estimates of Table 8 indicates that my calibration is obtained for a risk aversion of 8. This parameter lies within the range used by Wang (1993) and is below that used by Hong and Wang (2000). I choose to set the risk aversion parameter well below 8 to be consistent with Koijen (2012) who structurally estimates fund managers' relative risk aversion and reports a risk aversion of 5.51 and stock holdings of \$ 93 millions on average.

The volatility of ideas σ_S is chosen to be higher than σ_Π and both are set to the calibration of He and Wang (1995). A similar estimate for σ_Π is obtained in Banerjee (2010). The volatility of ideas is chosen so as to reflect that working ideas are diffuse but not too imprecise. Notice that respectively increasing γ, σ_Θ, σ_S and σ_Π or decreasing a_Θ would only make my results stronger.

To fix ideas regarding the chosen network thresholds, suppose one shuts down the price-learning channel so that $o_t^c \equiv \sigma_\Pi^2$. Then, at the chosen value for N, the marginal effect $\frac{1}{\sigma_\Pi^2}\frac{\partial}{\partial n}o_t^l(n)\Big|_{n=N} = -\frac{\sigma_\Pi^2\sigma_S^2}{(\sigma_S^2+\sigma_\Pi^2 n)^2}$ of an additional idea is less than -2%. In other words, the contribution of an additional idea becomes negligible: social interactions have mostly exhausted their informational role. Since learning from prices will make this contribution even weaker, N is a natural level for perfect knowledge to step in.

To further make sense of the chosen K, suppose an enforcement technology exists so that agents optimally pre-commit to switch after getting K ideas or more at a cost β. Assuming away time-inconsistency issues, they would compare the value $V_T^A = \sum_{n\in A}\mu_T^A(n)J^l(n,T)$ of Network A with that $e^{\gamma\beta}V_T^B = e^{\gamma\beta}\sum_{n\in B\cup\{\infty\}}\mu_T^B(n)J^l(n,T)$ of Network B at time T. The value K would then be optimal if $\beta = \frac{1}{\gamma}\log\left(\frac{E[V_T^A|\mathcal{F}_0^l]}{E[V_T^B|\mathcal{F}_0^l]}\right)$. In turn, an interaction intensity of one meeting twice per year ($\eta = 2$), every quarter ($\eta = 4$) and every two months ($\eta = 6$) imply a cost β of 0.34, 0.44 and 0.438, respectively.

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