The Low-Minus-High Portfolio and the Factor Zoo

Daniel Andrei, Julien Cujean, Mathieu Fournier

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- We will revisit this view

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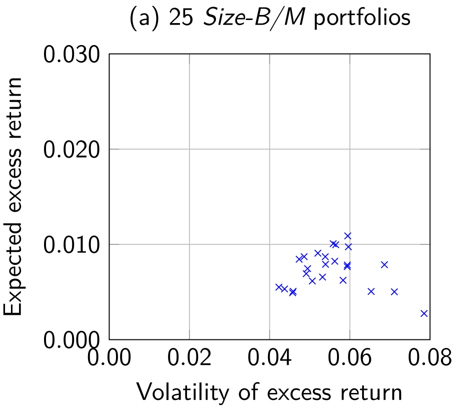
- We will begin with an example



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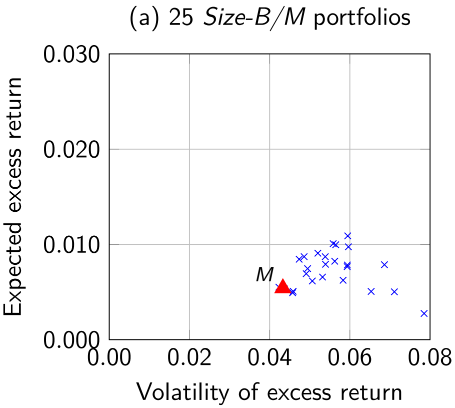

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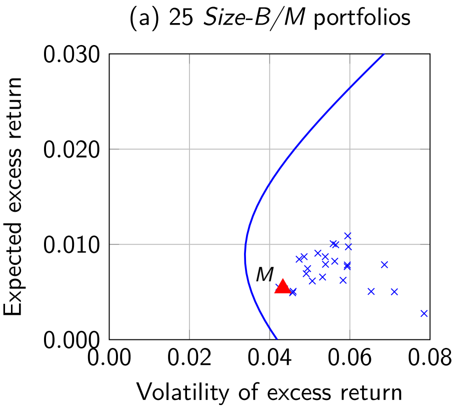

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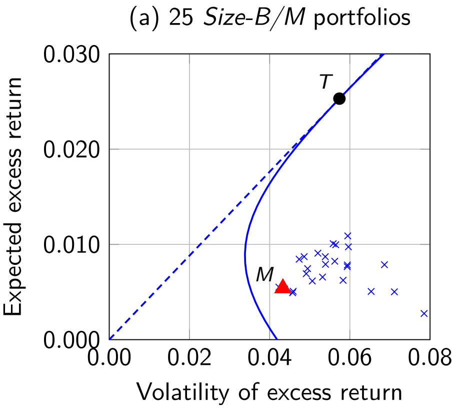

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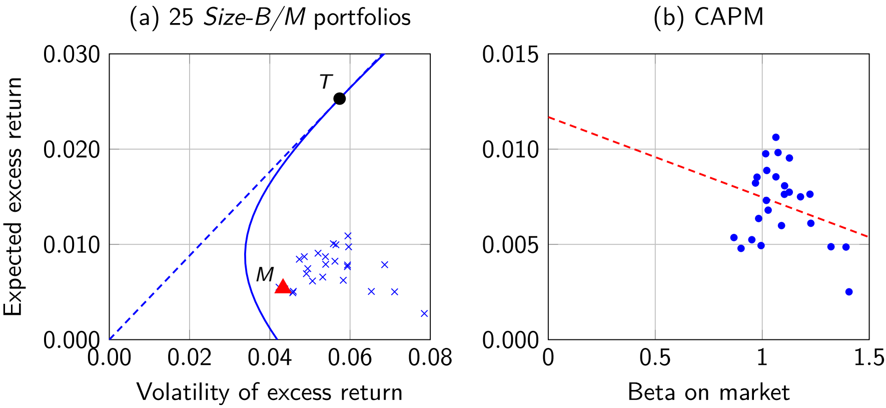

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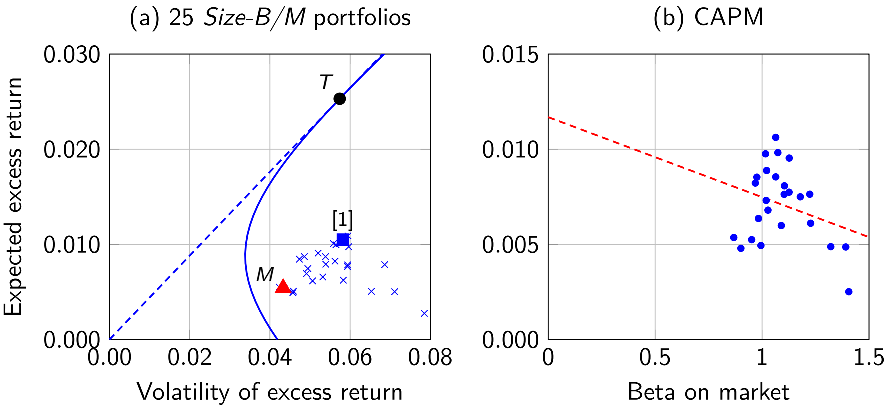

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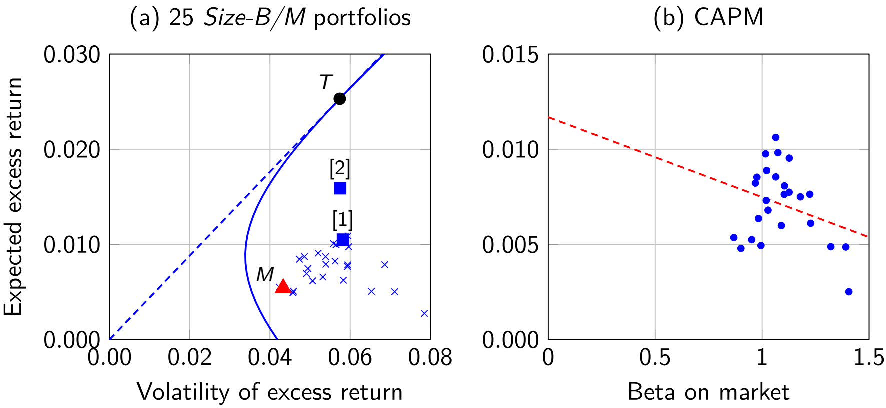

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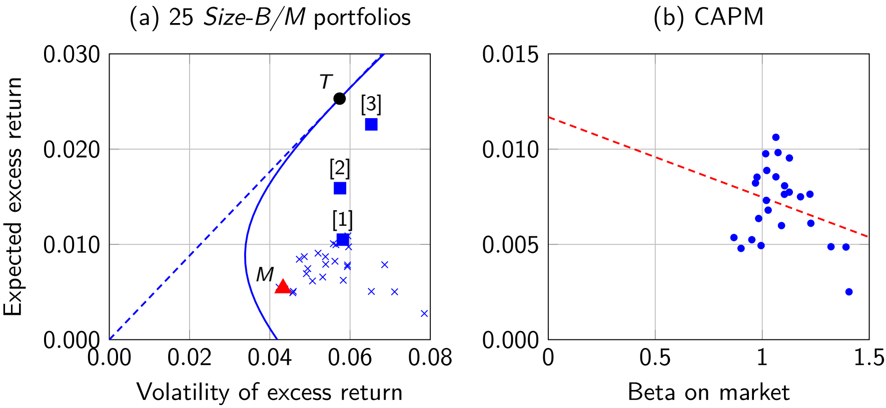

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Proposition 1

Expected excess returns for all assets are given by:

$$\mu = \frac{\mu_T \sigma_M^2}{\sigma_T^2} \beta + \frac{\mu_T \sigma_\Delta^2}{\sigma_T^2} \beta_\Delta$$

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- Using Δ_t, we compute the one-month ahead excess return of the low-minus-high portfolio (from t to $t + 1$)
- From 7/1963 to 12/2018, the LMH portfolio produces a mean monthly excess return of 1.49% and a monthly standard deviation of 7.6% $\Rightarrow$ the annualized Sharpe ratio is 0.68

Correlations

	LMH	MKT	HML	SMB	RMW	CMA	MOM
LMH		-0.40	0.45	-0.06	0.06	0.38	0.11
MKT	-0.40		-0.24	0.26	-0.19	-0.32	-0.09
HML	0.46	-0.24		-0.14	-0.20	0.68	-0.16
SMB	-0.02	0.27	-0.19		-0.27	-0.16	0.01
RMW	0.08	-0.21	0.06	-0.39		-0.20	0.17
CMA	0.43	-0.37	0.70	-0.17	-0.04		-0.08
MOM	0.17	-0.14	-0.19	0.00	0.11	-0.03	

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- Strong negative correlation with MKT
- Strong positive correlation with HML and CMA

Regressions: LMH on six factors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	0.0185	0.0109	0.0150	0.0142	0.0103	0.0128	0.0094
	(6.48)	(3.89)	(4.74)	(4.80)	(3.98)	(4.47)	(3.75)
MKT	-0.6937						-0.4812
	(-7.04)						(-4.97)
HML		1.2319					1.0812
		(6.28)					(6.40)
SMB			-0.0502				0.3952
			(-0.28)				(2.82)
RMW				0.2714			0.1374
				(0.80)			(0.55)
CMA					1.6169		0.3026
					(8.61)		(1.29)
MOM						0.3130	0.3759
						(2.40)	(3.86)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284	0.3569
Obs.	666	666	666	666	666	666	666

Regressions: six factors on LMH

	(1) MKT	(2) HML	(3) SMB	(4) RMW	(5) CMA	(6) MOM
Intercept	0.0086	0.0007	0.0022	0.0022	0.0011	0.0052
	(5.87)	(0.72)	(1.78)	(2.61)	(1.53)	(2.71)
LMH	-0.2288	0.1690	-0.0082	0.0224	0.1129	0.0954
	(-7.49)	(7.21)	(-0.28)	(0.83)	(6.88)	(2.17)
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- Are HML and CMA redundant? (Barillas and Shanken, 2017)

Corollary 1.1

Expected returns can be written:

$$\mu = \mu_M \widetilde{\beta} + \mu_\Delta \widetilde{\beta}_\Delta$$

where $\widetilde{\beta}$ *and* $\widetilde{\beta}_\Delta$ *are jointly estimated from multivariate time-series regressions.*

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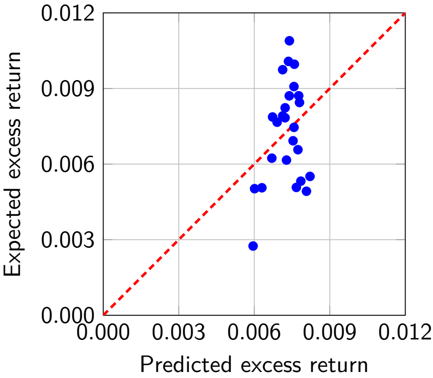
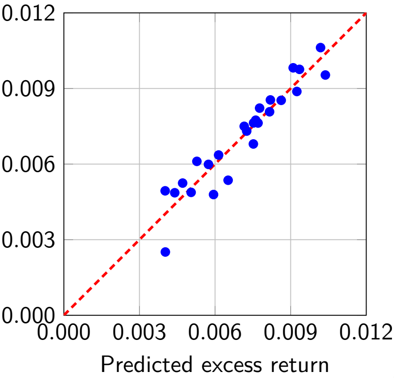
(a) CAPM



(b) Two-factor model



$$R_n(t) = \alpha_n + \widetilde{\beta}_n R_{MKT}(t) + \widetilde{\beta}_{\Delta,n} R_{LMH}(t) + \epsilon_n(t), \quad n = 1, ..., 25$$

	Low	2	3	4	High	Low	2	3	4	High
			$\alpha_n(\%)$					t-stat(α_n)		
Small	-0.26	0.06	0.07	0.19	0.21	-1.30	0.32	0.47	1.35	1.42
2	-0.04	0.08	0.13	0.10	0.07	-0.29	0.62	1.21	1.00	0.56
3	-0.07	0.08	0.08	0.12	0.20	-0.57	0.87	0.92	1.31	1.63
4	0.05	0.04	0.01	0.15	0.08	0.56	0.51	0.14	1.71	0.68
Big	0.07	0.06	-0.03	-0.06	0.07	1.08	1.02	-0.45	-0.61	0.53
			$\widetilde{\beta}_n$					t-stat$(\widetilde{\beta}_n)$		
Small	1.32	1.25	1.15	1.11	1.18	27.75	29.33	33.63	33.42	33.38
2	1.32	1.20	1.13	1.11	1.24	36.56	40.73	42.26	43.94	38.87
3	1.27	1.17	1.06	1.06	1.16	42.10	52.54	49.44	47.69	39.45
4	1.19	1.09	1.08	1.03	1.16	51.74	60.39	54.36	49.41	40.69
Big	0.96	0.94	0.91	0.93	1.00	64.48	66.71	48.86	39.86	32.13
			$\widetilde{\beta}_{\Delta,n}$					t-stat$(\widetilde{\beta}_{\Delta,n})$		
Small	-0.12	0.04	0.06	0.14	0.16	-4.53	1.47	3.25	7.27	7.95
2	-0.11	0.03	0.09	0.13	0.16	-5.21	1.79	5.78	9.21	8.57
3	-0.07	0.06	0.06	0.12	0.12	-4.13	4.41	5.05	9.40	7.10
4	-0.05	-0.01	0.07	0.09	0.08	-3.51	-0.52	6.19	7.51	4.89
Big	-0.05	-0.02	0.06	0.04	0.03	-5.94	-2.22	5.84	2.67	1.58

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
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β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

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	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Wald test (Intercept=0, β_{MKT}=0.0052, β_{LMH}=0.0149) $\Rightarrow$ p-value=35%

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
β_{CMA}					-0.0007	-0.0003	-0.0013	-0.0003
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GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
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					(-0.38)	(-0.19)	(-0.63)	(-0.18)
β_{MOM}							0.0254	0.0015
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
β_{RMW}					0.0049	0.0012	0.0057	0.0011
					(2.88)	(0.71)	(2.83)	(0.60)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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Obs.	25	25	25	25	25	25	25	25

Cross-sectional regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.0117	0.0005	0.0122	-0.0007	0.0098	0.0007	0.0028	0.0007
	(3.01)	(0.10)	(4.63)	(-0.18)	(3.31)	(0.17)	(0.66)	(0.18)
β_{MKT}	-0.0042	0.0049	-0.0067	0.0064	-0.0047	0.0049	0.0025	0.0048
	(-1.00)	(0.92)	(-2.16)	(1.43)	(-1.37)	(1.13)	(0.55)	(1.13)
β_{LMH}		0.0241		0.0284		0.0263		0.0264
		(4.57)		(5.56)		(6.25)		(5.96)
β_{HML}			0.0034	0.0033	0.0030	0.0032	0.0033	0.0032
			(3.09)	(2.91)	(2.70)	(2.88)	(2.98)	(2.87)
β_{SMB}			0.0015	0.0021	0.0020	0.0021	0.0021	0.0021
			(1.23)	(1.72)	(1.63)	(1.75)	(1.71)	(1.75)
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					(2.88)	(0.71)	(2.83)	(0.60)
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							(3.47)	(0.18)
Adj. R^2	0.0479	0.8852	0.6272	0.8898	0.7270	0.8900	0.8126	0.8835
GLS R^2	0.1087	0.6457	0.2838	0.6595	0.3352	0.6781	0.4636	0.6801
Obs.	25	25	25	25	25	25	25	25

Panel A: Sorts involving prior returns

	Momentum			ST Reversal			LT Reversal		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0170	-0.0035	0.0251	0.0022	-0.0048	-0.0003	0.0052	0.0035	0.0195
	(4.75)	(-0.42)	(3.17)	(0.68)	(-0.81)	(-0.03)	(1.68)	(0.97)	(1.47)
MKT	-0.0101	0.0114	-0.0169	0.0030	0.0099	0.0049	0.0010	0.0023	-0.0136
	(-2.55)	(1.32)	(-1.96)	(0.82)	(1.63)	(0.57)	(0.29)	(0.60)	(-1.04)
LMH		0.1058			0.0442			0.0124	
		(2.22)			(2.15)			(1.62)	
HML			-0.0093			0.0200			-0.0016
			(-1.44)			(2.20)			(-0.40)
SMB			0.0036			-0.0011			0.0064
			(0.61)			(-0.23)			(1.29)
Adj. R^2	0.1817	0.9502	0.8351	-0.0251	0.1233	0.7987	-0.1076	0.7976	0.9063
Obs.	10	10	10	10	10	10	10	10	10

Panel B: Sorts involving Investment, E/P, and CF/P

	Investment			Earnings/Price			Cashflow/Price		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	0.0096	-0.0042	0.0104	0.0102	-0.0006	0.0088	0.0114	-0.0009	0.0115
	(3.59)	(-0.73)	(1.54)	(2.27)	(-0.14)	(0.99)	(2.47)	(-0.17)	(1.54)
MKT	-0.0037	0.0096	-0.0048	-0.0042	0.0064	-0.0031	-0.0055	0.0067	-0.0058
	(-1.19)	(1.61)	(-0.70)	(-0.88)	(1.29)	(-0.35)	(-1.14)	(1.17)	(-0.76)
LMH		0.0441			0.0239			0.0252	
		(2.59)			(2.66)			(2.19)	
HML			0.0028			0.0023			0.0012
			(1.31)			(1.44)			(0.72)
SMB			0.0038			0.0071			0.0087
			(1.27)			(1.56)			(2.06)
Adj. R^2	0.0784	0.6816	0.4318	-0.0532	0.9071	0.8698	0.0585	0.6843	0.8627
Obs.	10	10	10	10	10	10	10	10	10

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Have we found, yet again, a better factor?

- LMH has an alpha of 1% per month, independently of the factors used as control variables

  The alphas of existing factors (Fama and French, 2015; Carhart, 1997) mostly disappear when controlling for LMH

  LMH commands a strongly positive risk premium

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  LMH has significant explanatory power in other portfolio sorts
- What are the macroeconomic risks (if any) underlying the LMH portfolio?
- To answer these questions, we appeal to theory

An equilibrium interpretation of the LMH portfolio

- N firms with productivities:

$$Z = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} (F + \widetilde{F}) + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix} \equiv \Phi(F + \widetilde{F}) + \epsilon$$

and final payoffs $D = KZ$

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- A continuum of investors with dispersed information and CARA utility over terminal wealth $\Rightarrow$

$$\omega_i = \frac{1}{\gamma} \widehat{\Sigma}^{-1} \widehat{\mu}_i$$

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  where $\widehat{\mu}_i \equiv \mathbb{E}[R|\mathcal{F}_i]$ and $\widehat{\Sigma} \equiv \text{Var}[R|\mathcal{F}_i]$
- The total supply of shares is noisy and unobservable: $M + \widetilde{M}$ (Grossman and Stiglitz, 1980)

Proposition 3

There exists a partially revealing rational expectations equilibrium in which the vector of market-to-book ratios, P/K, is given by

$$\frac{P}{K} = \Phi F + \xi_0 M + \alpha \widetilde{F} + gG + \xi \widetilde{M},$$

where the coefficients ξ_0 $(N \times N)$, α $(N \times 1)$, g $(N \times 1)$, and ξ $(N \times N)$ solve

$$\xi_0 = -\gamma \frac{\widehat{\Sigma}}{K}, \quad \alpha = \Phi \frac{\tau - \tau_F - \tau_G}{\tau}, \quad g = \Phi \frac{\tau_G}{\tau}$$
$$\xi = -\frac{\gamma K + \sqrt{\tau_M \tau_P}}{\tau} \Phi \Phi' - \frac{\gamma K}{\tau_\epsilon} \mathbb{I}_N,$$

and the parameter $\tau \equiv Var^{-1}[\widetilde{F} | \mathcal{F}_i]$ is a scalar determined in equilibrium as the unique positive solution of a cubic equation.

CAPM

Corollary 3.1

In this economy, an unconditional CAPM relation holds:

$$\mu = \frac{\widehat{\Sigma} M}{\widehat{\sigma}_M^2} \mu_M = \widehat{\beta}\, \mu_M,$$

where $\mu \equiv \mathbb{E}[\widehat{\mu}]$, $\widehat{\sigma}_M^2 \equiv M' \widehat{\Sigma} M$, *and* $\mu_M \equiv M' \mu$.

Average agent vs. econometrician

Corollary 3.2

Under the information set of the average agent, the Sharpe ratio of the market portfolio reaches its maximum attainable level in the economy:

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Corollary 3.3

Under the information set of the econometrician, the unconditional market portfolio is not mean-variance efficient:

$$\frac{\mu_M}{\sigma_M} < \sqrt{\mu' \Sigma^{-1} \mu}.$$

Thus, the econometrician rejects the CAPM.

Intuition for Corollary 3.3

For the average agent, returns are predictable:

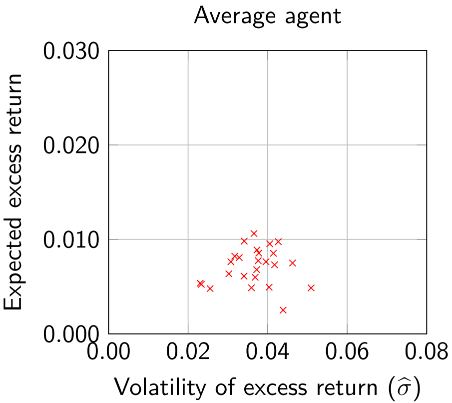
$$\widehat{\mathbb{E}}[R] = \gamma \widehat{\Sigma} M + \text{noise}$$

Take unconditional expectation:

$$\mathbb{E}[R] = \gamma \widehat{\Sigma} M$$

and therefore obtain the unconditional CAPM.

Intuition for Corollary 3.3



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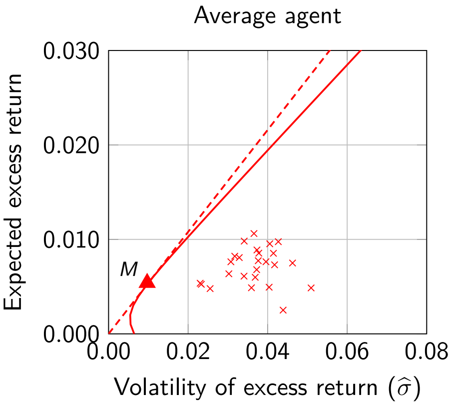

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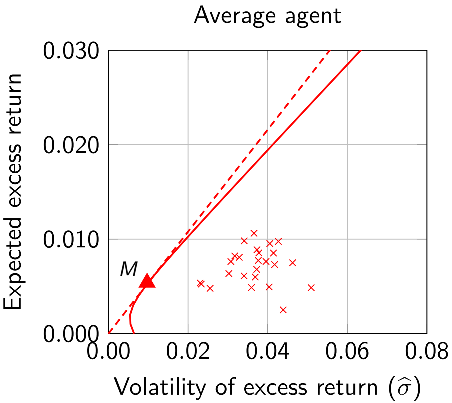

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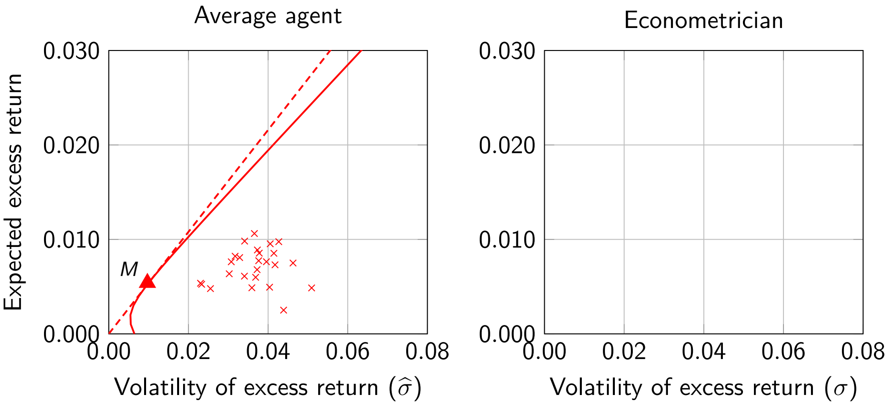

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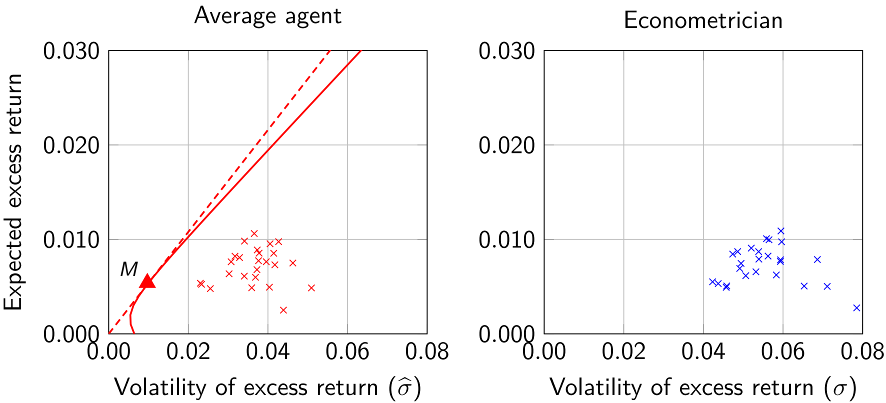

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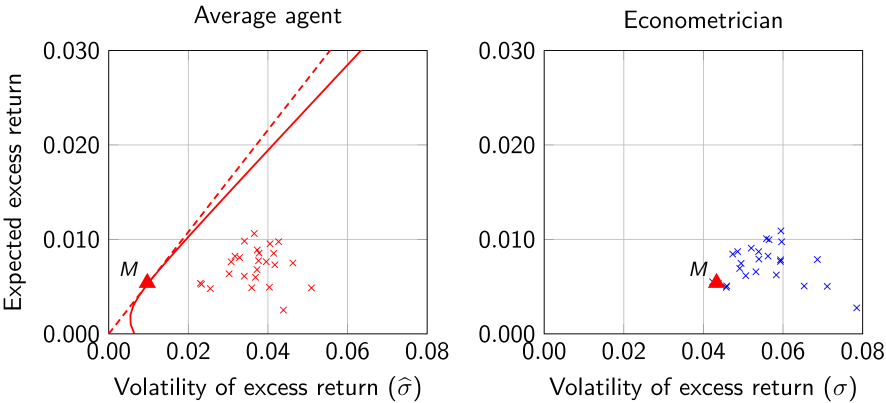

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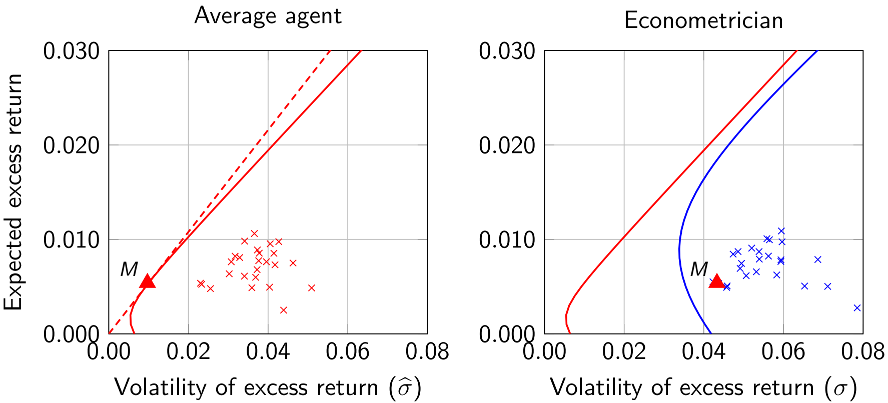

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The CAPM is rejected $\Rightarrow$ a perfect two-factor model

Corollary 3.4

For the econometrician, the portfolio $T = \Sigma^{-1}\mu/B$, *where* $B \equiv \mathbf{1}'\Sigma^{-1}\mu$, *is ex-post efficient. Since* $T \neq M$, *the econometrician can define* $\Delta \equiv T - M$ *and write the expected returns on all assets as*

$$\mu = \frac{\mu_T \sigma_M^2}{\sigma_T^2}\beta + \frac{\mu_T \sigma_\Delta^2}{\sigma_T^2}\beta_\Delta$$

where $\mu_T \equiv \mu' T$, $\sigma_T^2 \equiv T'\Sigma T$, $\sigma_\Delta^2 \equiv \Delta'\Sigma\Delta$, $\beta \equiv \Sigma M/\sigma_M^2$, *and* $\beta_\Delta \equiv \Sigma\Delta/\sigma_\Delta^2$.

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- The theory helps us understand how the returns of the LMH portfolio covary with observables (e.g., market-to-book ratios, investment rates, ...)

The LMH portfolio and the value factor

Proposition 4

In this economy, the cross-sectional variation in excess returns is entirely explained by firms' betas together with the vector of market-to-book ratios, $\mathbb{E}[P]/K$*:*

$$\mu = \lambda_1 \beta + \lambda_2 \frac{\mathbb{E}[P]}{K}, \text{ with } \lambda_1 > 0 \text{ and } \lambda_2 < 0.$$

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- Firms with low market-to-book ratios ("value" firms) have higher expected excess returns after controlling for market betas
- Yet, we have not assumed that these firms are riskier
- Mis-measurement in betas not only leads to the rejection of the CAPM, but also creates a "value factor."

In the data: a strong link between HML and LMH

	(1) MKT	(2) HML	(3) SMB	(4) RMW	(5) CMA	(6) MOM
Intercept	0.0086	0.0007	0.0022	0.0022	0.0011	0.0052
	(5.87)	(0.72)	(1.78)	(2.61)	(1.53)	(2.71)
LMH	-0.2288	0.1690	-0.0082	0.0224	0.1129	0.0954
	(-7.49)	(7.21)	(-0.28)	(0.83)	(6.88)	(2.17)
Adj. R^2	0.1575	0.2070	-0.0011	0.0046	0.1814	0.0284
Obs.	666	666	666	666	666	666

The LMH portfolio and the investment factor

- Ex-post profit of a firm n:

$$\Pi_n = \left[\phi_n(F + \widetilde{F}) + \epsilon_n\right](K + I_n) - I_n - \frac{a}{2}\left(\frac{I_n}{K}\right)^2 K$$

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Corollary 4.1

In this economy, the cross-sectional variation in excess returns is entirely explained by firms' betas together with the vector of investment ratios, $\mathbb{E}[I^*]/K$:

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The Factor Zoo

- Consider a model with $K > 1$ factors. Then:

Proposition 5

In the eyes of the empiricist expected returns satisfy the equilibrium relation:

$$\mu = \underbrace{\frac{\sigma_M^2}{\widehat{\sigma}_M^2}\left(1 + \frac{\gamma^2}{\tau_M \tau_\epsilon}\right)^{-1} \beta \mu_M}_{\text{distorted CAPM relation}} - \underbrace{\frac{\gamma \tau_M \tau_\epsilon}{\gamma^2 + \tau_M \tau_\epsilon} \sum_{k=1}^{K} \left(\sum_{j=1}^{K} \phi_j c_{kj} \right) \Phi_k}_{\text{The Factor Zoo}},$$

where $\phi_j \equiv \Phi_j' M$ denotes the average loading on factor j and the coefficients c_{jk} are scalars.

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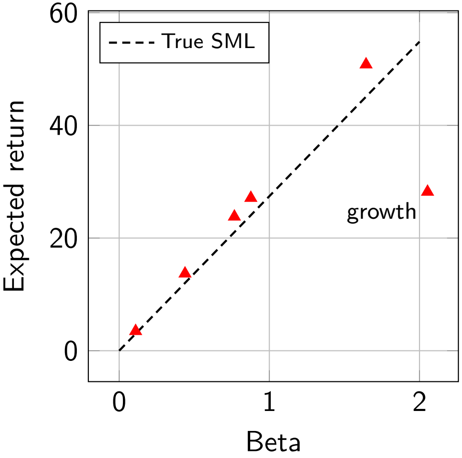
where $\phi_j \equiv \Phi_j' M$ denotes the average loading on factor j and the coefficients c_{jk} are scalars.

- Any observable variable (macroeconomic factor, or firm characteristic) that covaries with the last term joins the zoo

The factor zoo: example of a growth portfolio

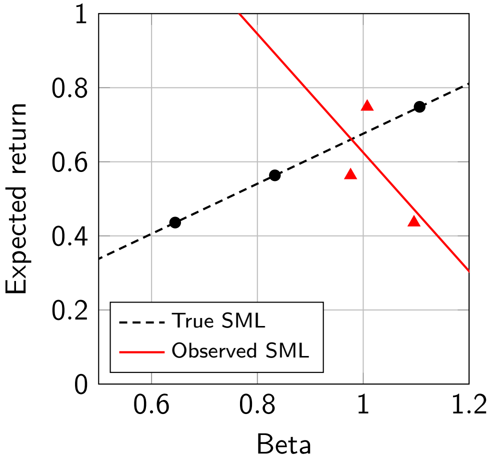
- Here we assume 2 factors and 6 assets with loadings:

$$\Phi \equiv \begin{pmatrix} 4 & 8 & 15 & 1 & 7 & -5 \\ 0 & 0 & 0 & 0 & 0 & 30 \end{pmatrix}'$$



The factor zoo: example of a downward sloping SML

- Here we go back to the one factor case and assume that assets with high factor loading are in low supply:



Further work

- Going back to

$$\mu = \frac{\mu_T \sigma_M^2}{\sigma_T^2}\beta + \frac{\mu_T \sigma_\Delta^2}{\sigma_T^2}\beta_\Delta,$$

if β and β_Δ are negatively related (in the data, the correlation between them is -0.79), then we should observe an inverted U-shape for the SML (Hong and Sraer, 2016). The cause is disagreement between econometrician and agents, rather than disagreement among agents.

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  if β and β_Δ are negatively related (in the data, the correlation between them is -0.79), then we should observe an inverted U-shape for the SML (Hong and Sraer, 2016). The cause is disagreement between econometrician and agents, rather than disagreement among agents.
- "All or Nothing" result: when there are multiple factors, the econometrician must control for **all** of them. If not $\Rightarrow$ omitted variable bias. Thus, looking for "the best factor" is useless.
- Further understand LMH vs. momentum

Conclusion

- Empirical asset pricing has identified hundreds of anomalies, and interpreted this as evidence against the CAPM
- When the CAPM is rejected, one can always build a long-short portfolio that explains, together with the market, the cross section of returns
- A statistical mirage
- Anomalies are silent about the true cause of the CAPM rejection

"We really don't know whether to believe the theory or the data... In the light of the uncertainty about the reasons for the difference between the theory and the data, the safest course may be to assume that the theory is correct."

— *Black and Scholes (1974, p. 405)*

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