The Low-Minus-High Portfolio and the Factor Zoo

Daniel Andrei (McGill), Julien Cujean (U of Bern), and Mathieu Fournier (HEC Montréal)

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- We present a counterexample to this view

Large literature trying to tame the zoo

- Barillas and Shanken (2018), Bryzgalova (2015), Chen and Zimmermann (2018), Chinco, Neuhierl, and Weber (2019), Chordia, Goyal, and Saretto (2017), Engelberg, McLean, and Pontiff (2018), Feng, Giglio, and Xiu (2019), Giglio and Xiu (2018), Giglio, Liao, and Xiu (2018), Harvey and Liu (2018), Kan and Zhang (1999), Lewellen, Nagel, and Shanken (2010), Linnainmaa and Roberts (2018), McLean and Pontiff (2016), Harvey (2017), Harvey, Liu, and Zhu (2016), Hou, Xue, and Zhang (2018), Romano and Wolf (2005), Smith (2018), Yan and Zheng (2017), ...

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- We propose a theory of the factor zoo
- We want to understand why certain factors are particularly important
- We argue that the "demographics" of the zoo have key implications for asset-pricing tests

Investors' view vs. empiricist's view

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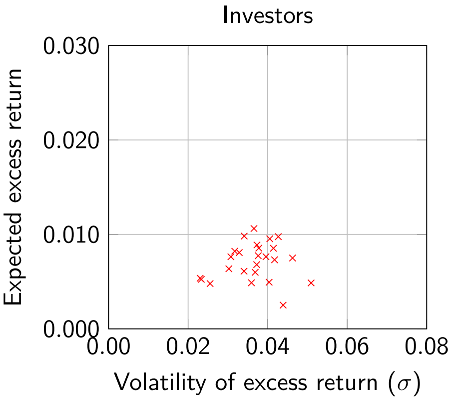

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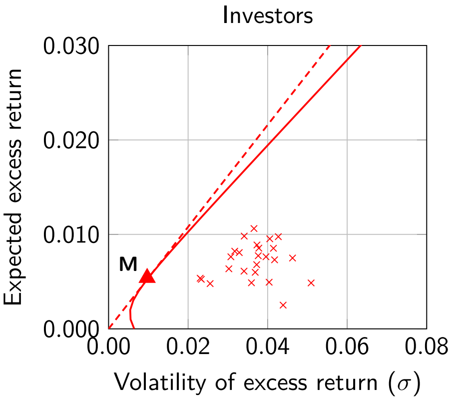

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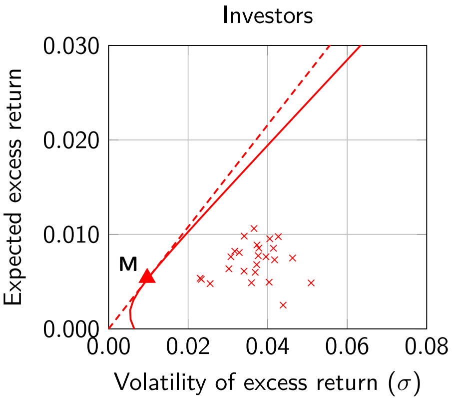

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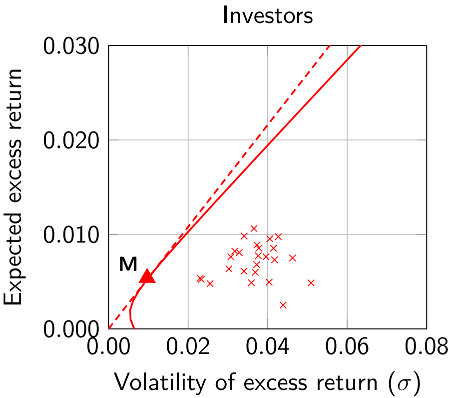

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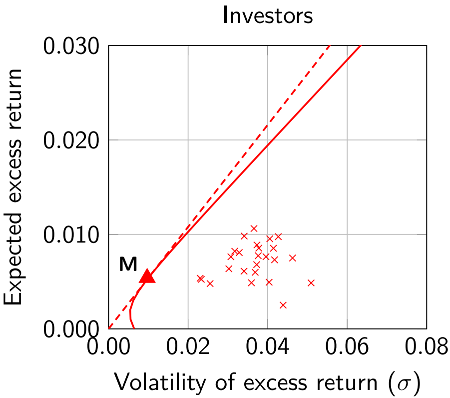

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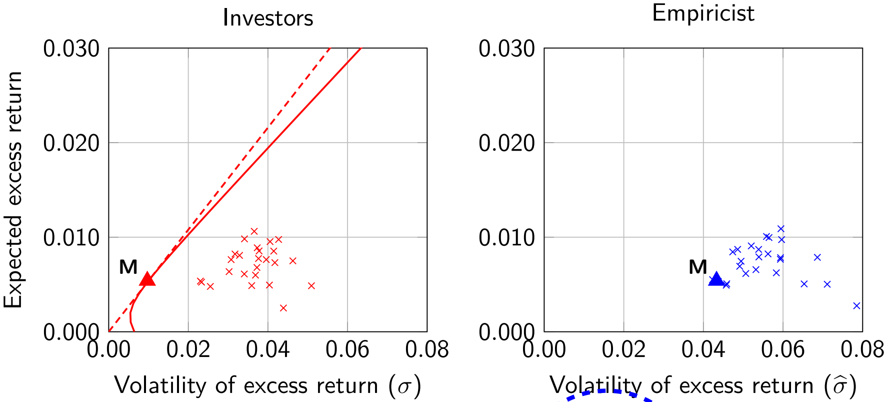

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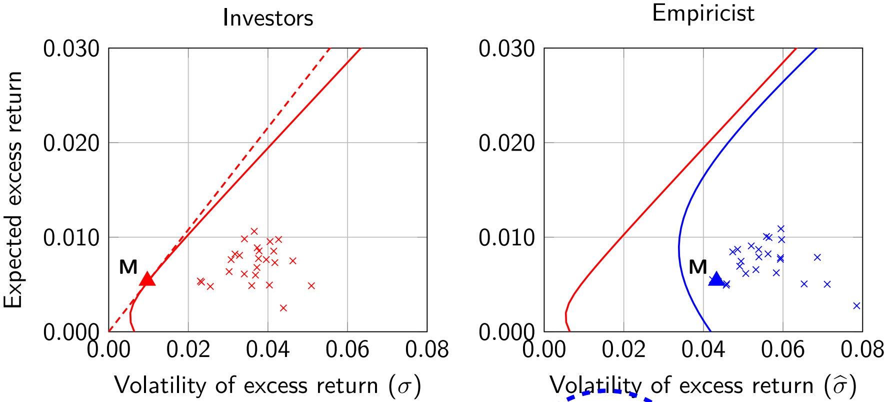

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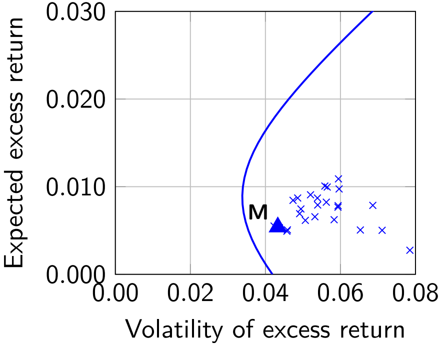
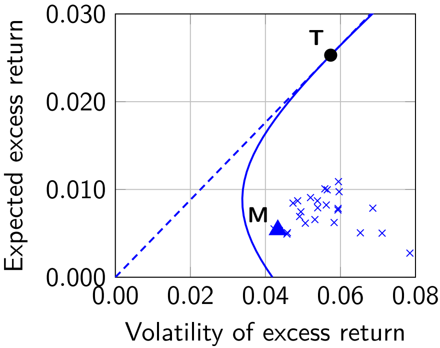

0.030
0.020
0.010
0.000
Expected excess return
M
0.00
0.02
0.04
0.06
0.08
Volatility of excess return



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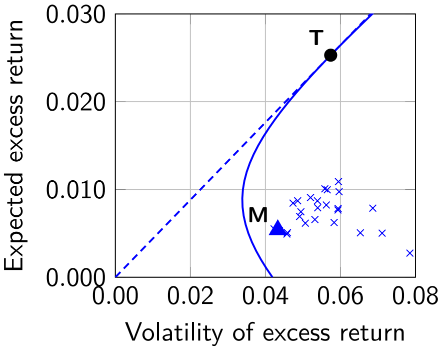

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- $\boldsymbol{\Delta}^{+}$ assets are under-invested (cheap)
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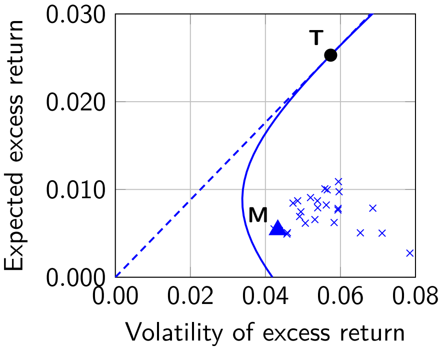

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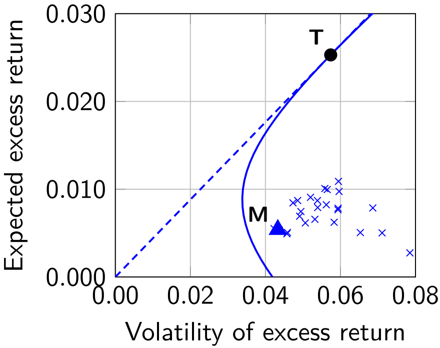
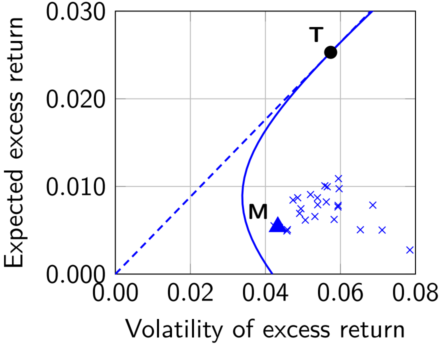
  $\Rightarrow$ $\boldsymbol{\Delta}$ is a **Low-Minus-High** (LMH) portfolio



Proposition 2

The expected returns on all assets satisfy

$$\mathbb{E}[\widetilde{\mathbf{R}}^e] = \mathbb{E}[\widetilde{R}^e_T]\widehat{\beta}_T \quad \Rightarrow \quad \mathbb{E}[\widetilde{\mathbf{R}}^e] = \frac{\mathbb{E}[\widetilde{R}^e_T]\widehat{\sigma}^2_M}{\widehat{\sigma}^2_T}\widehat{\beta} + \frac{\mathbb{E}[\widetilde{R}^e_M]\widehat{\sigma}^2_\Delta}{\widehat{\sigma}^2_T}\widehat{\beta}_\Delta$$



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FACTOR ZOO

An equilibrium interpretation of the LMH portfolio

- N firms with productivities driven by $J \leq N$ factors:

$$\widetilde{\mathbf{Z}} = \begin{bmatrix} \phi_{1,1} & \cdots & \phi_{1,J} \\ \vdots & \ldots & \vdots \\ \phi_{N,1} & \cdots & \phi_{N,J} \end{bmatrix} \left(\begin{pmatrix} F_1 \\ \vdots \\ F_J \end{pmatrix} + \begin{pmatrix} \widetilde{F}_1 \\ \vdots \\ \widetilde{F}_J \end{pmatrix} \right) + \begin{bmatrix} \tilde{\epsilon}_1 \\ \vdots \\ \tilde{\epsilon}_N \end{bmatrix}$$

$$\equiv \mathbf{\Phi}(\mathbf{F} + \widetilde{\mathbf{F}}) + \tilde{\boldsymbol{\epsilon}} \qquad (1)$$

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- A continuum of investors with private information and CARA utility over terminal wealth $\Rightarrow$

$$\boldsymbol{\omega}_i = \frac{1}{\gamma}\mathbf{\Sigma}^{-1}\,\mathbb{E}[\widetilde{\mathbf{R}}^e|\mathscr{F}_i] \tag{2}$$

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- Total supply of shares is: $\mathbf{M} = \underbrace{\mathbf{M} - \tilde{\mathbf{m}}}_{\text{informed}} + \tilde{\mathbf{m}}$ (liquidity traders have inelastic demand $\tilde{\mathbf{m}}$, He and Wang, 1995)

- Market clearing:

$$\int_i \boldsymbol{\omega}_i \mathrm{d}i = \int_i \frac{1}{\gamma} \boldsymbol{\Sigma}^{-1} \mathbb{E}[\widetilde{\mathbf{R}}^e | \mathscr{F}_i] di = \mathbf{M} - \tilde{\mathbf{m}} \tag{3}$$

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- Unconditional CAPM:

$$\mathbb{E}[\widetilde{\mathbf{R}}^e] = \frac{\boldsymbol{\Sigma} \mathbf{M}}{\mathbf{M}' \boldsymbol{\Sigma} \mathbf{M}} (\mathbf{M}' \mathbb{E}[\widetilde{\mathbf{R}}^e]) = \boldsymbol{\beta} \, \mathbb{E}[\widetilde{R}^e_M] \tag{7}$$

Equilibrium and the unconditional CAPM

- Standard linear equilibrium (Proposition 1)

Corollary 1.1

In this economy, an unconditional CAPM relation holds:

$$\mathbb{E}[\widetilde{\mathbf{R}}^e] = \underbrace{\frac{\boldsymbol{\Sigma}\mathbf{M}}{\mathbf{M}'\boldsymbol{\Sigma}\mathbf{M}}}_{\beta} \mathbb{E}[\widetilde{R}^e_M] \tag{8}$$

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- Equivalently:

$$\frac{\mu_M}{\sigma_M} = \sqrt{\boldsymbol{\mu}'\mathbf{\Sigma}^{-1}\boldsymbol{\mu}} \tag{9}$$

where $\boldsymbol{\mu} \equiv \mathbb{E}[\widetilde{\mathbf{R}}^e]$, $\sigma^2_M \equiv \mathbf{M}'\mathbf{\Sigma M}$, and $\mu_M \equiv \mathbb{E}[\widetilde{R}^e_M]$.

Predictability leaves a mark on empiricist's beta

- Investors view expected returns as a perturbation around the CAPM:

$$\mathbb{E}[\widetilde{\mathbf{R}}^e|\mathscr{F}^i] = \boldsymbol{\beta}\,\mathbb{E}[\widetilde{R}^e] + \boldsymbol{\epsilon}^i, \quad \boldsymbol{\epsilon}^i \sim \mathcal{N}(\mathbf{0}, \mathrm{Var}[\mathbb{E}[\widetilde{\mathbf{R}}^e|\mathscr{F}_i]]) \quad (10)$$

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- This mark has two origins:

$$\mathrm{Var}[\mathbb{E}[\widetilde{\mathbf{R}}^e|\mathscr{F}_i]] = \underbrace{\mathrm{Var}[\bar{\mathbb{E}}[\widetilde{\mathbf{R}}^e]]}_{\text{time-series variation}} + \underbrace{\mathrm{Var}[\mathbb{E}[\widetilde{\mathbf{R}}^e|\mathscr{F}_i] - \bar{\mathbb{E}}[\widetilde{\mathbf{R}}^e]]}_{\text{cross-sectional variation}} \tag{12}$$

Average agent vs. empiricist

Corollary 1.3

Under the information set of the empiricist, the unconditional market portfolio is not mean-variance efficient:

$$\frac{\mu_M}{\widehat{\sigma}_M} < \sqrt{\boldsymbol{\mu}'\widehat{\boldsymbol{\Sigma}}^{-1}\boldsymbol{\mu}} \tag{13}$$

Thus, the empiricist rejects the CAPM.

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- Follows from Cauchy-Schwarz inequality. The only (pathological) case with equality is:

$$\{\text{only one factor } F\} \wedge \{\mathbf{M} \propto \boldsymbol{\Phi}\} \tag{14}$$

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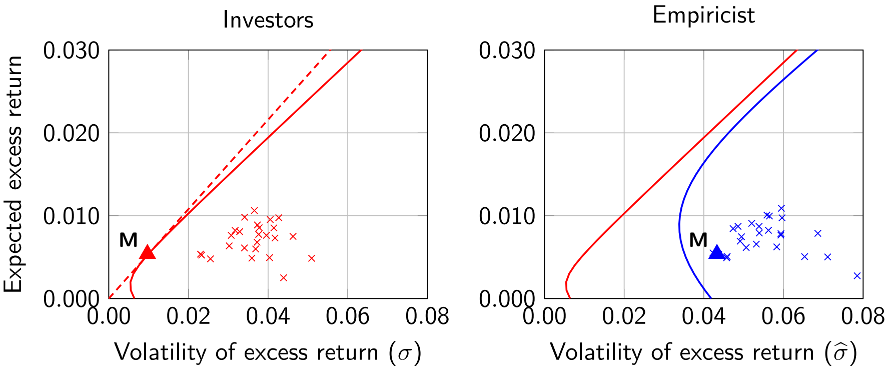

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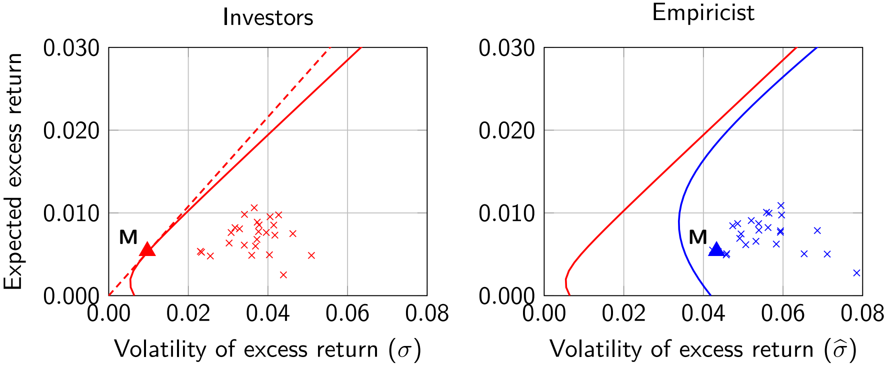

Proposition 2

Define $\boldsymbol{\Delta} \equiv \mathbf{T} - \mathbf{M}$ *and write the expected returns on all assets as*

$$\boldsymbol{\mu} = \frac{\mu_T \widehat{\sigma}^2_M}{\widehat{\sigma}^2_T} \widehat{\boldsymbol{\beta}} + \frac{\mu_T \widehat{\sigma}^2_\Delta}{\widehat{\sigma}^2_T} \widehat{\boldsymbol{\beta}}_\Delta$$

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$$\begin{aligned} \mathbf{\Sigma} &= \widehat{\mathbf{\Sigma}} - \text{Var}[\mathbb{E}[\widetilde{\mathbf{R}}^e | \mathscr{F}_i]] && (15) \\ &= c_1 \widehat{\mathbf{\Sigma}} - c_2 \mathbf{\Phi}\mathbf{\Phi}', \text{ with } c_1, c_2 > 0 && (16) \end{aligned}$$

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- Multiply with $\mathbf{M}$:

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- Multiply with $\mathbf{M}$:

$$\underbrace{\mathbf{\Sigma}\mathbf{M}}_{\propto \boldsymbol{\mu}} = \underbrace{c_1 \widehat{\mathbf{\Sigma}}\mathbf{M}}_{\propto \widehat{\beta}} - (c_2 \mathbf{\Phi}'\mathbf{M})\mathbf{\Phi} \tag{17}$$

Use the model to interpret the LMH portfolio

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- And thus:

$$\boldsymbol{\mu} = \lambda_1 \widehat{\beta} - \lambda_2 \frac{\mathbb{E}[\mathbf{P}]}{K}, \text{ with } \lambda_1, \lambda_2 > 0 \tag{18}$$

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$\frac{\mathbb{E}[\mathbf{P}]}{K}$, $\frac{\mathbb{E}[\mathbf{I}^*]}{K}$ and $\frac{\mathbf{M}}{\|\mathbf{M}\|^2}$ are observed ex-ante!

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$\Rightarrow$ characteristics are proxies for **unobservable** risk factors

Unleashing the factor zoo

- Consider now the case with $J > 1$ factors. Then:

$$\mathbb{E}[\widetilde{\mathbf{R}}^e] = \lambda_1 \widehat{\beta} - \sum_{k=1}^{J} \lambda_k \mathbf{\Phi}_k = \text{CAPM} + \text{Factor zoo} \qquad (22)$$

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Proposition 6

Denoting by $\widehat{B}_k = \mathbf{\Phi}_k' \widehat{\mathbf{\Sigma}}^{-1} \mathbf{1}$, beta on LMH satisfies:

$$\widehat{\beta}_\Delta = \sum_{k=1}^{J} c_k \widehat{\mathbf{\Sigma}} (\mathbf{M} - \underbrace{\widehat{\mathbf{\Sigma}}^{-1} \mathbf{\Phi}_k / \widehat{B}_k}_{\substack{\text{efficient portfolio} \\ \text{fully invested in factor } k}}) \tag{23}$$

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- A large economy: $N, J \to \infty$, $J/N \to \psi$ (and $T/N^{3/2} \to \varphi$)

The factor zoo in a large economy (cont'd)

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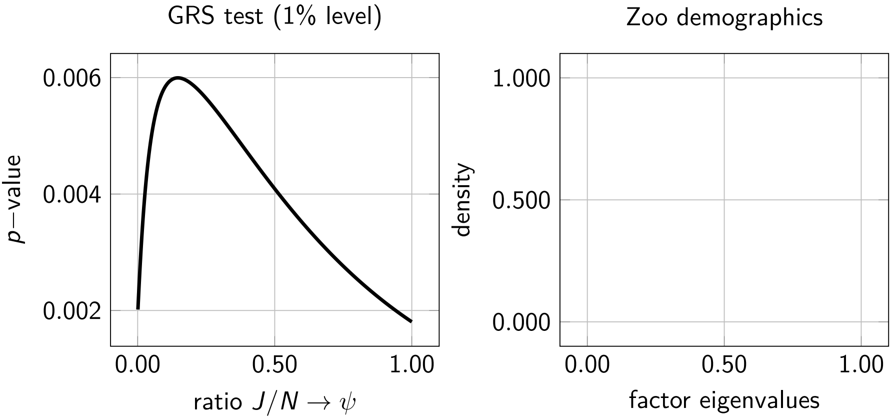
$\Rightarrow$ the eigenvalues of $\mathbf{\Phi}'\mathbf{\Phi}/N$ follow the Marchenko-Pastur law, which defines zoo demographics in this example

The factor zoo in a large economy (cont'd)



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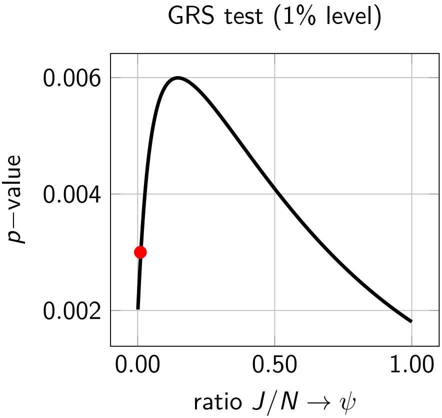

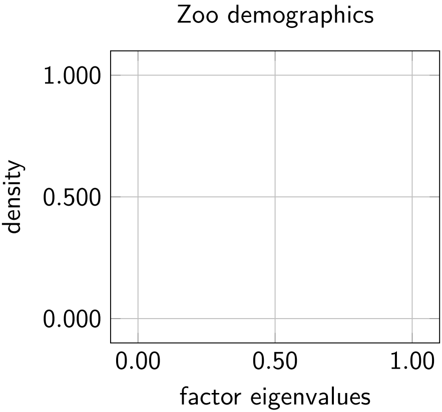

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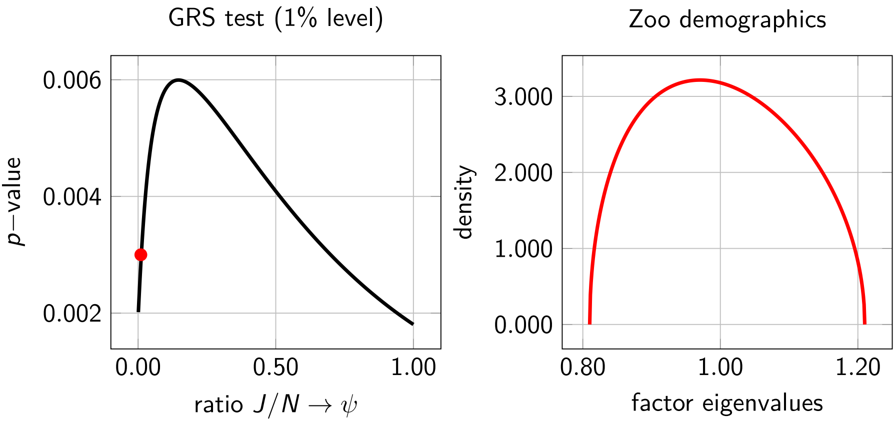

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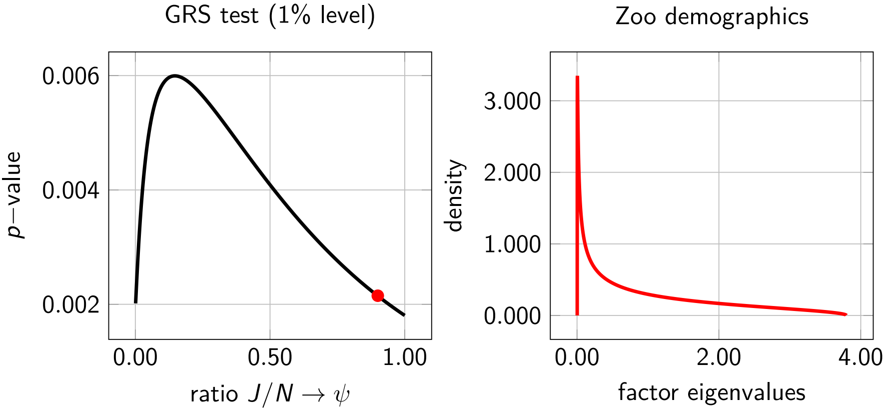

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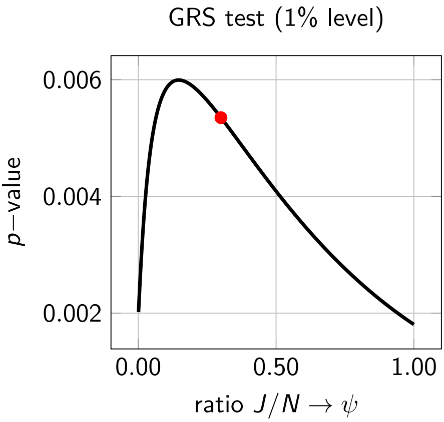

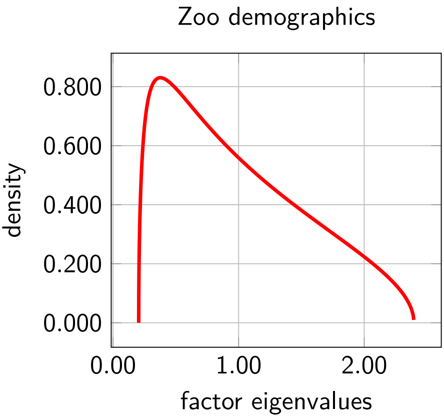

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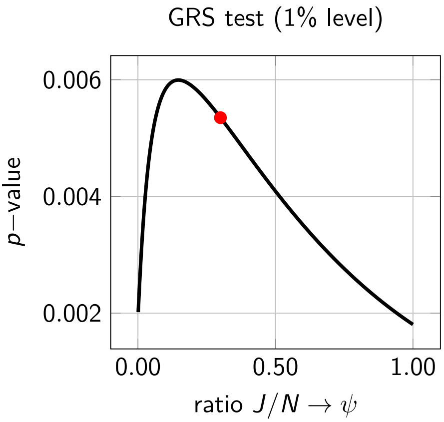

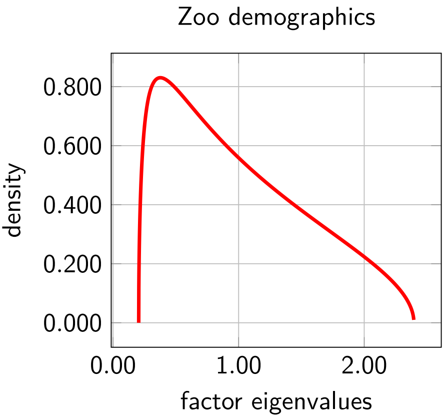

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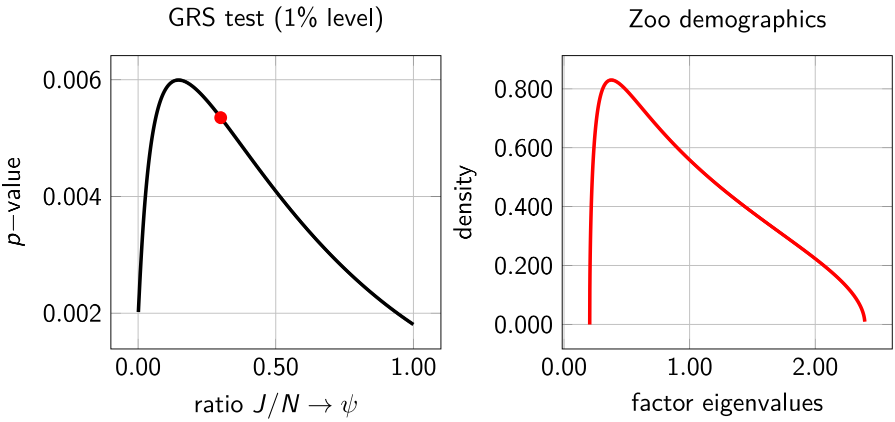

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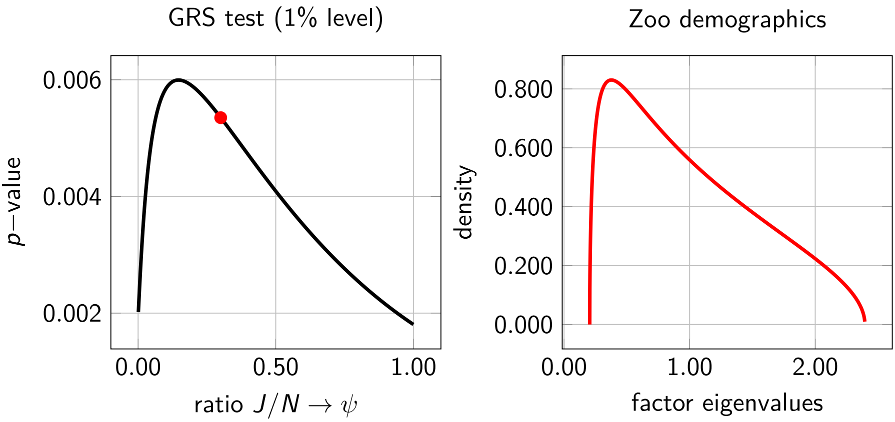

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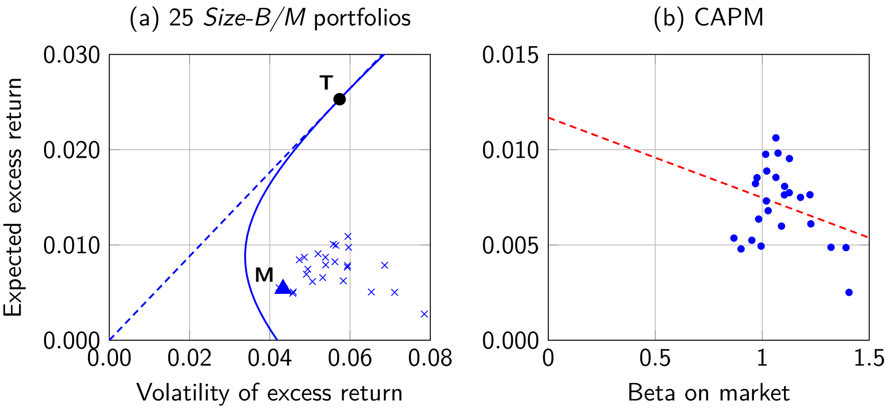
- in this example, a few highly predictable factors or many irrelevant factors is bad for the empiricist

Illustration in the *Size-B/M* portfolio space



- Monthly data, July 1963-December 2018, 666 months

Build the LMH portfolio

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HML	0.46	-0.24		-0.14	-0.20	0.68	-0.16
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Regressions: LMH on six factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.0185	0.0109	0.0150	0.0142	0.0103	0.0128	0.0094
	(6.48)	(3.89)	(4.74)	(4.80)	(3.98)	(4.47)	(3.75)
MKT	-0.6937						-0.4812
	(-7.04)						(-4.97)
HML		1.2319					1.0812
		(6.28)					(6.40)
SMB			-0.0502				0.3952
			(-0.28)				(2.82)
RMW				0.2714			0.1374
				(0.80)			(0.55)
CMA					1.6169		0.3026
					(8.61)		(1.29)
MOM						0.3130	0.3759
						(2.40)	(3.86)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284	0.3569
Obs.	666	666	666	666	666	666	666

Regressions: LMH on six factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.0185	0.0109	0.0150	0.0142	0.0103	0.0128	0.0094
	(6.48)	(3.89)	(4.74)	(4.80)	(3.98)	(4.47)	(3.75)
MKT	-0.6937						-0.4812
	(-7.04)						(-4.97)
HML		1.2319					1.0812
		(6.28)					(6.40)
SMB			-0.0502				0.3952
			(-0.28)				(2.82)
RMW				0.2714			0.1374
				(0.80)			(0.55)
CMA					1.6169		0.3026
					(8.61)		(1.29)
MOM						0.3130	0.3759
						(2.40)	(3.86)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284	0.3569
Obs.	666	666	666	666	666	666	666

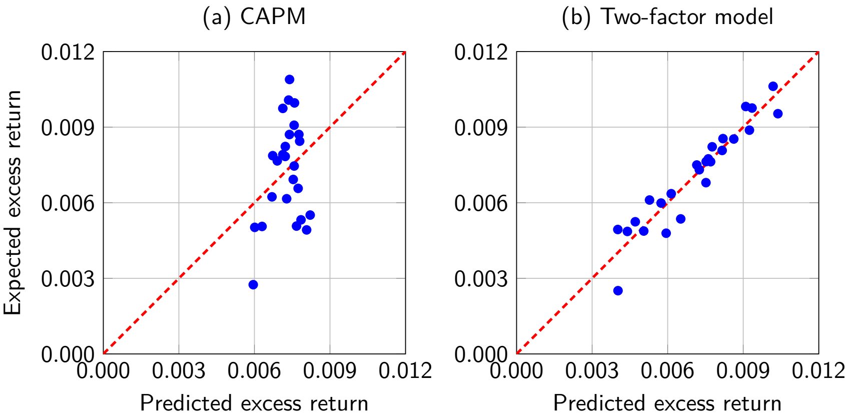
Regressions: six factors on LMH

	(1) MKT	(2) HML	(3) SMB	(4) RMW	(5) CMA	(6) MOM
Intercept	0.0086	0.0007	0.0022	0.0022	0.0011	0.0052
	(5.87)	(0.72)	(1.78)	(2.61)	(1.53)	(2.71)
LMH	-0.2288	0.1690	-0.0082	0.0224	0.1129	0.0954
	(-7.49)	(7.21)	(-0.28)	(0.83)	(6.88)	(2.17)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284
Obs.	666	666	666	666	666	666

Regressions: six factors on LMH

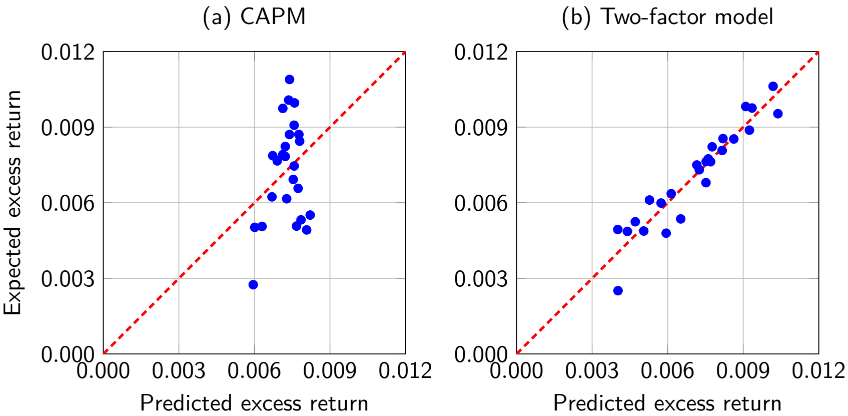
	(1) MKT	(2) HML	(3) SMB	(4) RMW	(5) CMA	(6) MOM
Intercept	0.0086	0.0007	0.0022	0.0022	0.0011	0.0052
	(5.87)	(0.72)	(1.78)	(2.61)	(1.53)	(2.71)
LMH	-0.2288	0.1690	-0.0082	0.0224	0.1129	0.0954
	(-7.49)	(7.21)	(-0.28)	(0.83)	(6.88)	(2.17)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284
Obs.	666	666	666	666	666	666

$$\mathbb{E}[R^e] = \widehat{\alpha} + \frac{\mathbb{E}[R^e_T]\widehat{\sigma}^2_M}{\widehat{\sigma}^2_T}\widehat{\beta} + \frac{\mathbb{E}[R^e_M]\widehat{\sigma}^2_\Delta}{\widehat{\sigma}^2_T}\widehat{\beta}_\Delta \quad (29)$$



0.0005 0.0049 0.0241 $R^2 = 0.89$

$$\mathbb{E}[R^e] = \widehat{\alpha} + \frac{\mathbb{E}[R^e_T]\widehat{\sigma}^2_M}{\widehat{\sigma}^2_T}\widehat{\beta} + \frac{\mathbb{E}[R^e_M]\widehat{\sigma}^2_\Delta}{\widehat{\sigma}^2_T}\widehat{\beta}_\Delta \tag{29}$$



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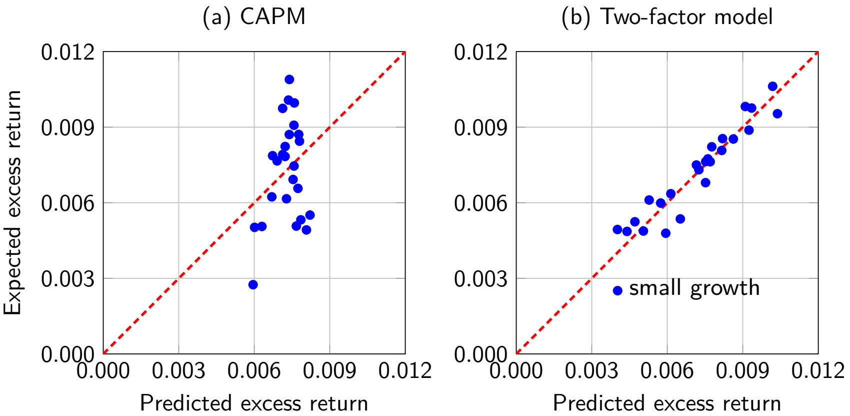
$$\mathbb{E}[R^e] = \widehat{\alpha} + \frac{\mathbb{E}[R^e_T]\widehat{\sigma}^2_M}{\widehat{\sigma}^2_T}\widehat{\beta} + \frac{\mathbb{E}[R^e_M]\widehat{\sigma}^2_\Delta}{\widehat{\sigma}^2_T}\widehat{\beta}_\Delta \tag{29}$$



$$R_n(t) = \alpha_n + \widetilde{\beta}_n R_{MKT}(t) + \widetilde{\beta}_{\Delta,n} R_{LMH}(t) + \epsilon_n(t), \quad n = 1, ..., 25$$

	Low	2	3	4	High	Low	2	3	4	High
			$\alpha_n(\%)$					t-stat(α_n)		
Small	-0.26	0.06	0.07	0.19	0.21	-1.30	0.32	0.47	1.35	1.42
2	-0.04	0.08	0.13	0.10	0.07	-0.29	0.62	1.21	1.00	0.56
3	-0.07	0.08	0.08	0.12	0.20	-0.57	0.87	0.92	1.31	1.63
4	0.05	0.04	0.01	0.15	0.08	0.56	0.51	0.14	1.71	0.68
Big	0.07	0.06	-0.03	-0.06	0.07	1.08	1.02	-0.45	-0.61	0.53
			$\widetilde{\beta}_n$					t-stat($\widetilde{\beta}_n$)		
Small	1.32	1.25	1.15	1.11	1.18	27.75	29.33	33.63	33.42	33.38
2	1.32	1.20	1.13	1.11	1.24	36.56	40.73	42.26	43.94	38.87
3	1.27	1.17	1.06	1.06	1.16	42.10	52.54	49.44	47.69	39.45
4	1.19	1.09	1.08	1.03	1.16	51.74	60.39	54.36	49.41	40.69
Big	0.96	0.94	0.91	0.93	1.00	64.48	66.71	48.86	39.86	32.13
			$\widetilde{\beta}_{\Delta,n}$					t-stat($\widetilde{\beta}_{\Delta,n}$)		
Small	-0.12	0.04	0.06	0.14	0.16	-4.53	1.47	3.25	7.27	7.95
2	-0.11	0.03	0.09	0.13	0.16	-5.21	1.79	5.78	9.21	8.57
3	-0.07	0.06	0.06	0.12	0.12	-4.13	4.41	5.05	9.40	7.10
4	-0.05	-0.01	0.07	0.09	0.08	-3.51	-0.52	6.19	7.51	4.89
Big	-0.05	-0.02	0.06	0.04	0.03	-5.94	-2.22	5.84	2.67	1.58

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
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			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

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Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

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	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
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					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
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GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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					(2.88)	(0.71)	(2.83)	(0.60)
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					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
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GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Wald test (Intercept=0, β_{MKT}=0.0052, β_{LMH}=0.0149) $\Rightarrow$ p-value=35%

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
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Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
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			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
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Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
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Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Panel A: Sorts involving prior returns

	Momentum			ST Reversal			LT Reversal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0170	-0.0035	0.0251	0.0022	-0.0048	-0.0003	0.0052	0.0035	0.0195
	(4.75)	(-0.42)	(3.17)	(0.68)	(-0.81)	(-0.03)	(1.68)	(0.97)	(1.47)
MKT	-0.0101	0.0114	-0.0169	0.0030	0.0099	0.0049	0.0010	0.0023	-0.0136
	(-2.55)	(1.32)	(-1.96)	(0.82)	(1.63)	(0.57)	(0.29)	(0.60)	(-1.04)
LMH		0.1058			0.0442			0.0124	
		(2.22)			(2.15)			(1.62)	
HML			-0.0093			0.0200			-0.0016
			(-1.44)			(2.20)			(-0.40)
SMB			0.0036			-0.0011			0.0064
			(0.61)			(-0.23)			(1.29)
Adj. R^2	0.1817	0.9502	0.8351	-0.0251	0.1233	0.7987	-0.1076	0.7976	0.9063
Obs.	10	10	10	10	10	10	10	10	10

Panel B: Sorts involving Investment, E/P, and CF/P

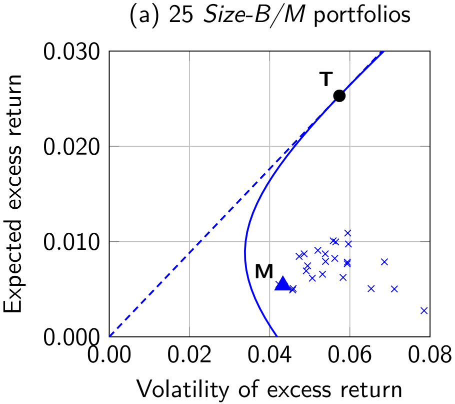
	Investment			Earnings/Price			Cashflow/Price		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0096	-0.0042	0.0104	0.0102	-0.0006	0.0088	0.0114	-0.0009	0.0115
	(3.59)	(-0.73)	(1.54)	(2.27)	(-0.14)	(0.99)	(2.47)	(-0.17)	(1.54)
MKT	-0.0037	0.0096	-0.0048	-0.0042	0.0064	-0.0031	-0.0055	0.0067	-0.0058
	(-1.19)	(1.61)	(-0.70)	(-0.88)	(1.29)	(-0.35)	(-1.14)	(1.17)	(-0.76)
LMH		0.0441			0.0239			0.0252	
		(2.59)			(2.66)			(2.19)	
HML			0.0028			0.0023			0.0012
			(1.31)			(1.44)			(0.72)
SMB			0.0038			0.0071			0.0087
			(1.27)			(1.56)			(2.06)
Adj. R^2	0.0784	0.6816	0.4318	-0.0532	0.9071	0.8698	0.0585	0.6843	0.8627
Obs.	10	10	10	10	10	10	10	10	10

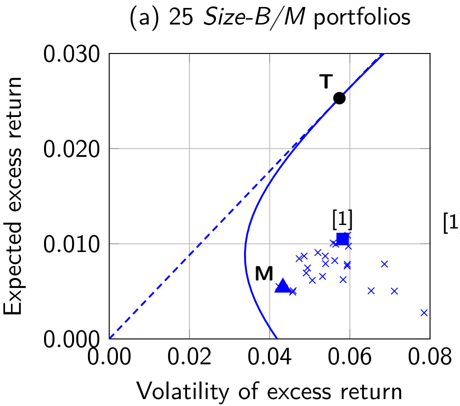
Panel A: Sorts involving prior returns

	Momentum			ST Reversal			LT Reversal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0170	-0.0035	0.0251	0.0022	-0.0048	-0.0003	0.0052	0.0035	0.0195
	(4.75)	(-0.42)	(3.17)	(0.68)	(-0.81)	(-0.03)	(1.68)	(0.97)	(1.47)
MKT	-0.0101	0.0114	-0.0169	0.0030	0.0099	0.0049	0.0010	0.0023	-0.0136
	(-2.55)	(1.32)	(-1.96)	(0.82)	(1.63)	(0.57)	(0.29)	(0.60)	(-1.04)
LMH		0.1058			0.0442			0.0124	
		(2.22)			(2.15)			(1.62)	
HML			-0.0093			0.0200			-0.0016
			(-1.44)			(2.20)			(-0.40)
SMB			0.0036			-0.0011			0.0064
			(0.61)			(-0.23)			(1.29)
Adj. R^2	0.1817	0.9502	0.8351	-0.0251	0.1233	0.7987	-0.1076	0.7976	0.9063
Obs.	10	10	10	10	10	10	10	10	10

Panel B: Sorts involving Investment, E/P, and CF/P

	Investment			Earnings/Price			Cashflow/Price		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0096	-0.0042	0.0104	0.0102	-0.0006	0.0088	0.0114	-0.0009	0.0115
	(3.59)	(-0.73)	(1.54)	(2.27)	(-0.14)	(0.99)	(2.47)	(-0.17)	(1.54)
MKT	-0.0037	0.0096	-0.0048	-0.0042	0.0064	-0.0031	-0.0055	0.0067	-0.0058
	(-1.19)	(1.61)	(-0.70)	(-0.88)	(1.29)	(-0.35)	(-1.14)	(1.17)	(-0.76)
LMH		0.0441			0.0239			0.0252	
		(2.59)			(2.66)			(2.19)	
HML			0.0028			0.0023			0.0012
			(1.31)			(1.44)			(0.72)
SMB			0.0038			0.0071			0.0087
			(1.27)			(1.56)			(2.06)
Adj. R^2	0.0784	0.6816	0.4318	-0.0532	0.9071	0.8698	0.0585	0.6843	0.8627
Obs.	10	10	10	10	10	10	10	10	10


(a) 25 *Size-B/M* portfolios
0.030
0.020
0.010
0.000
0.00
0.02
0.04
0.06
0.08
Expected excess return
Volatility of excess return
T
M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0.030
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Expected excess return
T
[1]
M
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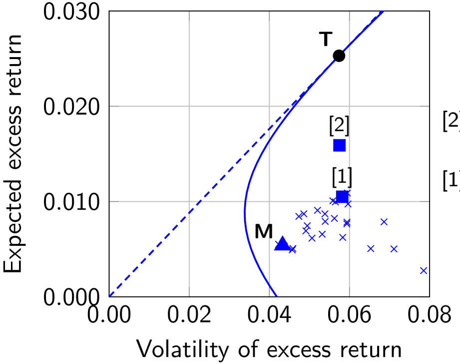
0.06
0.08
Volatility of excess return

[1] = MKT+HML+SMB

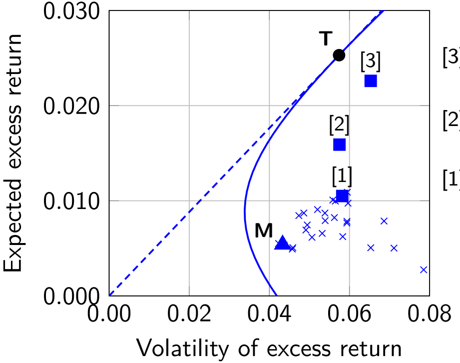
(a) 25 *Size-B/M* portfolios



[2] = MKT+HML+SMB+RMW+CMA

[1] = MKT+HML+SMB

(a) 25 *Size-B/M* portfolios

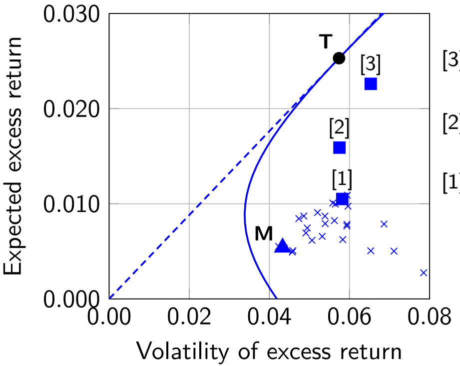


[3] = MKT+HML+SMB+RMW+CMA+MOM

[2] = MKT+HML+SMB+RMW+CMA

[1] = MKT+HML+SMB

(a) 25 *Size-B/M* portfolios

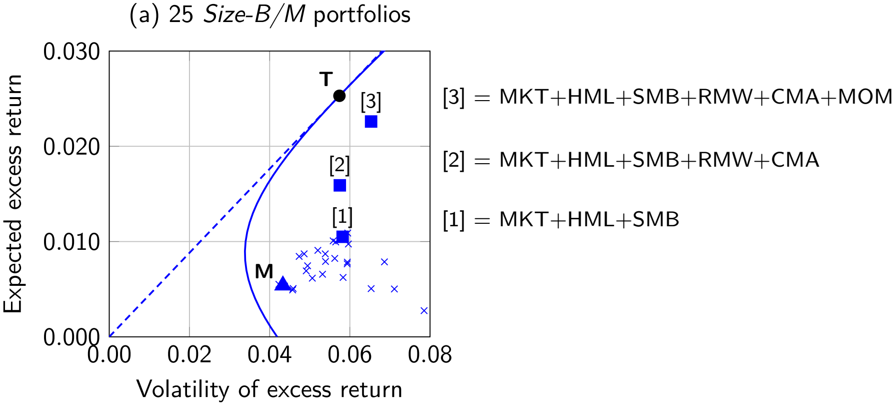


[3] = MKT+HML+SMB+RMW+CMA+MOM

[2] = MKT+HML+SMB+RMW+CMA

[1] = MKT+HML+SMB

- Hundreds of anomalies ⇒ evidence against the CAPM



- Hundreds of anomalies ⇒ evidence against the CAPM
- Before rejecting the CAPM, it is important to understand how expected returns are moving

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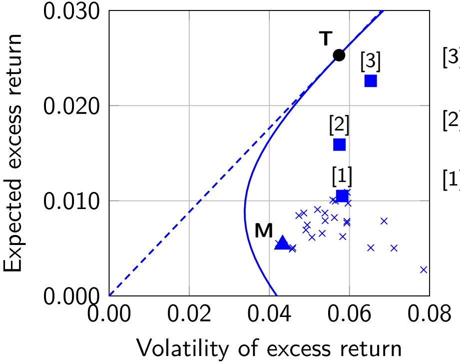

[3] = MKT+HML+SMB+RMW+CMA+MOM

[2] = MKT+HML+SMB+RMW+CMA

[1] = MKT+HML+SMB

- Hundreds of anomalies ⇒ evidence against the CAPM
- Before rejecting the CAPM, it is important to understand how expected returns are moving
- Anomalies are driven by movements in expected returns

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Empiricist's view (econometric intuition)

- Assume rational expectations:

$$R^e_{t+1} = \mathbb{E}_t[R^e_{t+1}] + \varepsilon_{t+1}; \quad \mathbb{E}_t[\varepsilon_{t+1}] = 0,\ \mathbb{E}_t[\varepsilon_{t+1}\varepsilon'_{t+1}] = \Sigma \quad (30)$$

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- Two ways to write the unconditional CAPM:

$$\mathbb{E}[R^e_{t+1}] = \underbrace{\frac{\Sigma M}{M'\Sigma M}}_{\beta} \mathbb{E}[R^e_{M,t+1}] \quad (31)$$

$$\underbrace{R^e_{t+1} - \mathbb{E}_t[R^e_{t+1}]}_{\varepsilon_{t+1}} = \beta \underbrace{\left(R^e_{M,t+1} - \mathbb{E}_t[R^e_{M,t+1}]\right)}_{M'\varepsilon_{t+1}} + z_{t+1} \quad (32)$$

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$$R^e_{t+1} = \mathbb{E}_t[R^e_{t+1}] + \varepsilon_{t+1}; \quad \mathbb{E}_t[\varepsilon_{t+1}] = 0,\ \mathbb{E}_t[\varepsilon_{t+1}\varepsilon'_{t+1}] = \Sigma \tag{30}$$

- Two ways to write the unconditional CAPM:

$$\mathbb{E}[R^e_{t+1}] = \underbrace{\frac{\Sigma M}{M'\Sigma M}}_{\beta} \mathbb{E}[R^e_{M,t+1}] \tag{31}$$

$$\underbrace{R^e_{t+1} - \mathbb{E}_t[R^e_{t+1}]}_{\varepsilon_{t+1}} = \beta \underbrace{\left(R^e_{M,t+1} - \mathbb{E}_t[R^e_{M,t+1}]\right)}_{M'\varepsilon_{t+1}} + z_{t+1} \tag{32}$$

- The empiricist observes only realized returns:

$$R^e_{t+1} = \widehat{\alpha} + \widehat{\beta} R^e_{M,t+1} + \widehat{z}_{t+1} \tag{33}$$

Empiricist's view (econometric intuition)

- Assume rational expectations:

$$R^e_{t+1} = \mathbb{E}_t[R^e_{t+1}] + \varepsilon_{t+1}; \quad \mathbb{E}_t[\varepsilon_{t+1}] = 0, \ \mathbb{E}_t[\varepsilon_{t+1}\varepsilon'_{t+1}] = \Sigma \tag{30}$$

- Two ways to write the unconditional CAPM:

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and thus

$$\widehat{\beta} = \beta + \frac{\mathrm{Var}[\,\mathbb{E}_t[R^e_{M,t+1}]\,]}{\mathrm{Var}[R^e_{M,t+1}]} \underbrace{\left(\frac{\mathrm{Cov}[\,\mathbb{E}_t[R^e_{t+1}]\,, R^e_{M,t+1}]}{\mathrm{Var}[R^e_{M,t+1}]} - \beta \right)}_{\neq 0} \tag{34}$$

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EIV rhs **EIV lhs**

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