Asymmetric Information and Inventory Concerns in Over-the-Counter Markets*

Julien Cujean†
R.H. Smith School of Business
University of Maryland

Rémy Praz‡
Swiss Finance Institute
EPFL

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Comments welcome

Abstract

We study how transparency, modeled as information about one's counterparty liquidity needs, affects the functioning of an over-the-counter market. In our model, investors hedge endowment risk by trading bilaterally in a search-and-matching environment. We construct a bargaining procedure that accommodates information asymmetry regarding investors' inventories. Both the trade size and the trade price are endogenously determined. Increased transparency improves the allocative efficiency of the market. However, it simultaneously increases inventory costs, and leads to a higher cross-sectional dispersion of transaction prices. For investors with large risk exposure, the increase of the inventory costs dominates the benefits of the market efficiency. We link the model's predictions to recent empirical findings regarding the effect of the TRACE reporting system on bond market liquidity.

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†Robert H. Smith School of Business, University of Maryland, 4466 Van Munching Hall, College Park, MD 20742, USA; +1 (301) 405 7707; www.juliencujean.com; jcujean@rhsmith.umd.edu .

‡Swiss Finance Institute at EPFL, Extranef #128, 1015 Lausanne, Switzerland Switzerland; +41 76 406 48 09; http://people.epfl.ch/remy.praz; remy.praz@epfl.ch.

1 Introduction

A common concern about over-the-counter (OTC) markets is their opaqueness—investors transact, often unaware of prices available from other counterparties and with little knowledge of trades negotiated recently.[1] Given the important role that OTC markets played in the global financial crisis, many regulators have attempted to shed some light on those so-called dark markets.[2] Perhaps the most notable reform aiming at an increased transparency was the Dodd-Frank Act.[3]

There are costs and benefits associated with an increased transparency. For instance, post-trade transparency—the availability of past transaction data—may lead to a more efficient asset allocation. It may, however, expose dealers to predatory behaviors and significantly reduce their incentives to take on inventory risk and provide liquidity. An important question is therefore to whom the costs and benefits of transparency accrue?

To address this question, we develop a general equilibrium model of an OTC market in which investors trade an asset to share endowment risk. Trading the asset first requires finding a counter-party, and we follow the search-and-matching approach developed by Duffie, Garleanu, and Pedersen (2005, 2007). Upon matching, agents bargain on the conditions of the transaction. To model transparency, we introduce *information asymmetry* among traders. We define *transparency* as a traders' ability to get information about their counterparties' inventory. Information asymmetry creates an adverse selection problem that makes it more difficult to execute large trades.[4] This aspect allows us to determine how regulatory requirements (e.g., TRACE) make inventories riskier.

In the presence of asymmetric information, the usual Nash bargaining solution characterizing bilateral trades is inadequate. We select an alternative bargaining protocol, which resembles the real-world mechanism used in the dealership market. Namely, one agent (say agent 1) posts a quote and the other (say agent 2) decides how many shares to buy or sell at that price.[5] We assume that, before posting the quote, agent 1 receives a signal about the inventory of agent 2. We view this signal as an attempt of agent 1 to extract information about agent 2's liquidity needs from past trading data, a natural outcome of post-trade transparency. The quality of this signal

[1] For a reference, see Duffie (2012).

[2] For example, the Financial Stability Board names transparency and the public dissemination of trade data as a main objective of introducing trade repositories for OTC derivatives trading. See, for instance, Board (2013) for more detail.

[3] The corresponding regulation for the US Bond market, the *Trade Reporting And Compliance Engine* (TRACE) exists since 2002. The corresponding European reform is the Market in Financial Instruments Directive (MiFID II).

[4] A quote request is usually followed by both a bid and an ask price being quoted. In this case, both prices would probably be tilted to disguise the attempted rip-off.

[5] Quotes on bond markets do not usually depend on the quantity exchanged. See, Li and Schüerhoff (2012).

precisely captures the notion of transparency we pursue: the more detailed trading data is available, the better the information about inventory concerns of any given trader is.[6]

Information asymmetry significantly complicates the analysis of the model. Our model, however, remains very tractable. We first solve for an investor's optimal trading strategies taking the cross-sectional distribution of inventories as given. Second, we endogenize this cross-sectional distribution. In particular, optimal trading strategies define the inventory dynamics, which, in turn, determine the cross-sectional distribution of inventories. We demonstrate that optimal strategies are linear in the inventory and signal of investors, making it possible to solve for a stationary equilibrium in closed form.

We show that an increase in transparency has 3 main implications for inventory costs and several dimensions of liquidity.[7] First, we show that transparency always increases inventory costs. This happens via two different channels. On the one hand, transparency exposes any given investor with large inventories to predatory pricing. On the other hand, as transparency improves the allocative efficiency of the market, it becomes more difficult for an investor with excessive exposure to find a counter-party with large and opposite liquidity needs. This second effect exacerbates the first one and is driven by the endogenous distribution of inventories.

Second, we get the more intuitive result that increased transparency always leads to a more efficient allocation of the asset, leading to less dispersion in inventory risk across the population of investors. As a consequence, we show that the cross-sectional variance of the trade sizes at any given moment is monotone decreasing in the degree of transparency of the OTC market.[8]

These 2 implications together imply that the effect of transparency on investors' value function is ambiguous. On average, investors benefit from an increased transparency and the resulting improvement in the allocative efficiency. In particular, transparency improves welfare. However, those investors with a sufficiently large (long or short) exposure find it increasingly costly to liquidate their position and would benefit from a more opaque market. We obtain an explicit expression for the exposure levels starting from which investors prefer opacity to transparency. This result is in line with the heterogeneous reactions to the introduction of TRACE, with

[6]The effects of post-trade transparency on inventory risk are particularly strong is markets with moderate/slow trading activity. In this case, even anonymized post-trade transparency can make it possible to infer traders' identities from post-trade data.

[7]We define the inventory costs as the reservation value of an investor who deviates from a zero risk exposure.

[8]A more natural measure of the trade sizes would be the average size. This is, actually, the average across all the trades of the *absolute value* of the quantity exchanged. Due to technical difficulties, we cannot characterize this quantity analytically.

negative reactions on the part of many institutional investors.[9] As our model predicts, transparency is detrimental to agents facing large exposures.

Finally, we show that the price dispersion—the cross-sectional variance of the transaction prices at a given moment—on the OTC market is increasing in the transparency of that market. Two opposite channels operate to generate this result. First, investors being risk averse, the price of a transaction depends on investors' reservation values for trading. As transparency tends to make risk sharing more efficient and to reduce the dispersion of the reservation values, there is a first channel whereby transparency reduces the cross-sectional dispersion of prices. Second, transparency simultaneously increases inventory costs, and the price of a given transaction will therefore drift away from the competitive price when transparency increases. In equilibrium, this second effect dominates, and the dispersion of prices increases with the transparency of the market.

Literature review Our model builds on the literature modeling OTC markets. This literature started, to a large extent, with Duffie et al. (2005) and Duffie et al. (2007). The bilateral trades in these models are characterized by the Nash bargaining solution and, as a result, not naturally suited to accommodate information asymmetry, inter-dealer market but no inventories. Furthermore, the transaction size is exogenously fixed, which prevents a discussion of the different costs and benefits to agents with moderate and large liquidity needs.

An alternative strand of literature considers the equilibrium effect of an intermittent, and sometimes costly, access to a centralized market. This literature started with Lagos and Rocheteau (2007) and Lagos and Rocheteau (2009). These models allow for portfolio decisions, but the inter-dealer is assumed to be competitive and dealers do not keep any inventories.

Our model is also related to classical references on inventory risk such as Ho and Stoll (1980) and Ho and Stoll (1981). These references do not consider, however, the feedback effect of the intermediation on the liquidity needs of the investors. This equilibrium effect is at the core of our analysis.

The explicit bargaining procedure that we devise means that our model is also related to Samuelson (1984), Grossman and Perry (1986), Mailath and Postlewaite (1990). In these references, just like in the classical references on inventory risk, there is no feedback effect of the quoting strategy on the distribution of valuations.

References such as Blouin and Serrano (2001), Duffie and Manso (2007), Duffie et al. (2009), and , Duffie et al. (2010) consider asymmetric information in decentralized markets. However, these references focus on common value asymmetry whereas we

[9] See, for instance, Decker (2007) and Bessembinder and Maxwell (2008).

analyze a setting with private value asymmetric information. Also the references do not consider portfolio decisions.

To obtain the equilibrium expressions in closed-form, we assume that agents are only risk-averse with respect to certain risks. The same procedure was used, for instance, by Biais (1993), Duffie et al. (2007), Vayanos and Weill (2008), Gârleanu (2009). Other references using "source-dependent" risk-aversions include Hugonnier et al. (2013) and Skiadas (2013).

Finally, in terms of formalism, the interaction between the distribution of types, the individual policies, and the value functions, means that our model is related to the literature on mean-field games, as introduced by Lasry and Lions (2007).

The outline of the paper is as follows. Section 2 describes the assets, investors, and other exogenous elements of our model. Section 3 solves for the optimal policy of an individual. Section 4 maps a certain trading pattern on the OTC market to a consistent cross-sectional distributional distribution of types. Section 5 solves for the equilibrium of the model and discusses its properties. Section 6 concludes.

2 Model

In this section, we present the various exogenous elements of our model economy.

Assets and Investors

In our model, investors trade bilaterally to share risks. Our model is based on Lo et al. (2004), from whom we borrow the specification of the trade motives, and on Duffie et al. (2005), from whom we borrow the meeting technology on the OTC market. The exact bargaining procedure that defines the trade details is original.

There are two assets. First, there is a risk-free bond freely traded and whose rate of return r is exogenously given. Second, there is a risky asset ("the stock") whose cumulated payouts

$$(D_t)_{t\geq 0}$$

is an arithmetic Brownian motion. Namely,

$$\mathrm{d}D_t = m_d \,\mathrm{d}t + \sigma_d \mathrm{d}B_t,\ t \geq 0,$$

with μ and σ being two constants and $(B_t)_{t\geq 0}$ being a Brownian motion.

The economy is populated by a normalized continuum of investors. The investors trade the stock for risk-sharing motives. Namely, each investor a receives an endowment

$$(\eta_t^a)_{t\geq 0}$$

whose dynamics are given by

$$\begin{cases} \mathrm{d}\eta_t^a &= Z_t^a \mathrm{d}D_t \\ \mathrm{d}Z_t^a &= \sigma_a \mathrm{d}B_t^a \end{cases},$$

with $(B_t^a)_{t\geq 0}$ being an "idiosyncratic" Brownian motion independent from the one driving the dividends of the stock. By *idiosyncratic* we mean that there is one such process per investor and that these processes are sufficiently independent for a version of the Strong Law of Large Numbers (SLLN) to hold cross-sectionally.

To sum up, the same aggregate risk factor drives the payouts of the stock and the endowment of the agents over the short-term. However, the level to which an endowment is exposed to this aggregate risk factor evolves in an idiosyncratic way.

Trading

Illiquidity on an OTC market materializes in that a counter-party is only infrequently available, does not necessarily want to trade the right quantity, and not necessarily at the right price. We capture the first of these aspects by assuming that an investor can only contact a counter-party at the jump times of a Poisson process with intensity λ. The counter-party who is contacted is drawn uniformly from across the population.

Once two investors are in contact, they must evaluate whether or not they wish to trade the stock and, if so, what the exact terms of the transaction should be.

On actual OTC markets, a common procedure to arrange a deal with a dealer is to ask the dealer for a quote and, assuming that the quote is deemed good enough, indicate how much of the asset one would like to either buy or sell. The quote usually consists of both a bid and an ask price.

For the trading in our model, we assume a stylized version of the previous procedure. Namely, once two investors are in contact, one of them quotes a binding price and the other one chooses a quantity to be exchanged at the quoted price. The exact bargaining procedure follows.

Bargaining 1:

Assume that the investors a (like "asks") just contacted the investor q (like "quotes"). These investors are identified with their "types" z_a, z_q, respectively. The distribution of the types across the population is μ.

(i) a asks q for a quote.

(ii) if *q finds it optimal to quote a price* **then**

1. q receives a signal s_a regarding the type of a. Namely,

$$s_a = Xz_a + (1 - X)\zeta, \tag{1}$$

   with $X \sim B(1, \tau)$ and $\zeta \sim \mu$ being independent of each other and of both z_a and z_q. Intuitively, the signal is exact with a probability τ which we call the *transparency* of the OTC market. In the other case, the signal is a pure noise.

2. q quotes a price $p = p(q, s_a)$ at which she is willing to trade with a

3. a chooses which quantity $q(a, p)$ she would like to buy (if $q \geq 0$) or sell (if $q < 0$).

4. q gives q units of the stock to a, a pays the amount $pq(a, p)$ to q

end

(iii) the two investors part ways.

Two comments are in order. First, when quoting the price p, the investor q may not be fully aware of the characteristics of a. This uncertainty regarding the valuation of one's counter-party is the type of opacity that our model captures.

Second, we assume that q quotes a unique price instead of both a bid and an ask price, as actual dealers would do. This assumption is made for the sake of tractability. This, said, as long as q has an accurate enough guess of a's valuation for the asset, q knows with some confidence whether the trade is going to be a buy or a sell. In particular, even if both a bid and an ask prices are quotes, only one of them is truly relevant.

Preferences

The investors maximizes their expected utility from consumption and have a utility function with constant absolute risk-aversion (CARA, or exponential, utility). Namely, an investor a solves the individual optimization problem

$$\sup_{(\tilde{c}_t)_{t\geq 0}} \mathrm{E}\left[\int_0^\infty e^{-\rho t} U\left(\tilde{c}_t\right)\, \mathrm{d}t\right] \tag{ip}$$

with the utility function

$$U(x) \triangleq -e^{-\gamma x}$$

over the "admissible" consumption policies. The constant $\gamma > 0$ is the coefficient of absolute risk-aversion. A consumption policy $(\tilde{c}_t)_{t\geq 0}$ is *admissible* if it satisfies two conditions.

1. The consumption policy can be financed, meaning that it satisfies the budget constraint

$$\mathrm{d}\tilde{w}_t^a = r\tilde{w}_t\mathrm{d}t - \tilde{c}_t\mathrm{d}t + \mathrm{d}\eta_t^a + \tilde{\theta}_t\mathrm{d}D_t - \tilde{P}_d\mathrm{d}\tilde{\theta}_t,\ t \geq 0. \tag{bc}$$

   In this last expression, the quantities with a tilde $(\tilde{\cdot})$ are endogenously chosen, whereas the other ones are fully exogenous. The interpretations of the endogenous quantities are as follows. $\tilde{w}_t^a$ denotes the amount invested in the bond and $\tilde{\theta}_t$ the number of stock shares held. The holdings $\tilde{\theta}_t$ can only be adjusted when another investor is met and, during such a meeting, both the change in holdings $\mathrm{d}\tilde{\theta}_t$ and the payment $\tilde{P}_d\ \mathrm{d}\tilde{\theta}_t$ are defined by the bargaining procedure described in Table 1. That is, $\tilde{P}_d$ is not a unique price in that it is contingent on the types of agents involved in a particular meeting.

2. The wealth process $(\tilde{w}_t)_{t\geq 0}$ satisfies the transversality condition

$$\lim_{T\to\infty} \mathrm{E}\left[e^{-r\gamma\tilde{w}_T}\right] = 0. \tag{tc}$$

   This regularity condition forbids the "financing" of consumption by an ever increasing amount of debt.

An agent is exposed to risky cash-flows both via her endowment and via her stock holdings. However, both of these exposures are driven by the same risk factor. For convenience and ease of interpretation, we thus define the *actual exposure* of the investor a as

$$z_t^a \triangleq Z_t^a + \theta_t$$

and rewrite the budget constraint as

$$\mathrm{d}\tilde{w}_t^a = r\tilde{w}_t\mathrm{d}t - \tilde{c}_t\mathrm{d}t + z_t^a\mathrm{d}D_t - P_d\mathrm{d}\theta_t.$$

The actual exposure follows a jump diffusion,

$$\mathrm{d}z_t^a = \ \mathrm{d}\theta_t + \sigma^a\ \mathrm{d}B_t^a, \tag{2}$$

with the jump part stemming from the trading and the diffusion part stemming form the idiosyncratic variation of the endowment exposure.

The actual exposure of an investor will define both her bargaining behavior and her value function. Consequently, the actual exposure of an investor is also referred to

as the *type* of an investor. Also, during the bargaining procedure, the signal received by the quoter is about the type of her counter-party.

3 Individual Problem

A given investor takes as given the aggregate quantities of the model and chooses her consumption and bargaining policies to solve her individual problem (ip). Aggregating these individual "best responses" yields new aggregate quantities. When we will solve for an equilibrium, we will solve for a fixed point over these aggregate quantities.

In this section, we solve for the individual policies of the agents and do so by the dynamic programming approach, meaning by solving a Hamilton-Jacobi-Bellman (HJB) equation.

First, we define the *value function* at time t of an investor a by

$$V(t,w,z) \triangleq \sup_{(\tilde{c}_s)_{s\geq t}} \mathrm{E}\left[\int_t^\infty e^{-\rho(s-t)}U\left(\tilde{c}_s\right)\,\mathrm{d}s \middle| w_t^a = w, z_t^a = z\right],$$

with w_t^a standing for a's wealth at time t and z_t^a standing for a's type at time t. Let us take as given an optimal consumption policy $(c_t)_{t\geq 0}$ and make two assumptions. First, the environment is stationary, meaning that the beliefs regarding the aggregate quantities are constant over time. Second, in terms of expected utility, an agent is fully described by her current wealth w_t^a and current type z_t^a.[10] Then, one can write

$$\left(\int_0^t e^{-\rho s}U\left(c_s\right)\,\mathrm{d}s + e^{-\rho t}V\left(w_t,z_t\right)\right)_{t\geq 0} = \left(\mathrm{E}\left[\int_0^\infty e^{-\rho s}U\left(c_s\right)\,\mathrm{d}s \middle| \mathcal{F}_t^a\right]\right)_{t\geq 0}, \quad (3)$$

with $\mathcal{F}_t^a$ standing for the information available to a at time t. We left out the time as an argument of the value function because of the stationarity assumption.

As the process on the right-hand side of (3) is a martingale, so is the one on the left-hand side, and its expected rate of change must be zero. Now, assuming that the value function is regular enough for Itô's lemma for jump-diffusions to hold, the expected rate of change of the process on the left-hand side is

$$\begin{aligned}
&\frac{1}{\mathrm{d}t}\mathrm{E}\left[\,\mathrm{d}\left(\int_0^t e^{-\rho s}U\left(c_s\right)\,\mathrm{d}s + e^{-\rho t}V\left(w_t,z_t\right)\right)\right] \\
&= e^{-\rho t}\left(\begin{array}{l}
U\left(c_s\right) - \rho V(w,z) + V_w(w,z)\left(rw - c + zm_d\right) \\
+\frac{1}{2}(V_{ww}(w,z)z^2\sigma^2 + V_{zz}(w,z)\sigma_z^2) \\
+\lambda\,\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\begin{pmatrix} V\left(w - qP\left(z_q,s_z\right),z+q\right) \\ -V\left(w,z\right)\end{pmatrix}\right] \\
+\lambda\left[\begin{array}{c}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[V\left(w + Q\left(z_a,p\right)p, z - Q\left(z_a,p\right)\right)\right] \\ -V\left(w,z\right)\end{array}\right]^+
\end{array}\right).
\end{aligned} \quad (4)$$

[10] These assumptions will be justified ex-post.

On the right-hand side, the first line corresponds to the utility from current consumption and the drift term of the value function. On this line, the consumption rate c_t is chosen by the investor.

The second line corresponds to the diffusion of the value function. There is no choice variable on this line.

The third line corresponds to the jump resulting from asking another investor for a quote. Namely, z_q is the type of the investor who was contacted and is drawn from the cross-sectional distribution of types μ. This distribution μ is taken as given by the individual investors. On this same line, s_z is the signal received by the potential quoter. In line with the definition (1),

$$s_z = X_z z + (1 - X_z)\zeta_z,$$

with the two random variables $X_z \sim B(1, \tau)$ and $\zeta_z \sim \mu$ being independent of each other and of all the other random quantities. The set A appearing in the indicator function represents the types of the investors who are ready to offer a quote. This set is also taken as given. The function

$$\begin{array}{rcl} A \times \mathbb{R} & \to & \mathbb{R} \\ (z_q, s_z) & \mapsto & P\left(z_q, s_z\right) \end{array}$$

represents the quoting strategy adopted by the other investors, and is also taken as given. On this third line, the purchase q is chosen by the investor after observing the quote $P(z_q, s_z)$.

The fourth line corresponds to the jump resulting from receiving a quote request. Namely, z_a is the type of the investor who asked for a quote and is also distributed according to μ. The signal regarding z_a, s_a is given by

$$s_a = X_a z_a + (1 - X_a)\zeta_a,$$

with $X_a \sim B(1, \tau)$, $\zeta_a \sim \mu$, and the same independence assumptions as above. The function

$$\begin{array}{rcl} \mathbb{R}^2 & \to & \mathbb{R} \\ (z_a, p) & \mapsto & Q\left(z_a, p\right) \end{array}$$

represents the purchase strategy adopted by the other investors, and is also taken as given. In this line, the quote p is chosen by the investor. The positive part in the expectation ("$[\cdot]^+$") represents the optimal decision of quoting or not.

Combining the martingale property of the processes in (3), the expected dynamics in (4), and the intuition that the optimal policy should be locally given by a maximization of these expected dynamics over the choice variables, we derive the HJB equation for

the individual problem (ip). Namely,

$$\begin{aligned}\rho V(w,z) = &\sup_{\tilde{c}} \left\{U\left(\tilde{c}_s\right) - V_w(w,z)\tilde{c}\right\} \\ &+ V_w(w,z)\left(rw + zm_d\right) \\ &+ \frac{1}{2}\left(V_{ww}(w,z)z^2\sigma^2 + V_{zz}(w,z)\sigma_z^2\right) \\ &+ \lambda \, \mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q \in A\}}\begin{pmatrix}\sup_{\tilde{q}} V\left(w - \tilde{q}P\left(z_q,s_z\right), z + \tilde{q}\right) \\ -V\left(w,z\right)\end{pmatrix}\right] \\ &+ \lambda \left[\begin{array}{c}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\sup_{\tilde{p}} \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[V\left(w + Q\left(z_a,\tilde{p}\right)\tilde{p}, z - Q\left(z_a,\tilde{p}\right)\right)\middle| s_a\right]\right] \\ -V\left(w,z\right)\end{array}\right]^+,\end{aligned} \tag{5}$$

with the random variables z_q, s_z, z_a, s_a, and set A satisfying the same distributional assumptions as above.

To analyze the HJB equation (5), we proceed in two steps. First, we assume a certain functional form ("Ansatz") for the solution to (5). Then, for tractability reasons, we focus on a certain asymptotic case.

Assumption 1. *The value function can be written as*

$$V(w,z) = -\exp\{-\alpha\left(w + v(z) + \bar{v}\right)\},$$

with $\alpha > 0$, $v \in \mathcal{C}^2$ *and* $\bar{v} \in \mathbb{R}$.

Such a functional form is common in models of consumption-portfolio choice with CARA investors. See, among many others, Wang (1994), Duffie et al. (2007), or Gârleanu (2009). Further, in an asymptotic case, this assumption will be justified ex-post by an explicit solution.

Note that the function $v(\cdot)$ and the constant $\bar{v}$ cannot be identified independently that $\bar{v}$ is only introduced for convenience.

With Assumption 1, the various derivatives appearing in (5) are all proportional to the value function itself. Namely,

$$\begin{aligned} V_w(w,z) &= (-\alpha)V(w,z) \\ V_{ww}(w,z) &= \alpha^2 V(w,z) \\ V_z(w,z) &= (-\alpha v'(z))\, V(w,z) \\ V_{zz}(w,z) &= \left((-\alpha v'(z))^2 - \alpha v''(z)\right) V(w,z). \end{aligned} \tag{6}$$

This homogeneity of the problem will simplify its treatment. First, we can characterize the optimal consumption.

Lemma 2. *The optimal consumption in the HJB equation (5) is*

$$c = \frac{\alpha}{\gamma}\left(w + v(z) + \bar{v}\right) - \frac{1}{\gamma}\log\left(\frac{\alpha}{\gamma}\right).$$

It corresponds to a utility

$$U(c) = \frac{\alpha}{\gamma}V(w,z),$$

which is thus proportional to the value function.

Proof. The proofs are in Appendix C.

Injecting the expressions for the derivatives of the value function in (6) and the optimal consumption stated in Lemma 2 into the HJB equation (5), and simplifying by $V(w,z)$, which is negative, yields

$$\begin{aligned}
\rho =& \frac{\alpha}{\gamma} + \alpha\left(\frac{\alpha}{\gamma}\left(w + v(z) + \bar{v}\right) - \frac{1}{\gamma}\log\left(\frac{\alpha}{\gamma}\right)\right) \\
&- \alpha\left(rw + zm_d\right) \\
&+ \frac{1}{2}\left(\alpha^2 z^2\sigma^2 + \left(\left(-\alpha v'(z)\right)^2 - \alpha v''(z)\right)\sigma_z^2\right) \\
&+ \lambda\,\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\left(\inf_{\tilde{q}}\frac{V\left(w - \tilde{q}P_d\left(z_q,s_z\right), z+\tilde{q}\right)}{V(w,z)} - 1\right)\right] \\
&+ \lambda\left[\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\inf_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\frac{V\left(w + Q\left(z_a,\tilde{p}\right)\tilde{p}, z - Q\left(z_a,\tilde{p}\right)\right)}{V(w,z)}\middle| s_a\right]\right] - 1\right]^+.
\end{aligned} \tag{7}$$

As this equation must hold for any wealth w and as, by Assumption 1, $\alpha > 0$,

$$\begin{aligned}
& \alpha\left(\frac{\alpha}{\gamma} - r\right)w = 0 \\
\Leftrightarrow \quad & \alpha = r\gamma.
\end{aligned} \tag{8}$$

Now, injecting (8) into (7), choosing

$$\bar{v} \triangleq \frac{1}{r\gamma}\left(\frac{\rho}{r} - 1 + \log(r)\right)$$

to get rid of the constant terms, and normalizing by $-\alpha = -r\gamma$, yields

$$\begin{aligned}
rv(z) =& zm_d - \frac{1}{2}\left(r\gamma z^2\sigma^2 + \left(r\gamma\left(v'(z)\right)^2 - v''(z)\right)\sigma_z^2\right) \\
&+ \frac{\lambda}{-r\gamma}\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\left(\inf_{\tilde{q}} e^{-r\gamma(-\tilde{q}P_d(z_q,s_z)+v(z+\tilde{q})-v(z))} - 1\right)\right] \\
&+ \frac{\lambda}{-r\gamma}\left[\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\inf_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[e^{-r\gamma(Q(z_a,\tilde{p})\tilde{p}+v(z-Q(z_a,\tilde{p}))-v(z))}\middle| s_a\right]\right] - 1\right]^+.
\end{aligned}$$

Rearranging the terms representing the trading now yields

$$\begin{aligned} rv(z) =& zm_d - \frac{1}{2}\left(r\gamma z^2\sigma^2 + \left(r\gamma\left(v'(z)\right)^2 - v''(z)\right)\sigma_z^2\right) \\ &+ \lambda\,\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\sup_{\tilde{q}}\frac{1-e^{-r\gamma(-\tilde{q}P_d(z_q,s_z)+v(z+\tilde{q})-v(z))}}{r\gamma}\right] \\ &+ \lambda\left[\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\sup_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\frac{1-e^{-r\gamma(Q(z_a,\tilde{p})\tilde{p}+v(z-Q(z_a,\tilde{p}))-v(z))}}{r\gamma}\middle| s_a\right]\right]\right]^+. \end{aligned} \tag{9}$$

In this last equation (9), the first line of the right-hand side balances the instantaneous benefits (expected payouts) and costs (variance of payouts scaled by the risk-aversion) of having a certain exposure z to the aggregate risk factor. The second and third line corresponds to the benefits induced by the possibility to adjust one's exposure by trading the stock.

Analyzing equation (9) is difficult because of the non-linearity of the terms related to trading, those on the second and third line. This non-linearity itself stems from the risk-aversion toward the non-fundamental risks.

For the sake of tractability, and following arguments in Duffie et al. (2007), Vayanos and Weill (2008), and, to mention two particularly transparent examples, Biais (1993) and Gârleanu (2009), we will make the investors risk-neutral with respect to the trading risks while maintaining the risk-aversion with respect to the fundamental risk.

The proper way to achieve this "focused risk-aversion" is to let the risk-aversion coefficient go to zero,

$$\gamma \to 0,$$

while scaling up the fundamental aggregate risk

$$\sigma \sim \frac{1}{\sqrt{\gamma}} \overset{\gamma\to 0}{\longrightarrow} +\infty.$$

As a result, the "quantity" of fundamental risk contained in any stock holding is maintained, but any other type of risk-aversion vanishes. The next assumption formalizes the asymptotic case that we will characterize explicitly.

Assumption 3. *The volatility of the dividends is inversely proportional to the risk-aversion of the investors. Namely,*

$$\sigma \triangleq \frac{1}{\sqrt{\gamma}}\bar{\sigma},$$

with $\bar{\sigma} > 0$. *Further, we assume that*

$$v(z) = v_0(z) + \mathcal{O}(\gamma),$$

implicitly assuming that a solution $v(\cdot)$ to (9) exists for any sufficiently small value of γ.

As it turns out, focusing on the asymptotic behavior of the model significantly simplifies the analysis. Combining the HJB equation (9) and Assumption 5 characterizes the asymptotic function $v_0(\cdot)$ as a solution to

$$\begin{aligned} rv_0(z) =& zm_d - \frac{1}{2}\left(r\gamma z^2\sigma^2 - v_0''(z)\sigma_z^2\right) \\ &+ \lambda \mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\sup_{\tilde{q}}\left(-\tilde{q}P_d\left(z_q,s_z\right)+v_0\left(z+\tilde{q}\right)-v_0(z)\right)\right] \\ &+ \lambda\left[\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\sup_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[Q\left(z_a,\tilde{p}\right)\tilde{p}+v_0\left(z-Q\left(z_a,\tilde{p}\right)\right)-v_0(z)\middle|\, s_a\right]\right]\right]^+. \end{aligned} \tag{10}$$

We can characterize $v_0(\cdot)$ explicitly and, following the examples of Biais (1993), Duffie et al. (2007), Vayanos and Weill (2008), and Gârleanu (2009), we will focus on the analysis of the asymptotic, but much more tractable, value function $v_0(\cdot)$ instead of the general $v(\cdot)$. For convenience, we will thus abuse our notations and, from now on, write $v(\cdot)$ for $v_0(\cdot)$.

At this stage, we would like to state more formally what we are searching for.

The equation (10) relies on beliefs regarding two types of aggregate quantities. These aggregate quantities are, first, the cross-sectional distribution of types μ and, second, the quoting and purchasing policies $P(\cdot,\cdot)$ and $Q(\cdot,\cdot)$ adopted by the other investors. The combination of these beliefs and the equation (5) define new, individually optimal, policies. Then, the aggregation of these individually optimal policy defines a certain type dynamics. Finally, a certain stationary distribution of types results from the type dynamics.

We want to solve for a rational expectations equilibrium of the model, meaning that we want the beliefs and the actual quantities to be consistent. We solve for such a rational expectations equilibrium in two steps. First, we take the type distribution as exogenous and ensure the rationality of the beliefs regarding the quoting and purchasing policies. We call such a solution, conditional on the type distribution, a "partial equilibrium" of the model. Then, we will ensure the rationality of the beliefs regarding the type distribution, and this will define an "equilibrium" of the model.

The formal definition of a partial equilibrium follows.

Definition 4 (Partial Equilibrium)**.** *Let a cross-sectional distribution of types μ be given. Then, a partial equilibrium of the model consists of a triplet of functions and a set $A \subset \mathbb{R}^2$. The three functions are*

$$z \mapsto v(z)$$

that describes how the value function of an investor depends on her type,

$$P : (z, s) \mapsto P(z, s),$$

that describes the quote provided by an investor of type z after receiving the signal s, and

$$Q : (z, p) \mapsto V(w, z)$$

that describes the number of shares purchased by an investor of type z after receiving a quote p.

The partial equilibrium quantities must be a "best-response" to themselves. Namely,

1. *$v(\cdot)$ satisfies the HJB equation (9), given A, $Q(\cdot,\cdot)$, and $P(\cdot,\cdot)$;*
2. *the purchasing policy $Q(\cdot,\cdot)$ satisfies*

$$Q(z,p) \in \arg\max_{\tilde{q}} \left(-\tilde{q} P_d\left(z_q, s_z\right) + v\left(z + \tilde{q}\right) - v(z)\right),$$

   meaning that it is optimal, given $v(\cdot)$;

3. *the set A satisfies*

$$A = \left\{ z : \mathrm{E}^{\mathcal{L}(z_a, s_a)} \left[\sup_{\tilde{p}} \mathrm{E}^{\mathcal{L}(z_a, s_a)} \left[Q\left(z_a, \tilde{p}\right) \tilde{p} + v\left(z - Q\left(z_a, \tilde{p}\right)\right) \middle| s_a \right] \right] \geq v(z) \right\},$$

   meaning that it contains the types of the investor who are willing to issue a quote, given $Q(\cdot,\cdot)$ and $v(\cdot)$;

4. *the quoting policy $P(\cdot,\cdot)$ satisfies*

$$P(z, s_a) \in \arg\max_{\tilde{p}} \mathrm{E}^{\mathcal{L}(z_a, s_a)} \left[Q\left(z_a, \tilde{p}\right) \tilde{p} + v\left(z - Q\left(z_a, \tilde{p}\right)\right) \middle| s_a \right]$$

   on the set $A \times \mathbb{R}$, meaning that it is optimal, given A, $Q(\cdot,\cdot)$ and $v(\cdot)$.

The equilibrium is partial because there is no connection between, on the one hand, the trading pattern induces by $P(\cdot,\cdot)$, $Q(\cdot,\cdot)$, and A and, on the other hand, the distribution of types μ.

Note that, strictly speaking, we are interested in the solutions to the individual problem (ip) and not in the solutions to the HJB equation (9). As these two sets need not be identical, this calls for a verification argument.

Proposition 5. *Let us assume that the transparency of the market is high enough, meaning that*

$$\tau \in \left[\frac{\sqrt{3}}{2}, 1 \right] \approx [0.866, 1], \tag{11}$$

and let us take as given the cross-sectional distribution of types μ and write $\mathcal{M}$ for its mean and $\mathcal{V}$ for its variance. Then, there exists a partial equilibrium for which the value functions are characterized by the quadratic function

$$v(z) = v_0 + v_1 z + v_2 z^2,$$

with

$$\begin{cases} v_0 &= \frac{\gamma\sigma^2\left(4\lambda\mathcal{M}^2\left((\tau-3)\tau^2+3\right)-9\sigma_z^2-4\lambda\left((\tau-4)\tau^2+2\right)\mathcal{V}\right)}{8\lambda((\tau-3)\tau^2+3)+18r} \\ v_1 &= \frac{\mu}{r} - \frac{4\gamma\lambda\mathcal{M}\sigma^2\left((\tau-3)\tau^2+3\right)}{4\lambda((\tau-3)\tau^2+3)+9r} \\ v_2 &= -\frac{9\gamma r\sigma^2}{8\lambda((\tau-3)\tau^2+3)+18r} \end{cases}.$$

The corresponding optimal purchasing policy is

$$Q : (z, p) \mapsto \frac{p - v_1}{2v_2} - z.$$

All the agents quote a price when being asked for one, meaning that

$$\begin{aligned} A &\overset{(\Delta)}{=} \left\{ z : \mathrm{E}^{\mathcal{L}(z_a, s_a)} \left[\sup_{\tilde{p}} \mathrm{E}^{\mathcal{L}(z_a, s_a)} \left[Q\left(z_a, \tilde{p}\right) \tilde{p} + v\left(z - Q\left(z_a, \tilde{p}\right)\right) \middle| s_a \right] \right] \geq v(z) \right\} \\ &= \mathbb{R}. \end{aligned}$$

And the function

$$P : (z, s_a) \mapsto v_1 + 2v_2 \left(\frac{1}{3} z + \frac{2}{3} \left(\tau s_a + (1-\tau)\mathcal{M} \right) \right)$$

describes the optimal quoting policy.

Remark 6. Without the assumption (11), no quadratic partial equilibrium exists. In the proof of Proposition 5, this assumption is critical in step (iv). If this assumption is relaxed, the term describing the utility benefits resulting from being asked for a quote will not be quadratic in the current type anymore, and the quadratic assumption in step (i) will not be consistent.

In order to characterize an equilibrium of the model, we still need the cross-sectional distribution of types or, as seen in Proposition 5, its two first moments. The next corollary is the first step in this direction.

Corollary 7. *The dynamics of the type z of a given agent a is*

$$\begin{aligned}
\mathrm{d}z_t = \sigma_z \, \mathrm{d}B_t \\
&+ \begin{pmatrix} X_{r,t} & \left(\frac{1}{3} z_{q,t} + \frac{2}{3} \left(\tau z_{t-} + (1-\tau)\mathcal{M}\right)\right) \\ +(1 - X_{r,t}) & \left(\frac{1}{3} z_{q,t} + \frac{2}{3} \left(\tau \zeta_{r,t} + (1-\tau)\mathcal{M}\right)\right) \\ - & z_{t-} \end{pmatrix} \mathrm{d}N^r_t \\
&+ \begin{pmatrix} X_{q,t} & \left(z_{r,t} + \frac{2}{3} \left(\tau z_{t-} - \left(\tau z_{r,t} + (1-\tau)\mathcal{M}\right)\right)\right) \\ +(1 - X_{q,t}) & \left(z_{r,t} + \frac{2}{3} \left(\tau z_{t-} - \left(\tau \zeta_{q,t} + (1-\tau)\mathcal{M}\right)\right)\right) \\ - & z_{t-} \end{pmatrix} \mathrm{d}N^q_t,
\end{aligned} \tag{12}$$

with the following distributional assumptions.

1. *N^r is a Poisson process with jump intensity λ. N^c jumps when a requests a quote from another investor.*
2. *$z_{q,t} \sim \mu$ is the type of the investor from whom a requests a quote at time t.*
3. *$X_{r,t} \sim B(1, \tau)$ is the Bernoulli random variable indicating whether the signal about a at time t is correct ($X_{a,t} = 1$) or uninformative ($X_{a,t} = 0$).*
4. *$\zeta_{r,t} \sim \mu$ is the uninformative signal about a.*
5. *N^q is a Poisson process with jump intensity λ. N^q jumps when a is asked for a quote by another investor.*
6. *$z_{r,t} \sim \mu$ is the type of the investor who requested a quote from a at t.*
7. *$X_{r,t} \sim B(1, \tau)$ is the Bernoulli random variable indicating whether a observes the current type $z_{r,t}$ or the agent who requested a quote at time t ($X_{r,t} = 1$) or an uninformative signal ($X_{r,t} = 0$).*
8. *$\zeta_r \sim \mu$ is the uninformative signal about the agent who requested a quote.*

Furthermore, the processes

$$B, N^r, N^q$$

and random variables

$$(z_{q,t}, X_{r,t}, \zeta_{r,t}, z_{r,t}, X_{q,t}, \zeta_{q,t})_{t \geq 0}$$

are all independent of each others.

4 Stationary Type Distribution

The calculation of the individual value functions in Section 3 relies on exogenous beliefs regarding the distribution of types across the investors. However, these individual

value functions themselves induce a certain trading pattern on the OTC market, and this trading pattern generates a certain type distribution. In this section we intend to make the beliefs regarding the type distribution rational. We formalize the rationality of the beliefs with the following definition.

Definition 8 (Consistent Type Distribution)**.** *A distribution of types μ is consistent if*

1. *the trading pattern induced by the partial equilibrium corresponding to μ generates a stationary distribution of types $\mu^{out}(\mu)$;*
2. *the assumed and actual distributions of types are identical, meaning that $\mu = \mu^{out}(\mu)$.*

As it turns out, there is a unique consistent distribution of types.[11]

Proposition 9. *There exists a unique equilibrium stationary distribution of types μ. μ solves the equation*

$$\hat{\mu}(w) = \frac{1}{1 + \frac{\frac{1}{2}\sigma_z^2}{2\lambda} w^2} \begin{pmatrix} \frac{\tau}{2} & e^{i\frac{2}{3}(1-\tau)\mathcal{M}w} & \hat{\pi}\left(\frac{1}{3}w\right) & \hat{\pi}\left(\frac{2}{3}tw\right) \\ +\frac{1-\tau}{2} & e^{i\frac{2}{3}(1-\tau)\mathcal{M}w} & \hat{\pi}\left(\frac{1}{3}w\right) & \hat{\pi}\left(\frac{2}{3}tw\right) \\ \frac{\tau}{2} & e^{-i\frac{2}{3}(1-\tau)\mathcal{M}w} & \hat{\pi}\left(\frac{2}{3}w\right) & \hat{\pi}\left(\left(1-\frac{2}{3}t\right)w\right) \\ +\frac{1-\tau}{2} & e^{-i\frac{2}{3}(1-\tau)\mathcal{M}w} & \hat{\pi}\left(\frac{2}{3}w\right) & \hat{\pi}\left(w\right)\hat{\pi}\left(-\frac{2}{3}tw\right) \end{pmatrix}, \ w \in \mathbb{R},$$

with $\hat{\mu}(w)$ being the Fourier transform of μ. Namely, if $z \sim \mu$, then

$$\hat{\mu}(w) \triangleq \mathrm{E}\left[e^{iwz}\right].$$

Furthermore, the first two moments of μ are

$$\mathrm{E}\left[z\right] = S$$
$$\mathrm{Var}\left[z\right] = \frac{9\sigma_z^2}{4\lambda\left(1+\tau^2\right)}.$$

At this stage, we can already describe certain characteristics of the distribution of trades.

Corollary 10. *The average transaction price is*

$$\frac{\mu}{r} - \gamma S \sigma^2.$$

The variance of the transaction prices is

$$\frac{81\gamma^2 r^2 \sigma^4 \sigma_z^2 (4\tau+1)}{\lambda\left(\tau^2+1\right)\left(8\lambda\left((\tau-3)\tau^2+3\right)+18r\right)^2}$$

[11] The exact sense in which the trading pattern "generates" a stationary type distribution is clear from the proof of the next proposition.

and is increasing in the transparency levels whenever

$$\lambda \geq \frac{6\left(131\sqrt{3}-101\right)}{20641}r \approx 0.037r$$

and the transparency τ is high enough. The variance of the transaction sizes is

$$\frac{\left(10-8\tau^2\right)\sigma_z^2}{4\lambda\left(1+\tau^2\right)}$$

and is decreasing over the relevant range of transparency levels.

Finally, an ordinary least-squares regression of the transaction prices against the transaction sizes yields the constant coefficient

$$\frac{\mu}{r}-\gamma S\sigma^2$$

and the slope coefficient

$$\frac{9}{10}\frac{2}{3}\left(\frac{1}{3v_2}\left(4\tau+1\right)-2v_2\tau^2\right).$$

The slope coefficient is increasing in τ over the relevant range when λ is not too small when compared to r.[12]

Interestingly, the variance of the transaction prices is typically increasing in the transparency of the market. It is true that a more transparent market leads to a more efficient allocation of the stock, to smaller liquidity needs, and to more homogeneous valuations across the population of investors. This should tend to decrease the cross-sectional variance of the transaction prices. However, more transparency leads to predatory quotes, which increases the inventory costs. This second effect increases the unit price of a transaction of a given size, increases the dispersion of prices, and dominates the first effect.

5 Equilibrium

Putting together the results of Section 3 regarding the individual optimality and Section 4 regarding the type distribution, the characterization of an "equilibrium" of the model is immediate. We first formally define our equilibrium concept.

[12] The exact condition is

$$\begin{aligned}&2\left(71+40\sqrt{3}\right)\lambda^3+108r^3\left(3\sqrt{3}\gamma^2\sigma^4-8\right)\\&\quad+9\lambda r^2\left(\left(48\sqrt{3}-9\right)\gamma^2\sigma^4-64\sqrt{3}+72\right)+120\left(3+\sqrt{3}\right)\lambda^2 r>0\end{aligned}$$

and is satisfied when we let either $\lambda\to\infty$ or $r\to 0$.

Definition 11 (Equilibrium)**.** *An equilibrium of the model consists of a consistent distribution of types μ, in the sense of Definition 8, and the corresponding partial equilibrium, in the sense of Definition 4.*

Corollary 12. *There exists exactly one quadratic equilibrium.*

Proof. Immediate from Proposition 5 and Proposition 8.

We can characterize the links between the expected utility of the investors and the transparency of the OTC market.

Corollary 13. *The relationship between transparency, as measured by the parameter τ measuring the quality of the signal, and expected utility, as measured by the function $v(\cdot)$, is ambiguous. Namely,*

$$\lim_{\tau\to 1}\frac{\partial}{\partial\tau}v(z)\begin{cases} >0 & , \quad |z-\mathcal{M}| < \frac{\sqrt{r\lambda(9r+\lambda)\sigma_z^2}}{\sqrt{6}r\lambda} \\ =0 & , \quad |z-\mathcal{M}| = \frac{\sqrt{r\lambda(9r+\lambda)\sigma_z^2}}{\sqrt{6}r\lambda} \\ <0 & , \quad |z-\mathcal{M}| > \frac{\sqrt{r\lambda(9r+\lambda)\sigma_z^2}}{\sqrt{6}r\lambda} \end{cases}.$$

In particular, more transparency benefits those investors with a moderate exposure to the aggregate risk but more opacity benefits those investors having either a sufficiently large or sufficiently low exposure. Overall, more transparency is socially desirable in the sense that

$$\partial_\tau \mathrm{E}^{\mu(z)}\left[v(z)\right] = \frac{9\gamma\sigma^2\sigma_z^2\tau}{4\lambda\left(1+\tau^2\right)^2} > 0.$$

The intuition behind the last proposition is that transparency increases the inventory costs.[13] This happens via two different channels. On the one hand, transparency exposes any given investor with large inventories to predatory pricing. On the other hand, as transparency improves the allocative efficiency of the market, it becomes more difficult for an investor with an excessive exposure to find a counterparty with large and opposite liquidity needs. This second effect exacerbates the first one and is driven by the endogenous distribution of types.

These results imply that the effect of transparency on the value function of the investors is ambiguous. First, on average, investors benefit from an increase of the transparency and the resulting improvement in the allocative efficiency. In particular, transparency is welfare improving. However, those investors with a sufficiently large (positive) or small (negative) exposure find it increasingly costly to liquidate this exposure and would benefit from a more opaque market. We characterize explicitly the exposure levels starting from which investors prefer opacity to transparency. This result is in line with the heterogeneous reactions to the introduction of TRACE, with

[13] We define the inventory costs as the reservation value of an investor who deviates from a zero risk exposure.

quite negative reactions of many institutional investors. Indeed, as our model predicts, transparency is a disadvantage for agents facing large inventory risk, such as, e.g., certain institutional investors.

6 Conclusion

We study a general equilibrium model in which agents share risks by trading on an OTC market. Both the transaction size and the transaction price are bargained bilaterally, and we analyze the equilibrium effect of asymmetric information regarding one's counter-party liquidity needs. We call the quality of this information the transparency of the market. We solve for both the value functions and the moments of the endogenous distribution of types in closed-form. Increased transparency has two main effects on the equilibrium of the model. On the one hand, it makes the asset allocation more efficient. On the other hand, it induces agents to adopt predatory quoting policies. These two effects both tend to increase the inventory costs, and we show how transparency is beneficial to those agents with moderate liquidity needs but detrimental to the rest of the population. We characterize the threshold starting from which investors value opacity in closed-form. Overall, however, more transparency is welfare improving. Our conclusions are in line with a number of sources documenting the mixed effects of transparency on the liquidity of certain OTC markets.[14]

One natural extension of our model is to consider two classes of agents. Agents in the first class, representing the dealers, trade both among themselves and with the agents of the second class. Agents in the second class, representing the end-users, can only trade with the dealers. In this setting, one can analyze the heterogeneous effects of transparency. Indeed, post-trade transparency makes the valuations of the asset across the end-users more homogeneous, but, for the dealers and as in the current model, it makes the information regarding one's counter-party inventory more accurate. Transparency may impact the entry decision of dealers. Understanding the interaction between these two sides of transparency is critical if one wants to evaluate the new regulatory reforms regarding the transparency of OTC markets.

[14] See Appendix A.

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Appendices

A TRACE

The Trade Reporting And Compliance Engine (TRACE) is a program initiated in 2002 by the National Association of Security Dealers (NASD). This program collects and disseminates anonymized bond transaction data. This program is aimed at introducing post-trade transparency on US bond markets, a major OTC market. A number of empirical studies find that TRACE made it possible for all investors to trade at prices closer to the inter-dealer price. TRACE is thus generally considered to have been a positive development, and similar programs have been initiated.[15,16]

However, some recent evidence indicates that TRACE may have reduced trading volumes for certain types of bonds. Hence, on certain bond markets, post-trade transparency moved two popular measures of liquidity in opposite directions. Namely, bid-ask spreads dropped, as did trading volumes. Furthermore, certain bond market participants expressed the view that TRACE had been detrimental to market liquidity. A transparent OTC market, these market participants argue, reduces the incentives to hold inventories and "make the market."[17]

B OTC Nash Bargaining

Nash solution to bargaining. First: when is there a surplus? Let (w_a, z_a) and (w_b, z_b) meet (wlog: $z_a < z_b$). There is a surplus exactly when

$$\begin{aligned}\emptyset \neq &\left\{(P,\Delta_z)\in\mathbb{R}^2:\ \begin{array}{ll}\alpha(w_a-P)+v(z_a+\Delta_z)\geq & \alpha w_a+v(z_a)\\ \alpha(w_b+P)+v(z_b-\Delta_z)\geq & \alpha w_b+v(z_b)\end{array}\right\}\\ =&\left\{(P,\Delta_z)\in\mathbb{R}^2:\ v(z_a+\Delta_z)-v(z_a)\geq \alpha P\geq v(z_b)-v(z_b-\Delta_z)\right\}\end{aligned}$$

Picking $\Delta_z = \frac{1}{2}(z_a + z_b)$ the concavity of v ensures that this set is never empty.

[15] Apparently, the reduction of the bid-ask spreads after the introduction of TRACE was driven by the improved bargaining power of the bond investors. See Goldstein et al. (2007), Bessembinder et al. (2006), and Edwards et al. (2007) for empirical discussing TRACE and bid-ask spreads. Bessembinder and Maxwell (2008) provides a non technical discussion on the same topic, with a number of institutional details. Green et al. (2007) discuss bid-ask spreads on the OTC market for municipal bonds.

[16] For example, the Financial Industry Regulatory Authority (FINRA) now collects transaction data for securitized products. See Hollifield et al. (2012) for an empirical analysis of opacity on the markets for securitized products.

[17] Asquith et al. (2013) find that TRACE decreased the trading volumes by up to 41.3% for certain categories of bonds. Das et al. (2013) argues that TRACE made bond markets less liquid. Bessembinder and Maxwell (2008) reports complaints by bond market investors about the introduction of TRACE and its adverse effect on liquidity. Finally, bond dealers lobbied against the introduction of TRACE. Their main arguments are summarized in NASD (2006) and largely overlap with the complaints in Bessembinder and Maxwell (2008).

In particular, any two agents will always trade, and the terms of the transaction will be dictated (under symmetric bargaining powers) by

$$\left(V(w_a - P, z_a + \Delta_z) - V(w_a, z_a)\right)\left(V(w_b + P, z_b - \Delta_z) - V(w_b, z_b)\right) \to \max_{P,\Delta_z}$$

The first order necessary conditions w.r.t. P yields

$$0 = \frac{V_w(w_a - P, z_a + \Delta_z)(-1)}{(V(w_a - P, z_a + \Delta_z) - V(w_a, z_a))} + \frac{V_w(w_b + P, z_b - \Delta_z)}{(V(w_b + P, z_b - \Delta_z) - V(w_b, z_b))},$$

as

$$V_w = -\alpha V,$$

this becomes

$$-v(z_a) + v(z_a + \Delta_z) - \alpha P = -v(z_b) + v(z_b - \Delta_z) + \alpha P \tag{13}$$

The FOC on Δ_z is

$$0 = \frac{V_w(w_a - P, z_a + \Delta_z)v'(z_a + \Delta_z)}{(V(w_a - P, z_a + \Delta_z) - V(w_a, z_a))} + \frac{V_w(w_b + P, z_b - \Delta_z)\left(-v'(z_b - \Delta_z)\right)}{(V(w_b + P, z_b - \Delta_z) - V(w_b, z_b))},$$

From the FOC on P, this simplifies to

$$v'(z_a + \Delta_z) = v'(z_b - \Delta_z),$$

and from the concavity of v, this means that

$$z_a + \Delta_z = z_b - \Delta_z = \frac{1}{2}(z_a + z_b).$$

Injecting this into (13) yields

$$\alpha P = \frac{1}{2}\left(v(z_b) - v(z_a)\right).$$

As expected both terms of the exchange are independent of the wealth.

Note: this "we move together to the center" is reminiscent of Afonso & Lagos, Trade Dynamics in the Market for Federal Funds (WP). What is critical is that the population dynamics can be determined without any specific reference to the function v.

C Proofs

Proof 14 (Proof of Lemma 2)**.** In the HJB equation (5), the first-order condition for the optimization over the consumption rate $\tilde{c}$ is

$$\begin{array}{rrcl} & U'(c) - V_w(w,z) & = & 0 \\ \Leftrightarrow & -\gamma U(c) & = & -\alpha V(w,z) \\ \Leftrightarrow & U(c) & = & \frac{\alpha}{\gamma} V(w,z), \end{array}$$

which is the second statement above. Solving for the optimal consumption now yields

$$\begin{array}{rrcl} & U(c) & = & \frac{\alpha}{\gamma} V(w,z) \\ \Leftrightarrow & -e^{-\gamma c} & = & -\frac{\alpha}{\gamma} e^{-\alpha(w+v(z)+\bar{v})} \\ \Leftrightarrow & c & = & \frac{\alpha}{\gamma}\left(w + v(z) + \bar{v}\right) - \frac{1}{\gamma}\log\left(\frac{\alpha}{\gamma}\right). \end{array}$$

Also, the concavity of the utility function $U(\cdot)$ ensures that the solution of the first-order condition is a point of maximum. Q.E.D.

Proof 15 (Proof of Proposition 5)**.** The proof of Proposition 5 is in six steps and proceeds along the lines of Definition 4, the definition of a partial equilibrium.

step (i) We first assume that the function $v(\cdot)$ that characterizes the value functions is quadratic. Namely,

$$v(z) = v_0 + v_1 z + v_2 z^2$$

defines a partial equilibrium. Let us also assume that $v_2 < 0$, meaning that $v(\cdot)$ is strictly concave.

step (ii) We derive the optimal purchasing policy, given $v(\cdot)$. Namely, a z-agent who was offered to trade at the price p solves the optimization

$$\sup_{\tilde{q}} \left\{ v\left(z + \tilde{q}\right) - \tilde{q}p \right\}.$$

The first-order condition yields a unique candidate,

$$q = \left(v'\right)^{(-1)}(p) - z = \frac{p - v_1}{2v_2} - z \triangleq Q(z,p).$$

By the concavity assumption on $v(\cdot)$, this candidate is a point of maximum, and $Q(\cdot,\cdot)$ is the optimal purchase policy.

step (iii) We derive the optimal quoting policy given the purchasing policy $Q(\cdot,\cdot)$ and $v(\cdot)$. Let us consider a z-investor who was contacted by a z_a-investor, accepted to provide a quote, and observes the signal

$$s_a = X z_a + (1 - X)\zeta$$

with

$$X \sim B(1,\tau); \quad \zeta \sim \mu; \quad X,\, \zeta : \text{ independent.} \tag{14}$$

The optimal quote solves the maximization

$$\sup_{\tilde{p}} \mathrm{E}^{\mathcal{L}(z_a,s_a)} \left[Q\left(z_a,\tilde{p}\right)\tilde{p} + v\left(z - Q\left(z_a,\tilde{p}\right)\right) \middle| s_a \right].$$

The first-order condition for this maximization is

$$\begin{aligned}
& \mathrm{E}^{\mathcal{L}(z_a,s_a)} \left[\begin{array}{l} Q_p\left(z_a,p\right)p + Q\left(z_a,p\right) \\ \quad + v'\left(z - Q\left(z_a,p\right)\right)\left(-Q_p(z_a,p)\right) \end{array} \middle| s_a \right] = 0 \\
\Leftrightarrow \quad & \mathrm{E}^{\mathcal{L}(z_a,s_a)} \left[\begin{array}{l} \frac{1}{2v_2}p + \left(\frac{p-v_1}{2v_2} - z_a\right) \\ \quad + \left(v_1 + 2v_2\left(z - \left(\frac{p-v_1}{2v_2} - z_a\right)\right)\right)\left(-\frac{1}{2v_2}\right) \end{array} \middle| s_a \right] = 0 \\
\Leftrightarrow \quad & \mathrm{E}^{\mathcal{L}(z_a,s_a)} \left[\frac{1}{2v_2}p + \frac{p-v_1}{2v_2} - z_a - \frac{v_1}{2v_2} - z + \frac{p-v_1}{2v_2} - z_a \middle| s_a \right] = 0 \\
\Leftrightarrow \quad & \frac{3}{2v_2}(p - v_1) - z - 2\,\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| s_a \right] = 0.
\end{aligned} \tag{15}$$

Now, given the definition of the signal in (14) and choosing $s \in \mathbb{R}$, Bayes' rule yields

$$\begin{aligned}
& \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| s_a = s \right] \\
=& \mathrm{P}\left[X=1\right] \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| s_a = s,\, X=1 \right] \\
& \quad + \mathrm{P}\left[X=0\right] \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| s_a = s,\, X=0 \right] \\
=& \tau\, \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| z_a = s,\, X=1 \right] \\
& \quad + (1-\tau)\, \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \middle| \zeta = s,\, X=0 \right] \\
=& \tau s + (1-\tau)\, \mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[z_a \right] \\
=& \tau s + (1-\tau)\mathcal{M}.
\end{aligned} \tag{16}$$

Injecting (16) into (15) and solving for p yields

$$p = v_1 + 2v_2\left(\frac{1}{3}z + \frac{2}{3}\left(\tau s_a + (1-\tau)\mathcal{M}\right)\right) \triangleq P\left(z, s_a\right).$$

Again, the concavity of $v(\cdot)$ makes this candidate a point of maximum, and $P(\cdot,\cdot)$ is the optimal quoting policy.

step (iv) We derive the expected benefits resulting from providing a quote. Namely, let us assume that a z-investor was asked for a quote by a z_a-investor. If the z-investor accept to issue a quote, she will receive the signal

$$s_a(\omega) = X(\omega) z_a + (1 - X(\omega))\zeta = s \tag{17}$$

and the optimal quote will be $P(z,s)$, as defined in the step (iii). As a result, the expected benefits resulting from providing a quote are

$$\begin{aligned}
&\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\sup_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[Q\left(z_a,\tilde{p}\right)\tilde{p}+v\left(z-Q\left(z_a,\tilde{p}\right)\right)-v(z)\middle|\, s_a=s\right]\right]\\
=\;&\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[Q\left(z_a,P(z,s)\right)P(z,s)+v\left(z-Q\left(z_a,\tilde{p}\right)\right)-v_0(z)\middle|\, s_a=s\right]\right]\\
=\;&-\frac{v_2}{3}\,\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\begin{array}{l}\left(4\mathcal{M}^2(1-\tau)^2-3(1-\tau)\left(\mathcal{M}^2+\mathcal{V}\right)-2\mathcal{M}(1-\tau)z+z^2\right)\\+2\tau(4\mathcal{M}(1-\tau)-z)s_a\\+(4\tau-3)\tau s_a^2\end{array}\right]\\
\overset{\Delta}{=}\;&(*).
\end{aligned}$$

Given the definition of the signal s_a in (17), the law of s_a is μ, the cross-sectional distribution of types.[18] Plugging in the moments of s_a, we can rewrite $(*)$ as

$$(*)=-\frac{v_2}{3}\left(4\mathcal{M}^2\left(1-\tau^2\right)+\left(\mathcal{M}^2+\mathcal{V}\right)\left(4\tau^2-3\right)-2\mathcal{M}z+z^2\right),$$

which is a quadratic expression in z with a positive leading coefficient and a determinant equal to

$$-\frac{4}{9}\left(4\tau^2-3\right)v_2^2\mathcal{V}.$$

In particular, under the assumption (11), this determinant is always non-positive and the expected benefits from quoting are always non-negative. In particular,

$$A\overset{(\Delta)}{=}\left\{z:\;\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[\sup_{\tilde{p}}\mathrm{E}^{\mathcal{L}(z_a,s_a)}\left[Q\left(z_a,\tilde{p}\right)\tilde{p}+v\left(z-Q\left(z_a,\tilde{p}\right)\right)\middle|\, s_a\right]\right]\geq v(z)\right\}=\mathbb{R}$$

and, in the third line of the right-hand side of the HJB equation (10), taking the positive part has no effect.

step (v) We derive the expected benefits resulting from asking for a quote. Combining the results from steps (ii), (iii), and (iv), we calculate the expected benefits

[18] Choosing $x\in\mathbb{R}$,

$$\begin{aligned}
\mathrm{P}\left[s_a\leq x\right]=&\;\mathrm{E}\left[\mathrm{P}\left[s_a\leq x\middle|\, X\right]\right]\\
=&\;\mathrm{P}\left[X=1\right]\mathrm{E}\left[\mathrm{P}\left[s_a\leq x\middle|\, X=1\right]\right]+\mathrm{P}\left[X=0\right]\mathrm{E}\left[\mathrm{P}\left[s_a\leq x\middle|\, X=0\right]\right]\\
=&\;\tau\,\mathrm{P}\left[z_a\leq x\right]+(1-\tau)\,\mathrm{P}\left[\zeta\leq x\right]\\
=&\mu\left((-\infty,x]\right).
\end{aligned}$$

As a result, $s_a\sim\mu$.

to a z-agent who just asked another agent for a quote to be

$$
\begin{aligned}
& \mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathbf{1}_{\{z_q\in A\}}\sup_{\tilde{q}}\left(-\tilde{q}P\left(z_q,s_z\right)+v\left(z+\tilde{q}\right)-v(z)\right)\right] \\
= \; & \mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\begin{array}{l} -Q(z_q,P\left(z_q,s_z\right))P\left(z_q,s_z\right) \\ \quad +v\left(z+Q(z_q,P\left(z_q,s_z\right))\right)-v(z)\end{array}\,\middle|\, X\right]\right] \\
= \; & \mathrm{P}\left[X=1\right]\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\begin{array}{l} -Q(z_q,P\left(z_q,s_z\right))P\left(z_q,s_z\right) \\ \quad +v\left(z+Q(z_q,P\left(z_q,s_z\right))\right)-v(z)\end{array}\,\middle|\, X=1\right] \\
& \quad +\mathrm{P}\left[X=0\right]\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\begin{array}{l} -Q(z_q,P\left(z_q,s_z\right))P\left(z_q,s_z\right) \\ \quad +v\left(z+Q(z_q,P\left(z_q,s_z\right))\right)-v(z)\end{array}\,\middle|\, X=0\right], \\
= \; & \tau\,\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\begin{array}{l} -Q(z_q,P\left(z_q,z_a\right))P\left(z_q,z_a\right) \\ \quad +v\left(z+Q(z_q,P\left(z_q,z_a\right))\right)-v(z)\end{array}\right] \\
& \quad +(1-\tau)\,\mathrm{E}^{\mathcal{L}(z_q,s_z)}\left[\begin{array}{l} -Q(z_q,P\left(z_q,\zeta\right))P\left(z_q,\zeta\right) \\ \quad +v\left(z+Q(z_q,P\left(z_q,\zeta\right))\right)-v(z)\end{array}\right] \\
= \; & -\frac{1}{9}v_2\begin{pmatrix} \mathcal{M}^2\left(4(\tau-3)\tau^2+9\right)+\mathcal{V}\left(4(1-\tau)\tau^2+1\right) \\ -\mathcal{M}\left(4(\tau-3)\tau^2+9\right)z \\ +\left(4(\tau-3)\tau^2+9\right)z^2\end{pmatrix}
\end{aligned}
$$

which is quadratic in the current type z.

step (vi) Using the assumption regarding $v(\cdot)$ in step (i) along with results in steps (iv) and (v), we rewrite the HJB equation (10) as

$$
\begin{aligned}
0 = & \frac{1}{9}v_2\left(-4\lambda\mathcal{M}^2\left((\tau-3)\tau^2+3\right)+9\sigma_z^2+4\lambda\left((\tau-4)\tau^2+2\right)\mathcal{V}\right)-rv_0 \\
& +\left(\mu+\frac{8}{9}\lambda\mathcal{M}\left((\tau-3)\tau^2+3\right)v_2-rv_1\right)z \\
& +\left(-\frac{1}{2}\gamma r\sigma^2-\frac{1}{9}v_2\left(4\lambda\left((\tau-3)\tau^2+3\right)+9r\right)\right)z^2.
\end{aligned}
$$

For the equation to hold for any type z, it must be that

$$
\left\{\begin{array}{rcl} 0 & = & \frac{1}{9}v_2\left(-4\lambda\mathcal{M}^2\left((\tau-3)\tau^2+3\right)+9\sigma_z^2+4\lambda\left((\tau-4)\tau^2+2\right)\mathcal{V}\right)-rv_0 \\ 0 & = & \mu+\frac{8}{9}\lambda\mathcal{M}\left((\tau-3)\tau^2+3\right)v_2-rv_1 \\ 0 & = & -\frac{1}{2}\gamma r\sigma^2-\frac{1}{9}v_2\left(4\lambda\left((\tau-3)\tau^2+3\right)+9r\right)\end{array}\right.
$$

This system is linear in the coefficients v_0, v_1, and v_2 and admits the unique solution

$$
\left\{\begin{array}{rcl} v_0 & = & \frac{\gamma\sigma^2\left(4\lambda\mathcal{M}^2\left((\tau-3)\tau^2+3\right)-9\sigma_z^2-4\lambda\left((\tau-4)\tau^2+2\right)\mathcal{V}\right)}{8\lambda\left((\tau-3)\tau^2+3\right)+18r} \\ v_1 & = & \frac{\mu}{r}-\frac{4\gamma\lambda\mathcal{M}\sigma^2\left((\tau-3)\tau^2+3\right)}{4\lambda\left((\tau-3)\tau^2+3\right)+9r} \\ v_2 & = & -\frac{9\gamma r\sigma^2}{8\lambda\left((\tau-3)\tau^2+3\right)+18r}\end{array}\right. .
$$

In particular, for these values of v_0, v_1, and v_2, the function $v(\cdot)$ defined in step (i), the function $Q(\cdot,\cdot)$ defined in step (ii), the function $P(\cdot,\cdot)$ defined in step (iii), and the set $A=\mathbb{R}$ defined in step (iv) satisfy all the conditions in Definition 4 and, thus, define a partial equilibrium of the model.

Proof 16 (Proof of Corollary 7)**.** The type dynamics of a are given by (2), meaning by

$$\mathrm{d}z_t = \sigma\ \mathrm{d}B_t + \ \mathrm{d}\theta_t, \tag{18}$$

with the first term being the idiosyncratic variation of the exposure and the second term being the discontinuous changes of exposure induced by trading.

Combining the trading strategy summarized in Section 2 and the equilibrium quoting and purchasing strategies stated in Proposition 5, we characterize the possible jumps as follows.

1. When a requests a quote at time t and the signal about her is correct, a's type becomes

$$z_{t-} + Q\left(z_{t-}, P\left(z_q, z_{t-}\right)\right) = \frac{1}{3}z_q + \frac{2}{3}\left(\tau z_{t-} + (1-\tau)\mathcal{M}\right),$$

   with z_q being the type of the quoter.

2. When a requests a quote at time t and the signal about her is uninformative, a's type becomes

$$z_{t-} + Q\left(z_{t-}, P\left(z_q, \zeta_r\right)\right) = \frac{1}{3}z_q + \frac{2}{3}\left(\tau\zeta_r + (1-\tau)\mathcal{M}\right),$$

   with z_q being the type of the quoter and ζ_r being the uninformative signal.

3. When a is asked for a quote at time t and receives an informative signal, a's type becomes

$$z_{t-} - Q\left(z_r, P\left(z_{t-}, z_r\right)\right) = z_r + \frac{2}{3}\left(z_{t-} - \left(\tau z_r + (1-\tau)\mathcal{M}\right)\right),$$

   with z_r being the type of the investor who requested a quote.

4. When a is asked for a quote at time t and receives an uninformative signal, a's type becomes

$$z_{t-} - Q\left(z_r, P\left(z_{t-}, \zeta_q\right)\right) = z_r + \frac{2}{3}\left(\tau z_{t-} - \left(\tau\zeta_q + (1-\tau)\mathcal{M}\right)\right),$$

   with z_r being the type of the investor who requested a quote and ζ_q being the uninformative signal.

Combining (18), the four possible jumps we just characterized, and the distributional assumptions in Section 2 yields the dynamics (12). Q.E.D.

D Figures

notation	parameter	value
r	risk-free rate	0.037
μ	expected dividend of the risky asset	0
σ	volatility of the dividends of the risky asset	0.285
σ_z	volatility of the idiosyncratic exposure	1
γ	absolute coefficient of risk-aversion	5
M_1	net supply of the risky asset	0
λ	meeting intensity on the OTC market	50

Table 1: Baseline parameter values for the numerical examples.

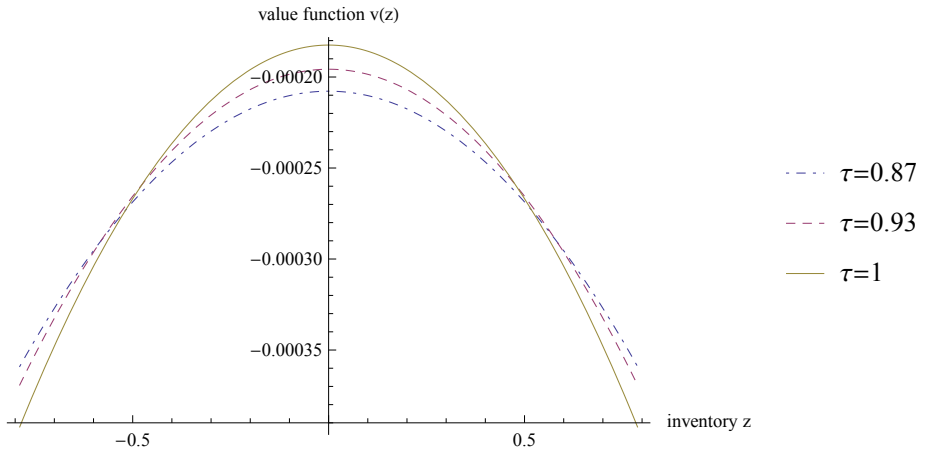

Figure 1: Value function as a function of the inventories for three levels of transparency τ. Note the non-monotone effect of transparency on the value function. The baseline parameters value are in Table 1.

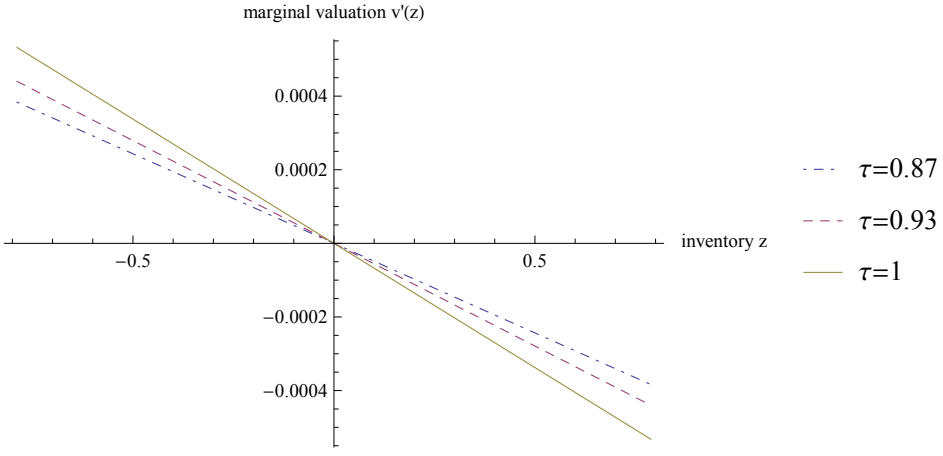

Figure 2: Marginal valuation of the risky asset as a function of the inventories for three levels of transparency τ. Transparency increases the dispersion of the valuations. The baseline parameters value are in Table 1.