Knowledge Cycles and Corporate Investment

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Motivation - the increasing importance of knowledge

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  - Low investment *despite* high valuations – Q (Gutierrez and Philippon (2017) and Crouzet and Eberly (2018))
  - Weak relation between investment and Q (Peters and Taylor (2017))
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- Open (relevant) questions:
  - How does knowledge interact with physical investment and valuation?
  - Does knowledge affect the investment-Q relation? How?

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    - Hard to buy directly (unlike physical capital)
    - Difficult to accumulate and retain (the human part)
    - Can depreciate very fast (obsolescence)
    - Effect on profits is highly uncertain

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- Our view: Knowledge results from **exploration and experimentation**

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- Knowledge = Information about current technology's quality
  - Passive learning-by-doing (i.e., passage of time)
  - Active **Experimentation** – learning-by-investing
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- **Explore** new tech when firms are confident that current tech is
  - Poor quality (unprofitable)
  - High quality (more entry eroding profits) – need for innovation

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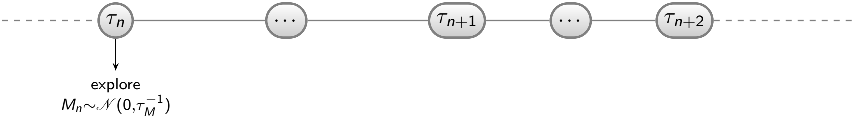

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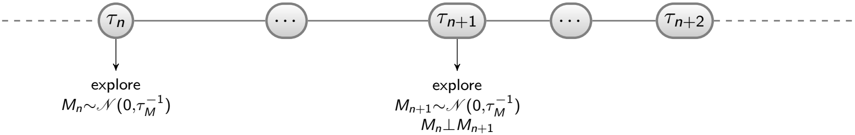

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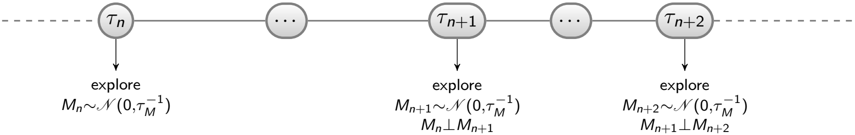

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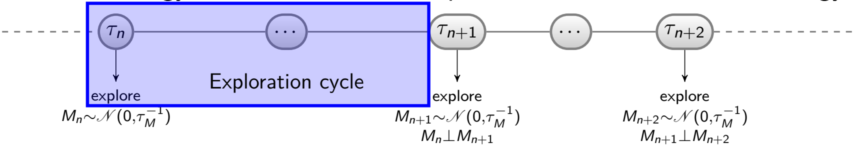

Investment as Knowledge Acquisition

- The firm can actively learn by "experimenting through investment". Investment ($i_t \equiv I_t / K_t > 0$) creates an informative signal flow:

$$\mathrm{d}S_t = \mathbf{1}_{i_t > 0} \left(\frac{\tau_S^{1/2} i_t^{1/2}}{\Omega_{t-}^{1/2}} M + \epsilon_t \right), \quad \epsilon_t \sim \mathcal{N}(0,1), \quad \text{i.i.d.}$$

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- Based on passive $(A_t)_{t\geq 0}$ and active signals $(S_t)_{t\geq 0}$ the firm obtains an estimate $\widehat{M}_t = \mathbb{E}_t[M]$ of quality, with error variance, $\Omega_t = \mathbb{V}_t[M]$

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- The $t-$stat ratio, $Z \equiv \widehat{M}/\Omega^{1/2}$, is a sufficient statistic for the firm's stock of knowledge about its current technology:

$$\mathrm{d}Z_t = \underbrace{\tau_A/2 Z_t \mathrm{d}t + \tau_A^{1/2}\mathrm{d}\widehat{B}_t}_{\text{passive learning}} + \underbrace{Z_{t-}\left(\left(1+\tau_S i_t\right)^{1/2} - 1\right) + \left(\tau_S i_t\right)^{1/2}\widehat{\epsilon}_t}_{\textit{experimenting}}$$
$$- \underbrace{Z_{t-}\mathbf{1}_{t=\tau}}_{\text{knowledge reset}}$$

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- Exploration is also costly; upon exploring a new technology, a fraction ω of the firm's capital K becomes obsolete:

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- To introduce competitive pressures we assume that new firms enter the industry as knowledge about the current technology improves:

$$\mathrm{d}N_t / N_t = \phi Z_t^2 \mathrm{d}t$$

Firm Value

- Firm is risk-neutral and discounts its cash flow at rate r; its value is:

$$\max_{\{\tau_n\}, i_{t+s} \geq 0} \mathbb{E}_t \left[\int_0^\infty e^{-rs} \Pi_{t+s} ds - \sum_{k \geq 0} \gamma_{t+\theta_k} e^{-r\theta_k} \right]$$

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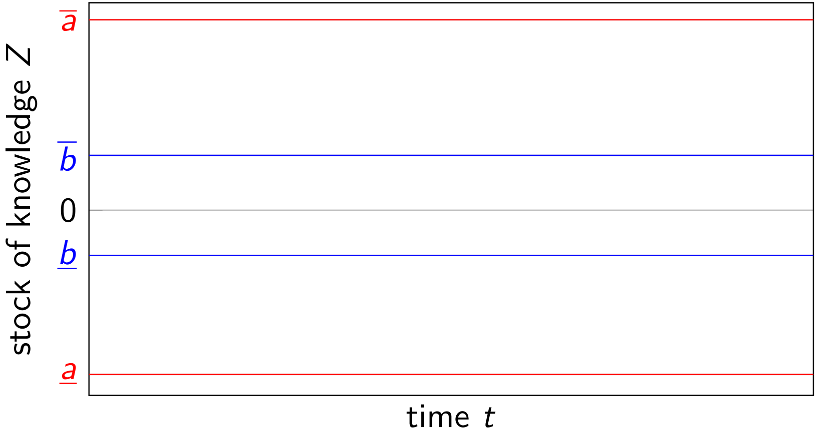

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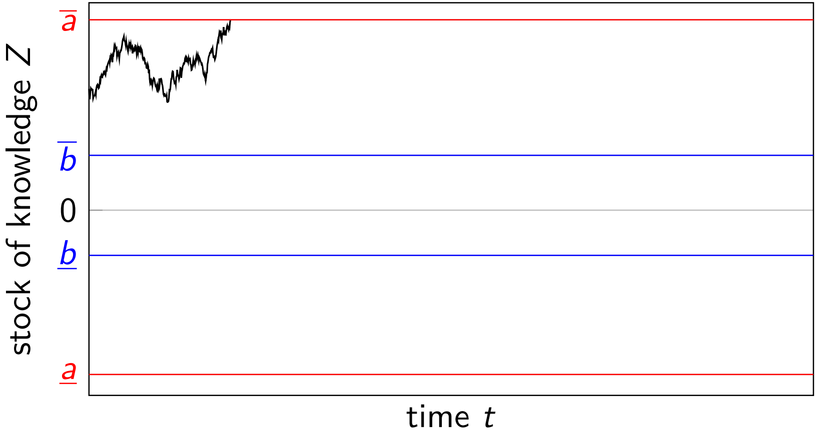

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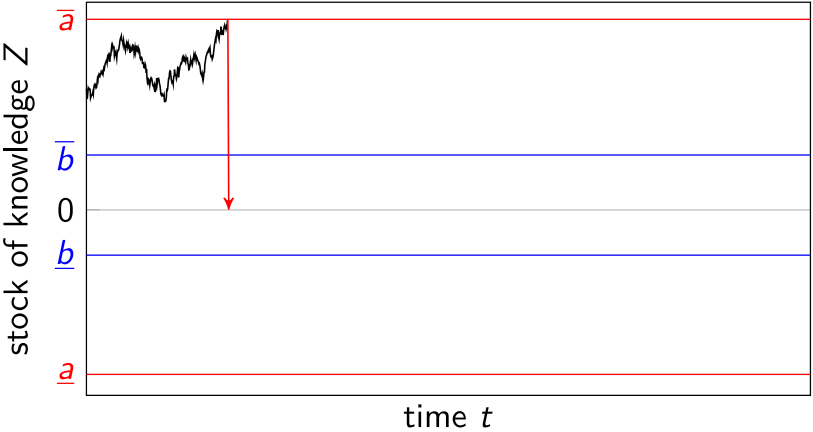

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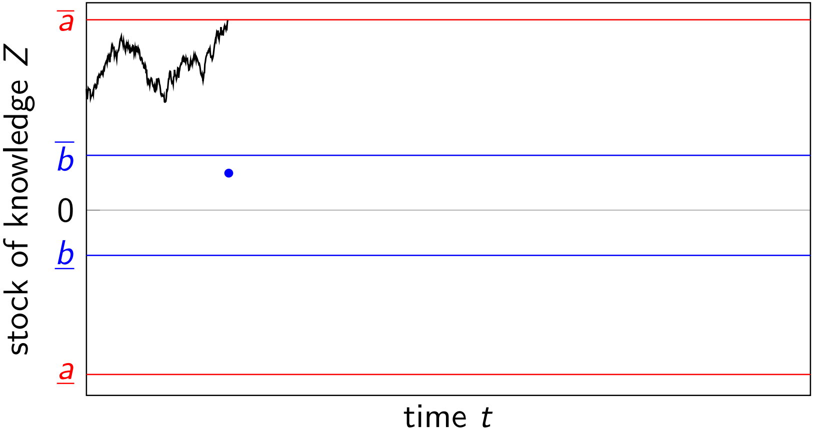

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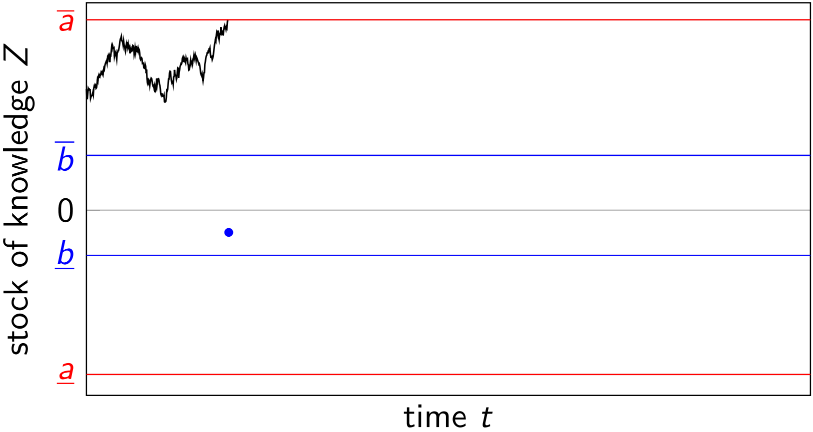

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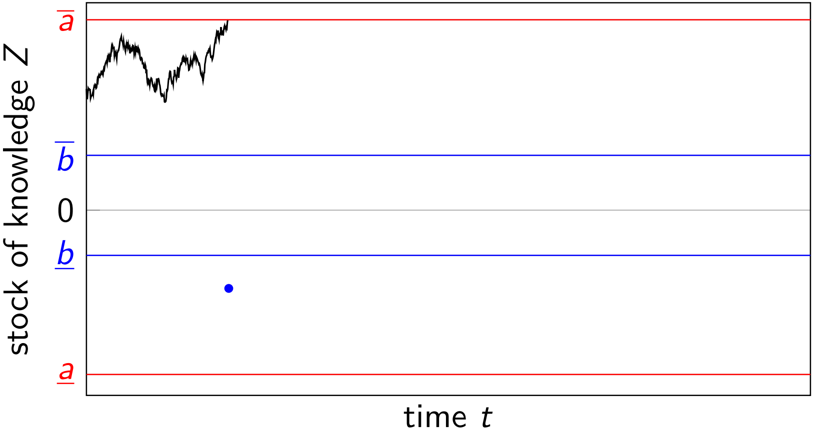

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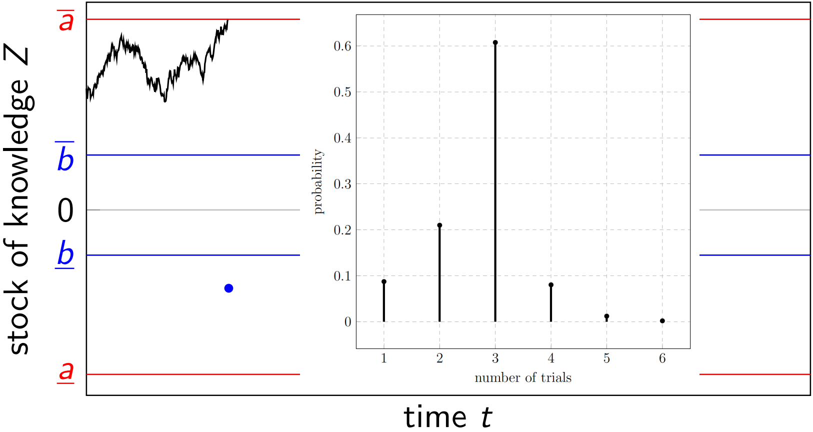

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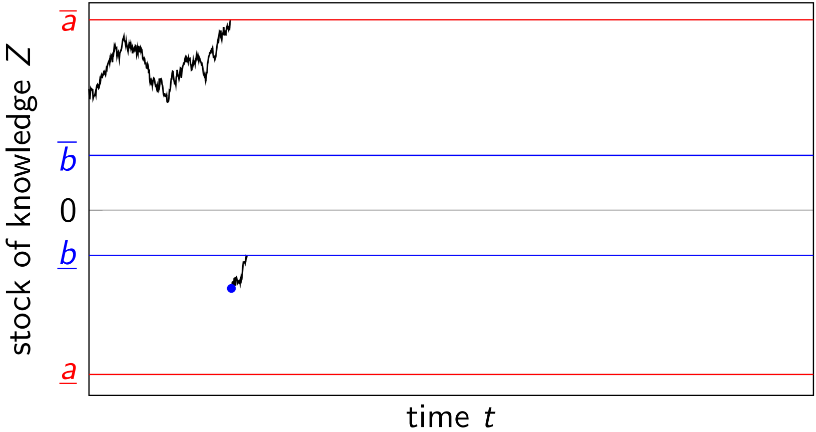

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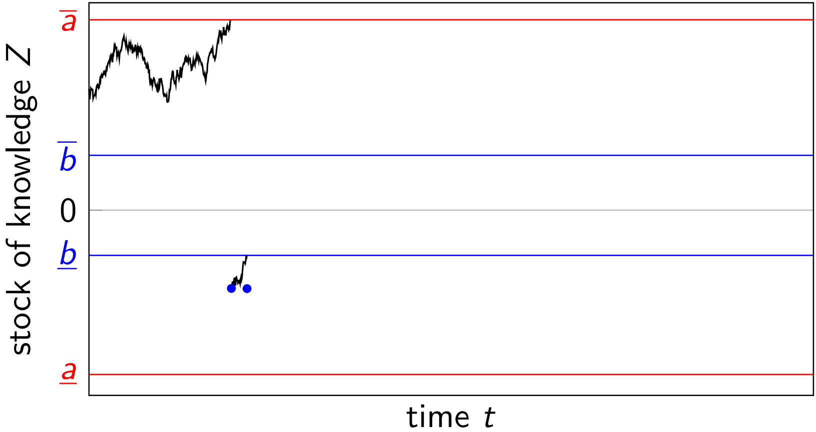

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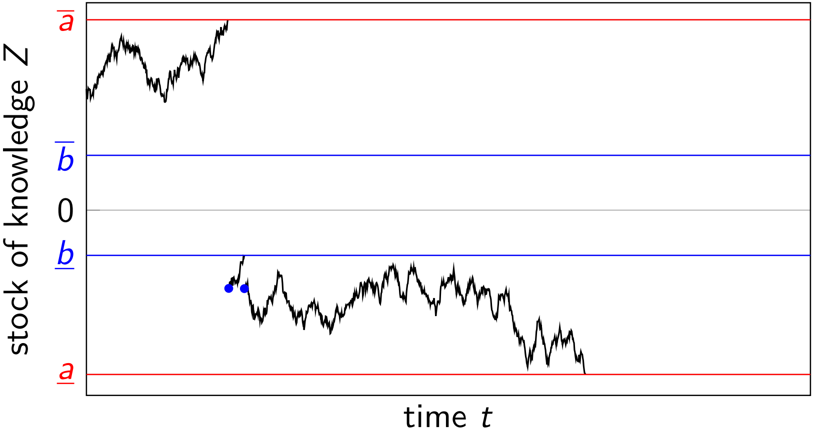

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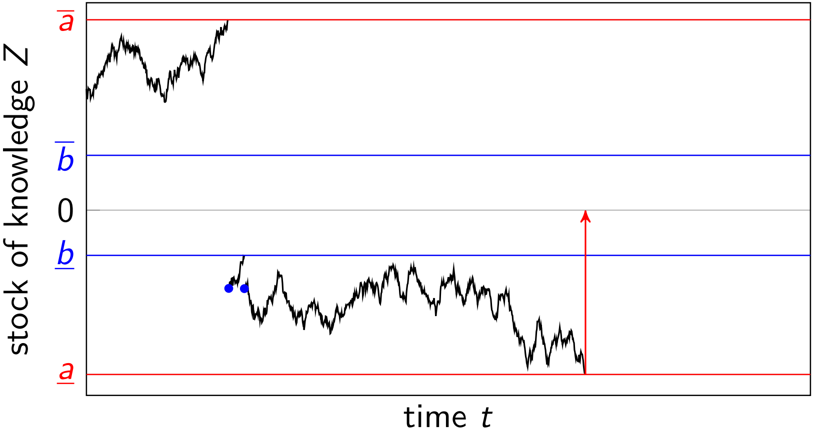

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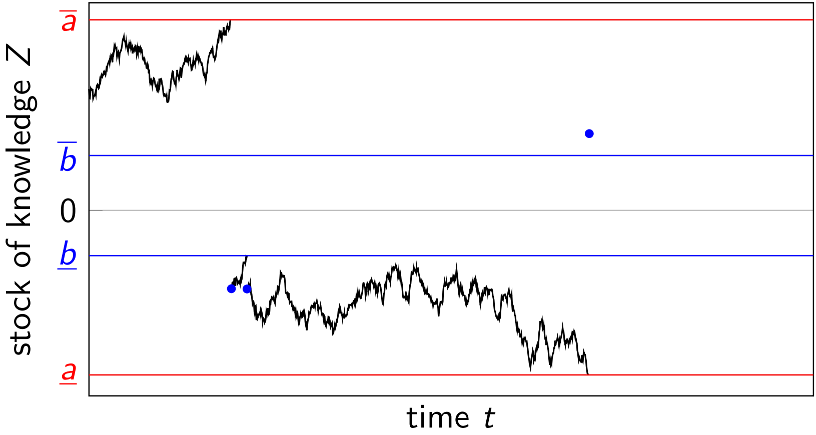

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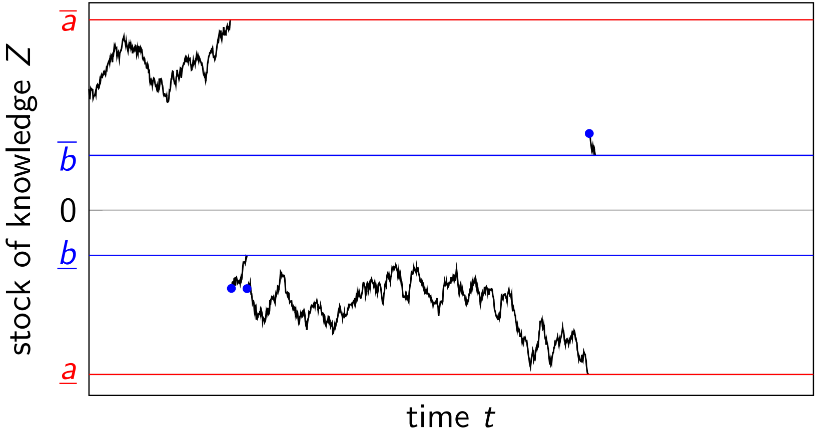

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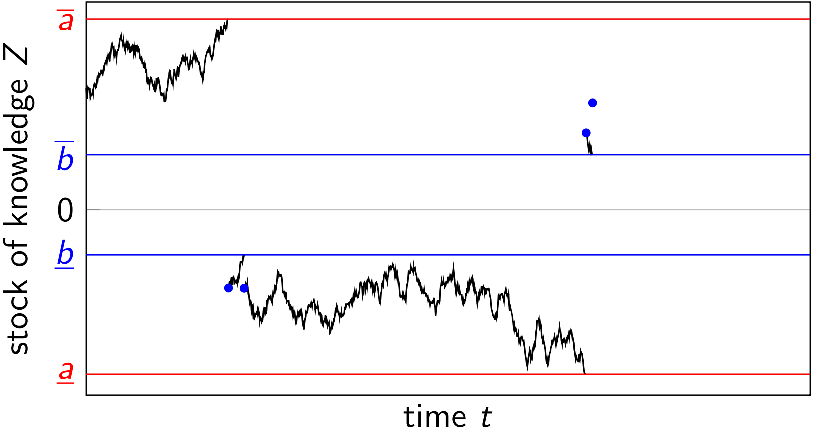

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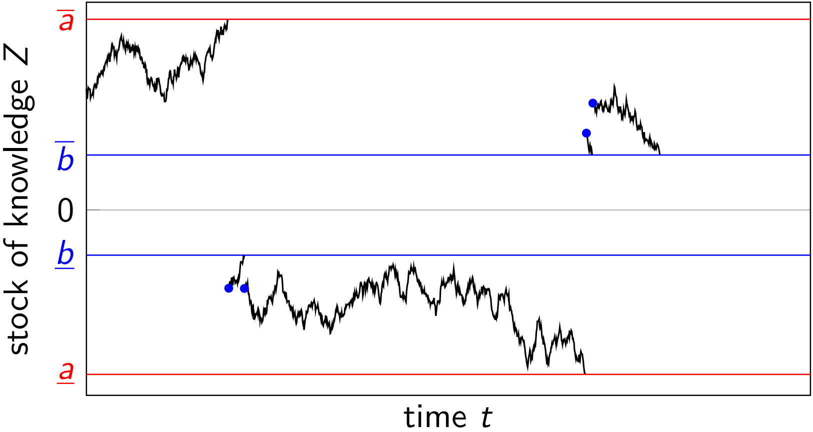

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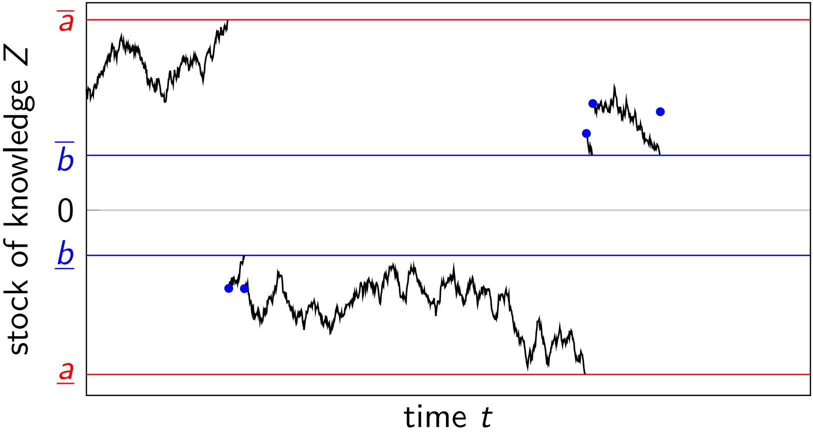

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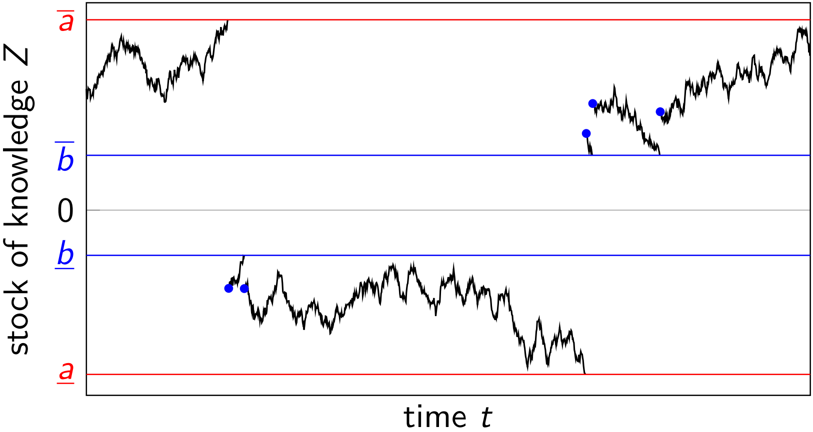

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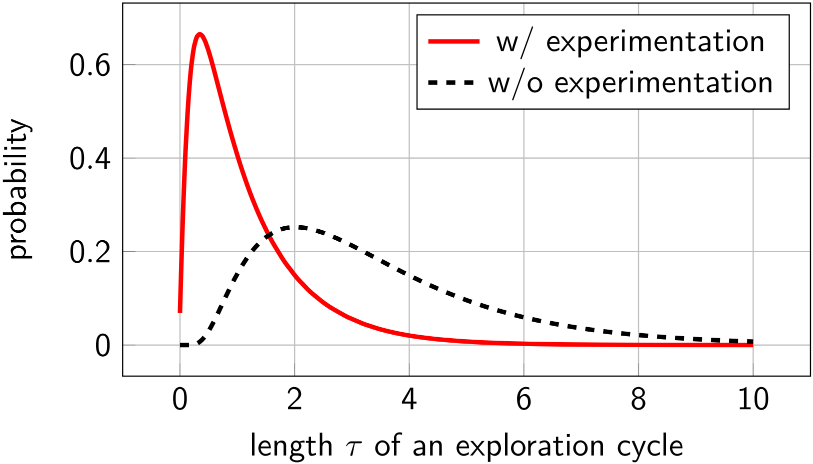
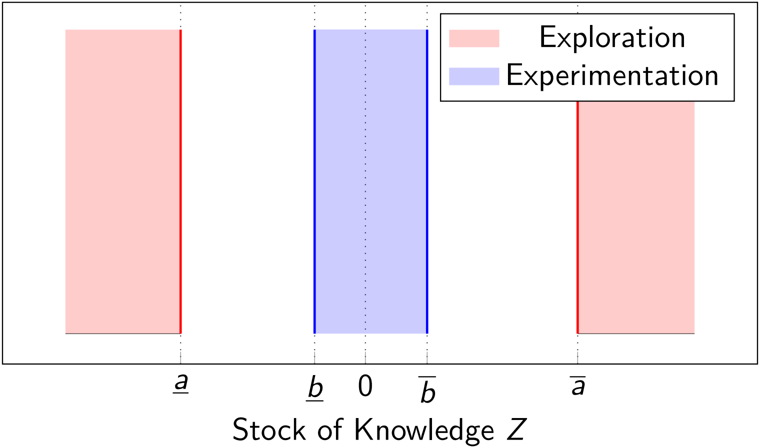


Firm Value

We compute the intensive firm value piecewise:

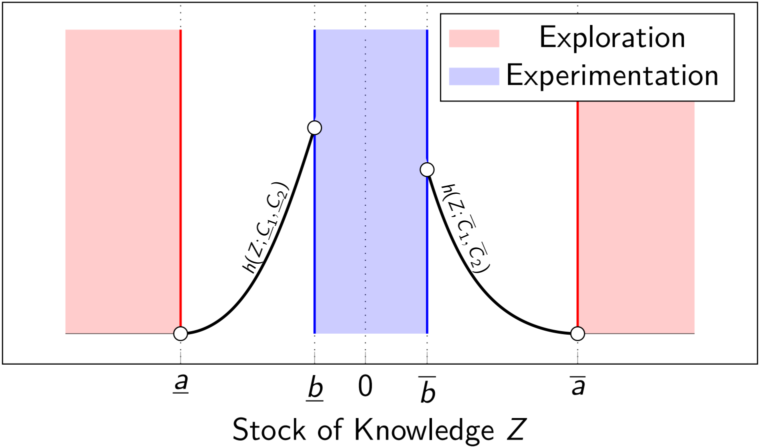
$v(Z) =$



Firm Value

We compute the intensive firm value piecewise when idle:

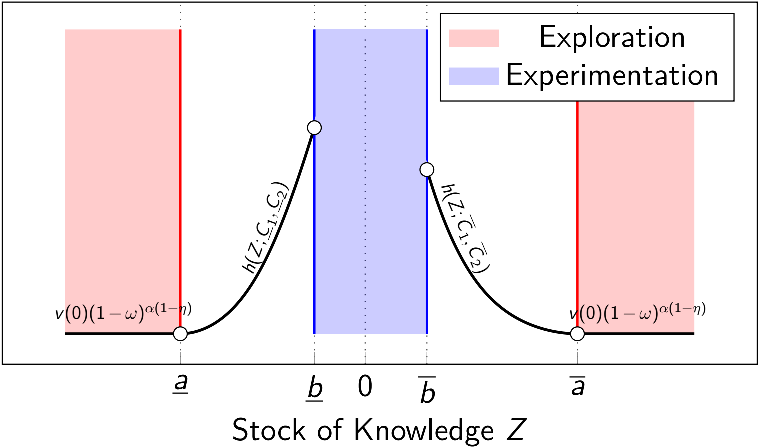
$$v(Z) = v_P(Z) + e^{g_1 Z + g_2 Z^2}(C_1 H_n(Z) + C_2 M(Z)) \equiv h(Z; C_1, C_2)$$



Firm Value

We compute the intensive firm value piecewise when exploring:

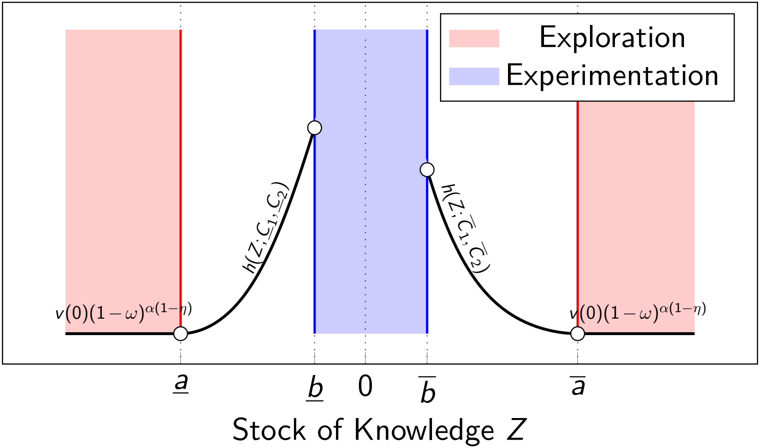
$$v(Z) = v(0)(1-\omega)^{\alpha(1-\eta)} \equiv g(0)$$



Firm Value

Firm value solves a Bellman equation when experimenting:

$V(Z) = \max_i -\kappa - \gamma/2 i^2$



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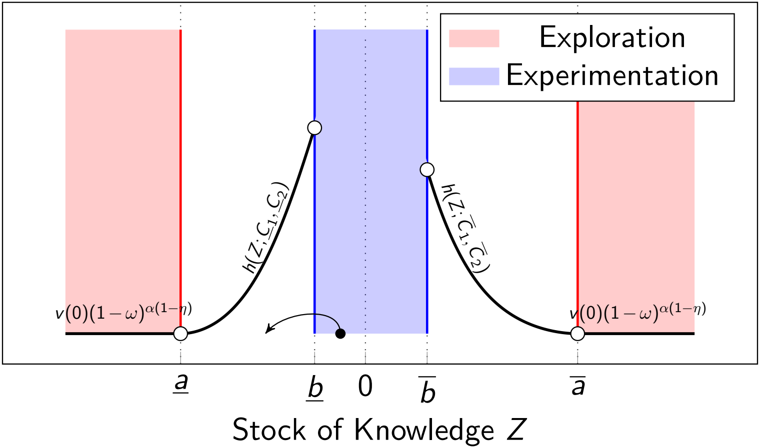
$V(Z) = \max_i -\kappa - \gamma/2 i^2 + (1 + i)^{\alpha(1-\eta)}\Big(\int_{\underline{a}}^{\underline{b}} h(x;\cdot) d\Phi(x;Z)$



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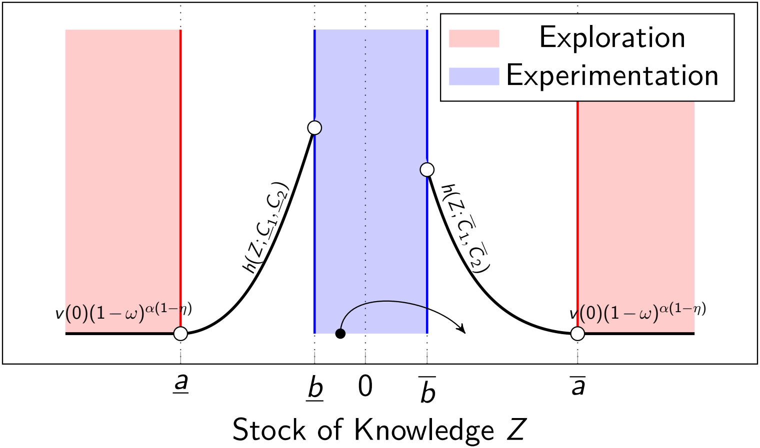

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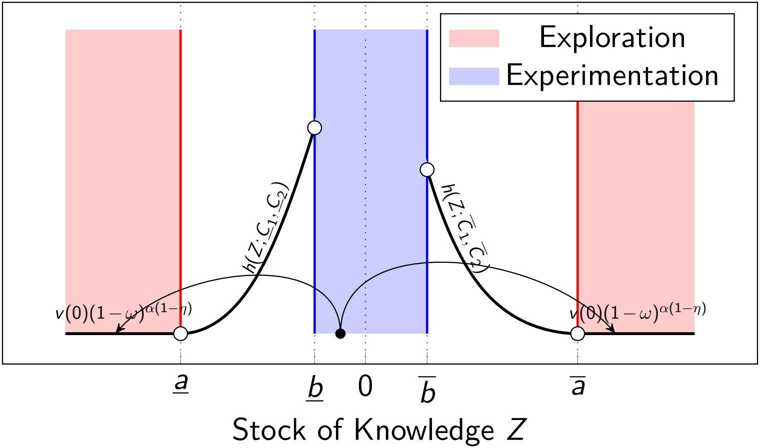

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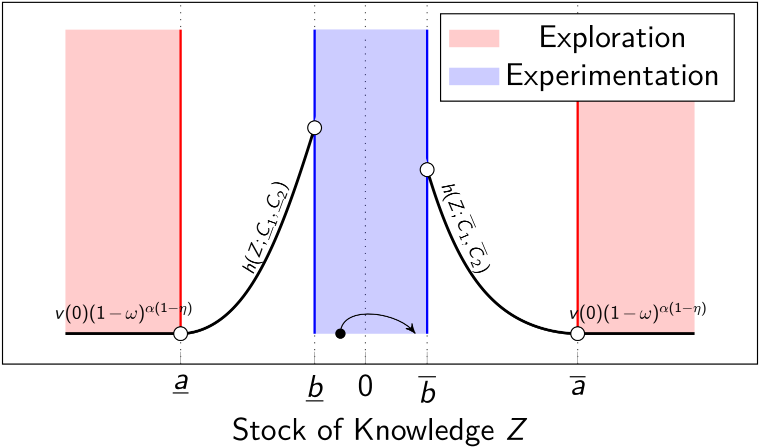

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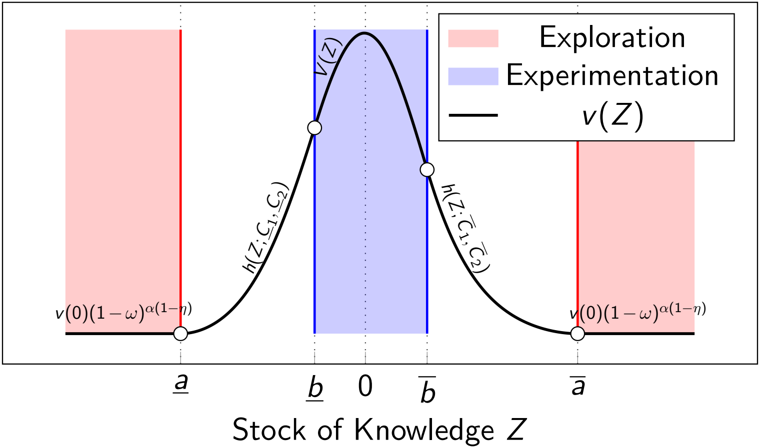

Corporate Investment and the Knowledge Channel

FOC for the optimal knowledge-contingent investment plan is:

$$i(Z) = \frac{1}{\gamma}\Big(q(Z) + \int_{\mathbb{R}} \frac{V(\cdot, K+I, x)}{\Pi(\cdot, K)} \frac{\mathrm{d}}{\mathrm{d}i}\varphi(x, i, Z)\mathrm{d}x\Big),$$

where $q(Z) \equiv \int_{\mathbb{R}} \frac{V_K(\cdot, K+I, x)}{\Pi(\cdot, K)}\varphi(x, i, Z)\mathrm{d}x$ (Abel & Blanchard (1986))

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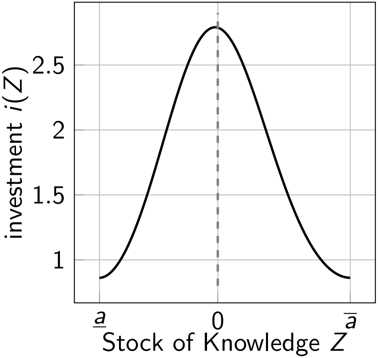
benchmark ($\tau_S = 0$ and $\kappa = 0$)



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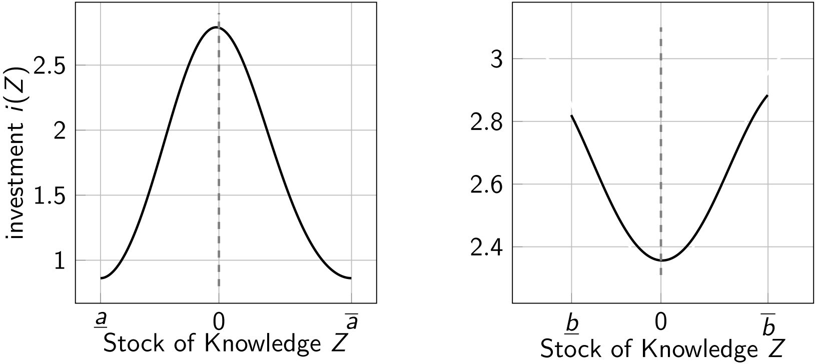

Corporate Investment and the Knowledge Channel

Use Stein's lemma to decompose knowledge channel:

$$\underbrace{\int_{\mathbb{R}} \frac{V(\cdot, K+I, x)}{\Pi(\cdot, K)} \frac{\mathrm{d}}{\mathrm{d}i} \varphi(x, i, Z) \mathrm{d}x}_{\text{knowledge channel } \propto \tau_S} = \frac{\tau_S}{2}(1+i)^{\alpha(1-\eta)} \Big(\underbrace{\frac{Z}{\sqrt{1+\tau_S i}} \int_{\mathbb{R}} v'(x) \varphi(x, i, Z) \mathrm{d}x}_{\text{marginal product of knowledge}} + \underbrace{\int_{\mathbb{R}} v''(x) \varphi(x, i, Z) \mathrm{d}x}_{\text{attitude towards risk}} \Big)$$

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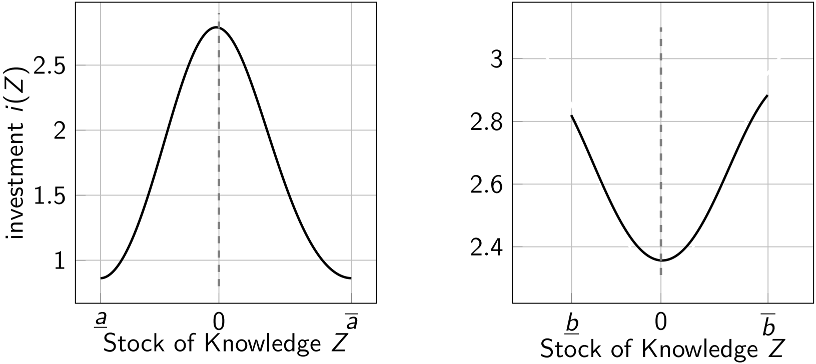

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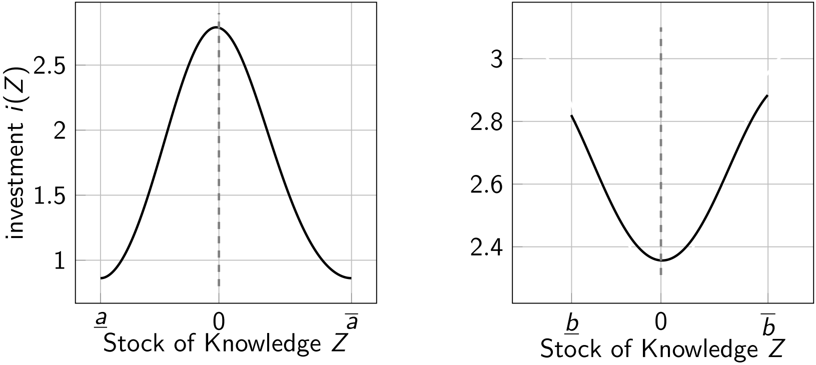

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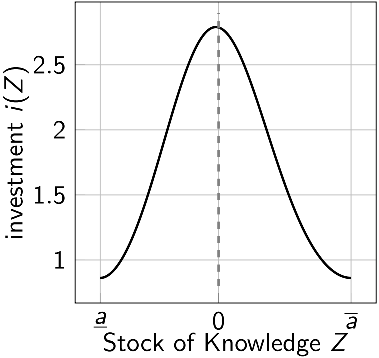
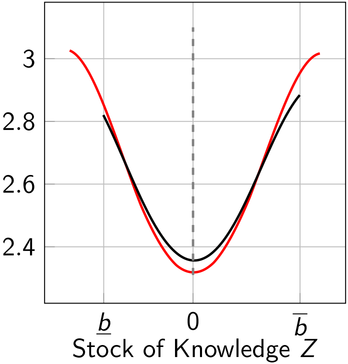


$\nearrow$ competitive pressures



Q-Investment Relation and Knowledge Cycles

Typical investment-average $Q(Z) \equiv V(K, Z, \cdot)/\Pi(K, \cdot)$ regression:

$$i(Z_t) = \alpha_k + \beta_k Q(Z_t) + \epsilon_t, \quad |Z_t| \in \text{quartile}_k, \quad k = 1, ..., 10$$

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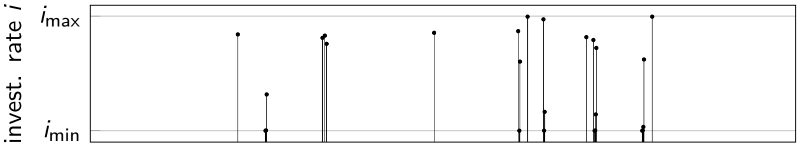

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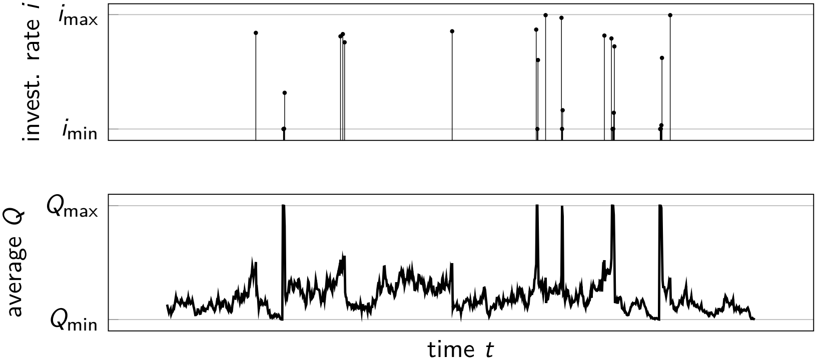

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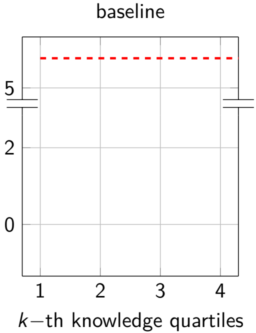

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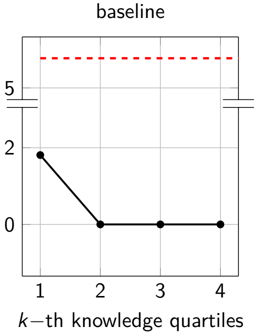

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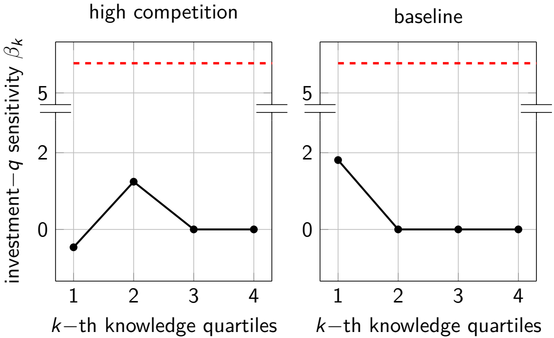

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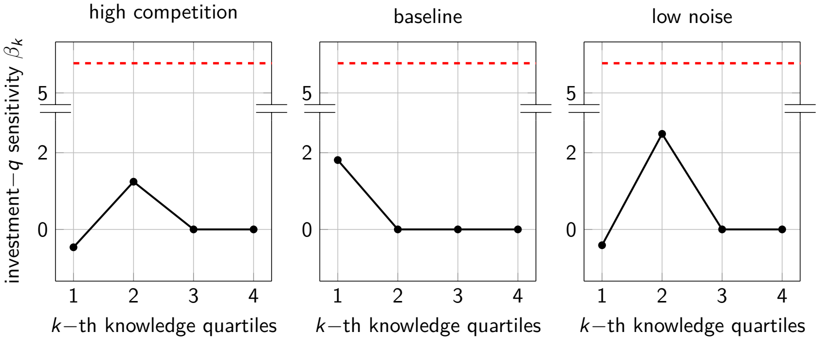

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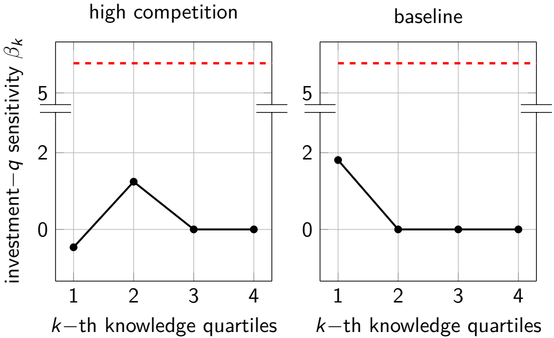

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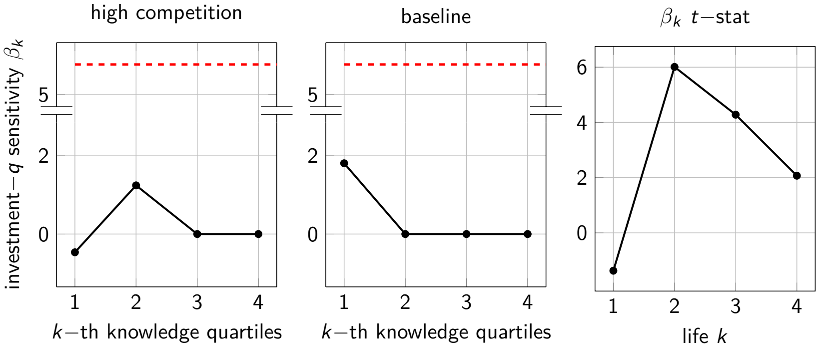

Hoberg and Maksimovic (2019)
Q on CAPX across product cycle

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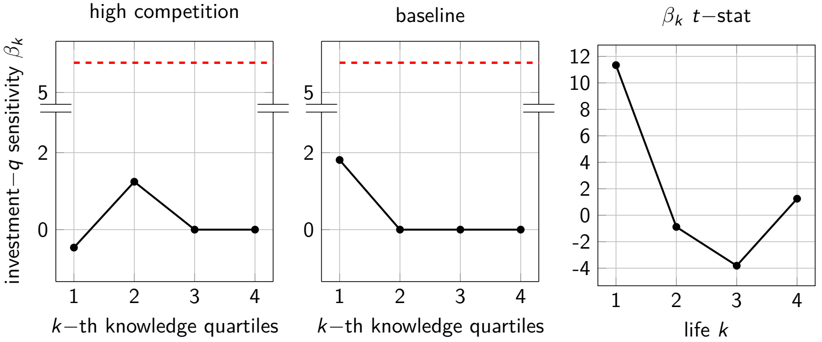

Hoberg and Maksimovic (2019)
Q on R&D across product cycle

Ideas for testing the model

- Measuring knowledge cycles from patents' text
  - Appearance and disappearance of technological words
  - Similar to Bowen et al. (2019)
  - Exploit variation in investment (and Q) across industries
- Measuring knowledge cycles from product descriptions
  - Similar to Hoberg and Maksimovic (2019)
  - Exploit variation in investment (and Q) across firms
- Empirical relevance and economic quantification (structural?)

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