

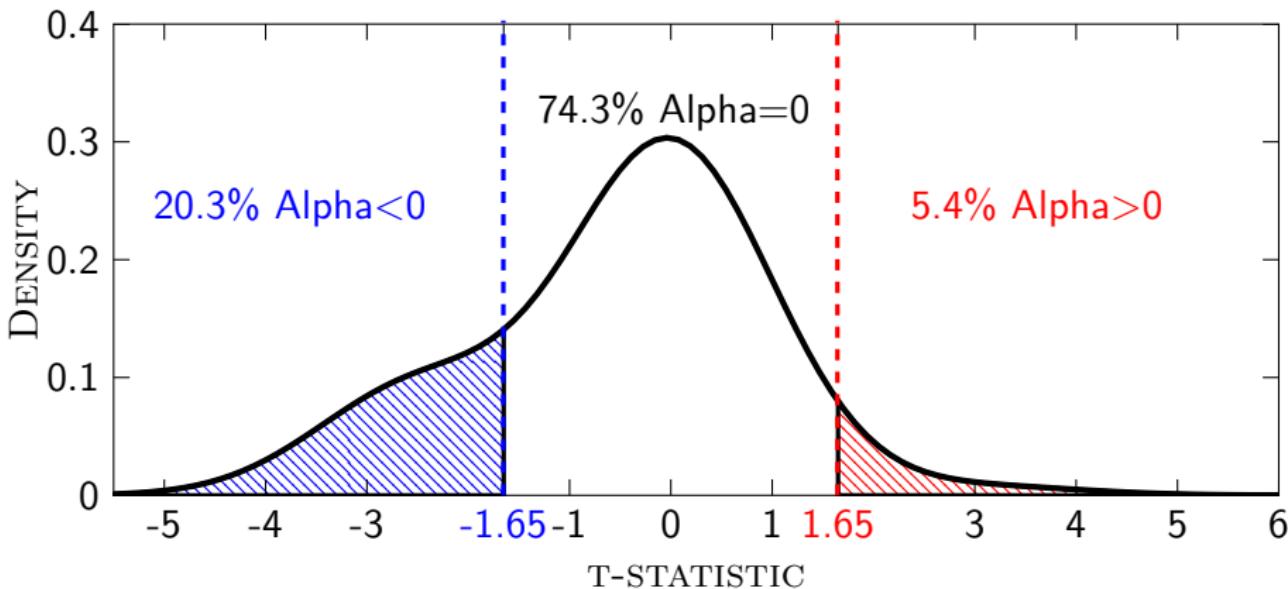
# Social Interactions and the Performance of Mutual Funds

Julien Cujean



LAEF 2016, Santa Barbara

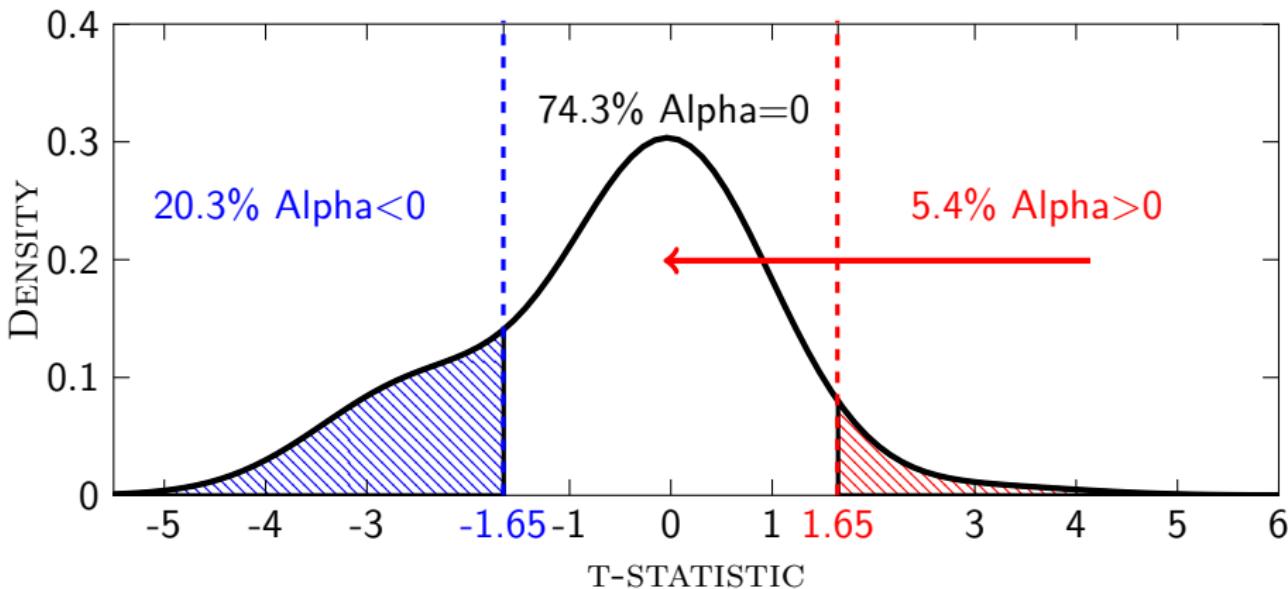
## A pervasive fact in the mutual fund industry



Cross-sectional distribution of alpha's t-stats  
*Skilled managers merely represent 2% of the population*

SOURCE: BARRAS, SCAILLET & WERMERS (JF, 2010)

## A pervasive fact in the mutual fund industry

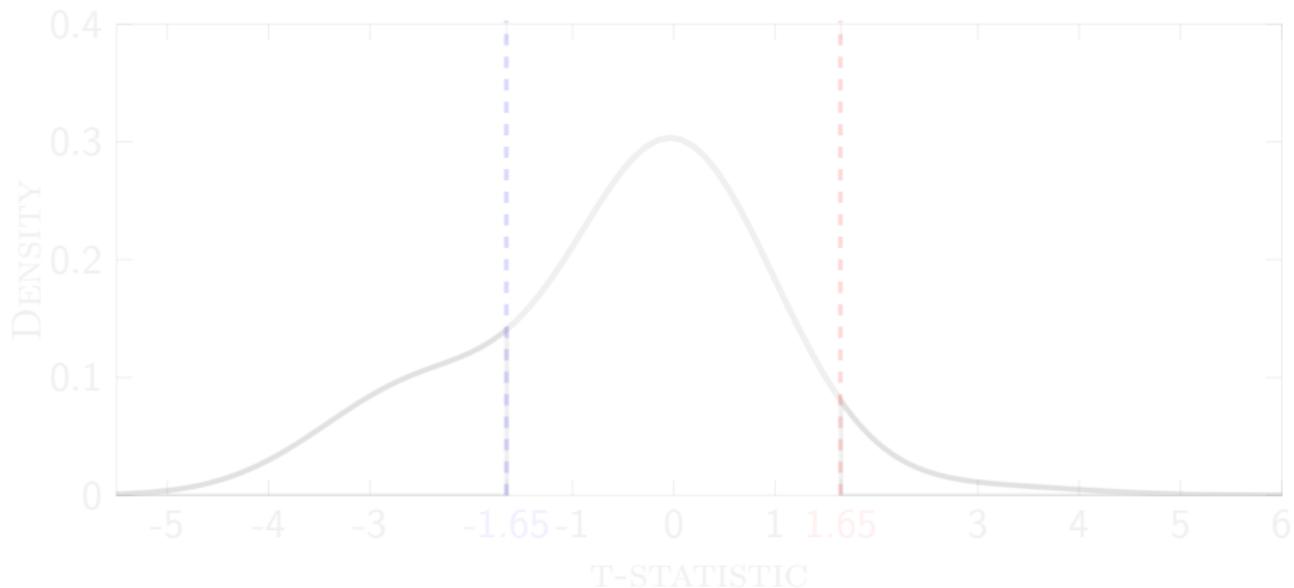


Cross-sectional distribution of alpha's t-stats

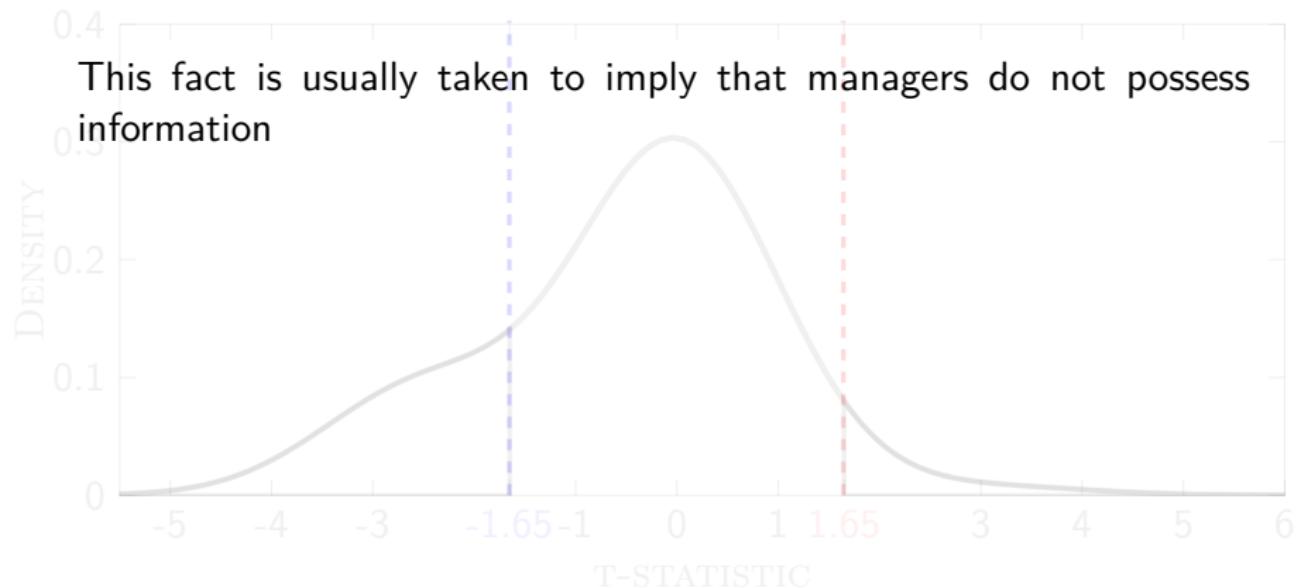
*Managers do not maintain their performance*

SOURCE: BARRAS, SCAILLET & WERMERS (JF, 2010)

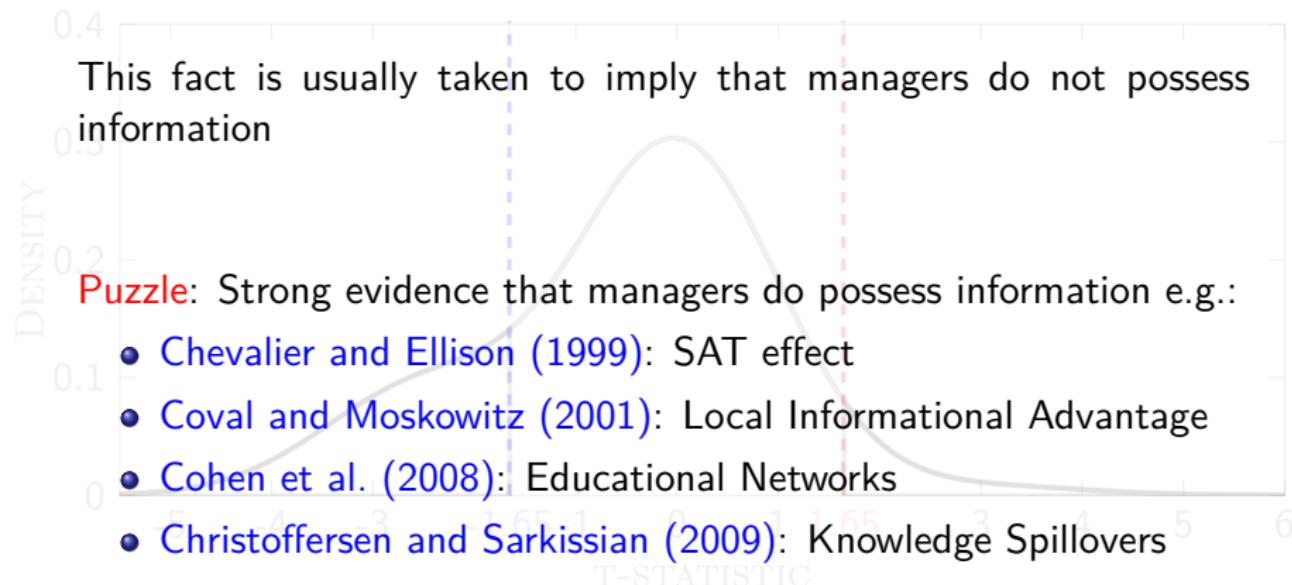
# Puzzling Evidence



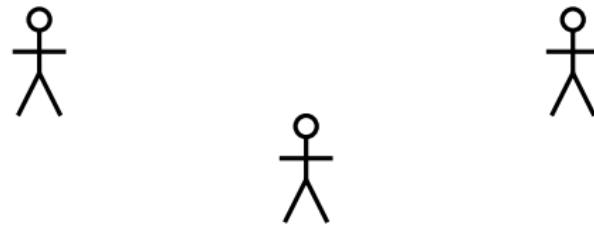
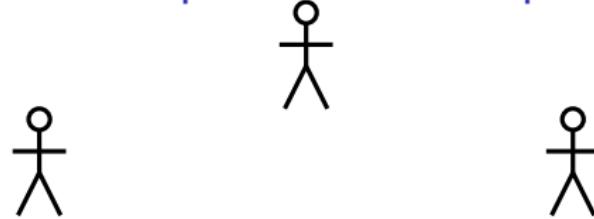
## Puzzling Evidence



# Puzzling Evidence

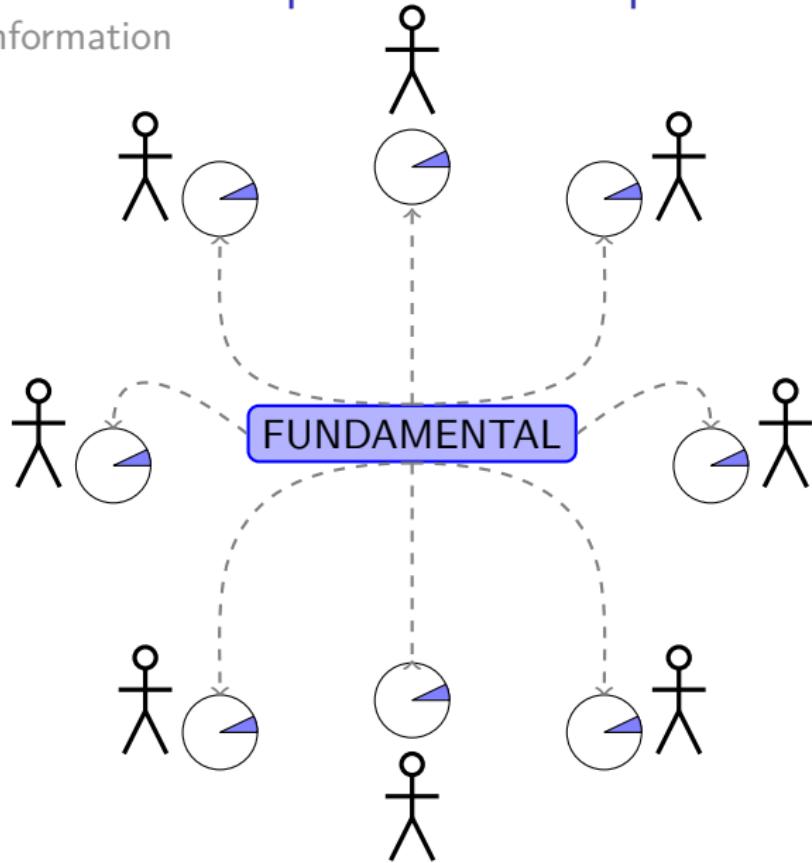


# A Standard Rational-Expectations Setup



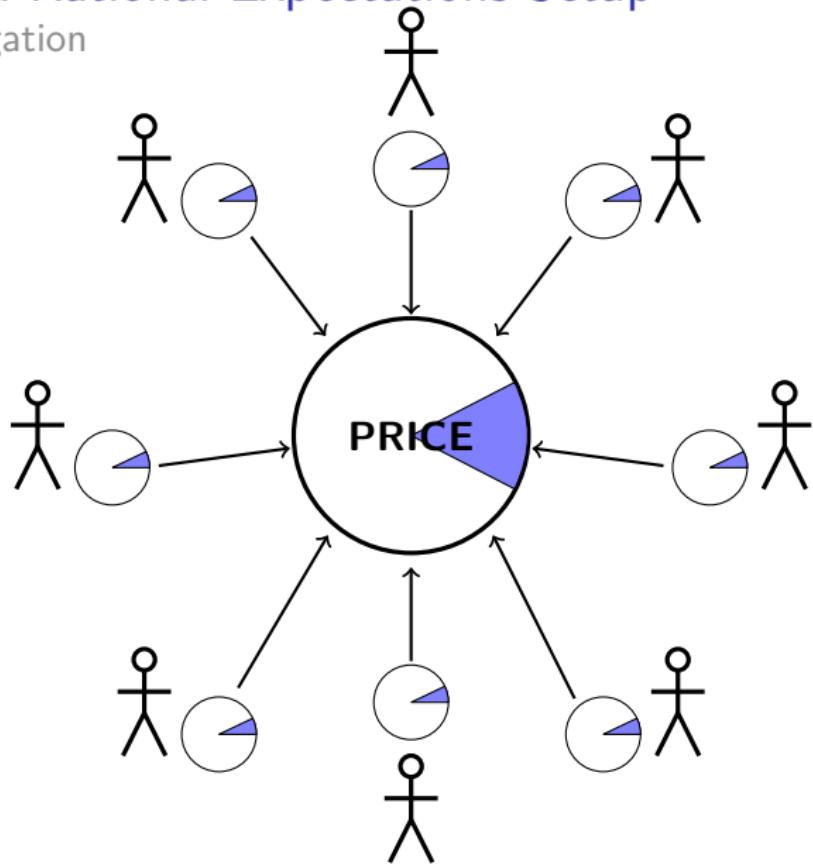
# A Standard Rational-Expectations Setup

Dispersed Information



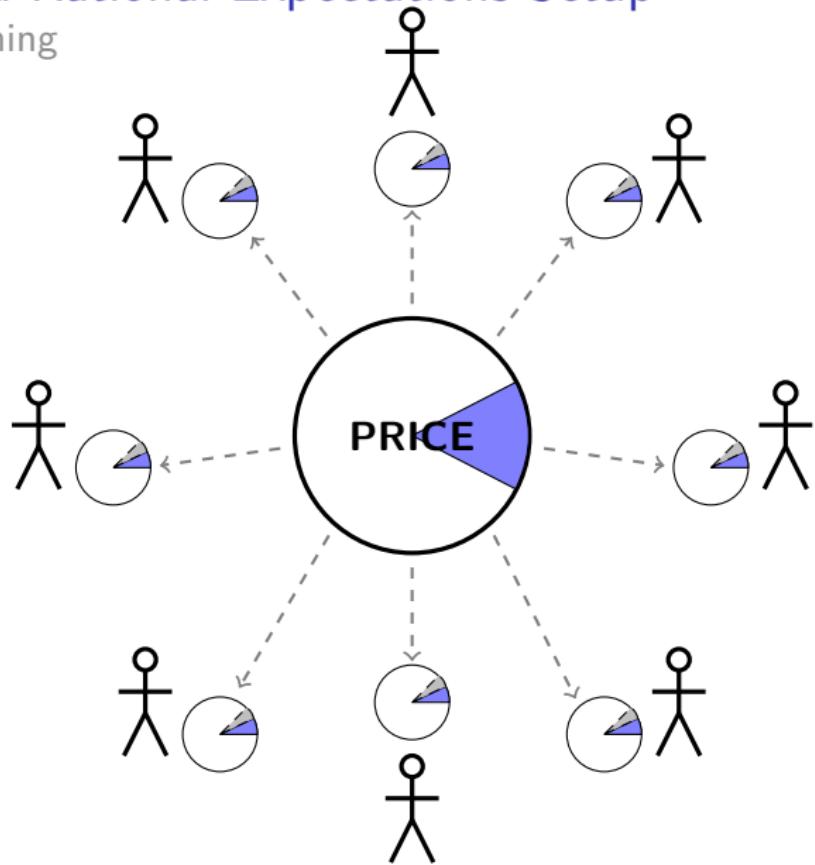
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Aggregation



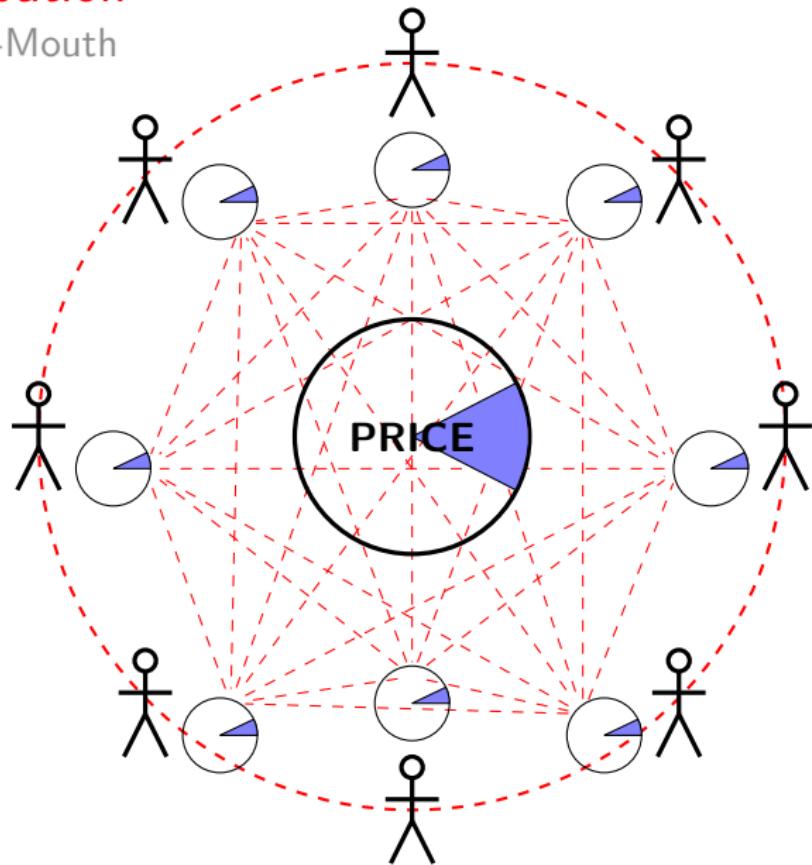
# A Standard Rational-Expectations Setup

Learning



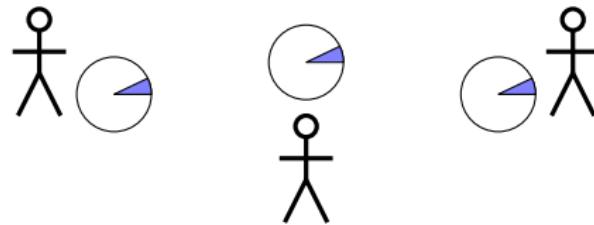
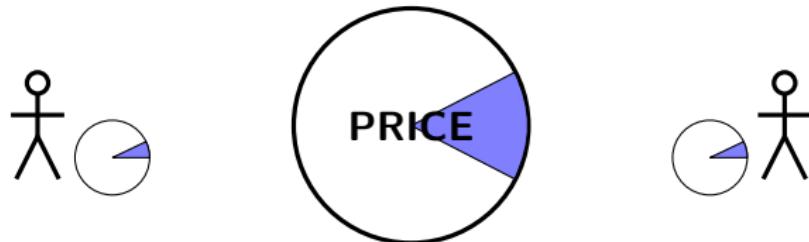
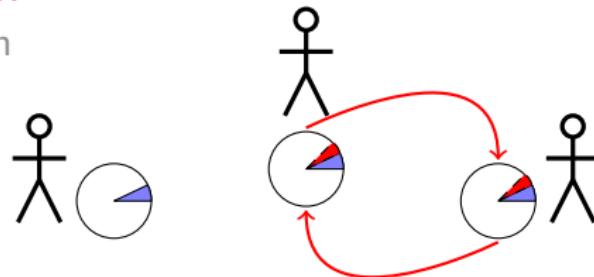
# My Contribution

Word-of-Mouth



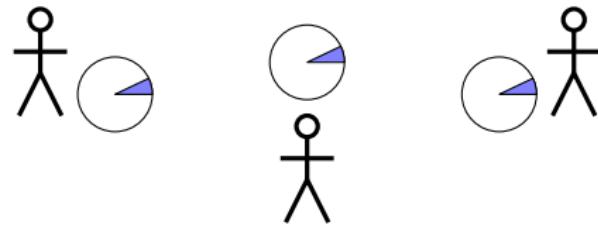
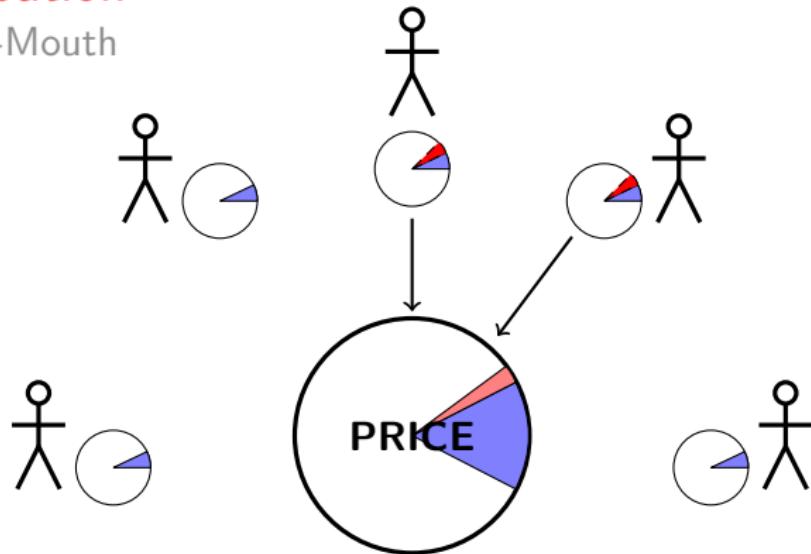
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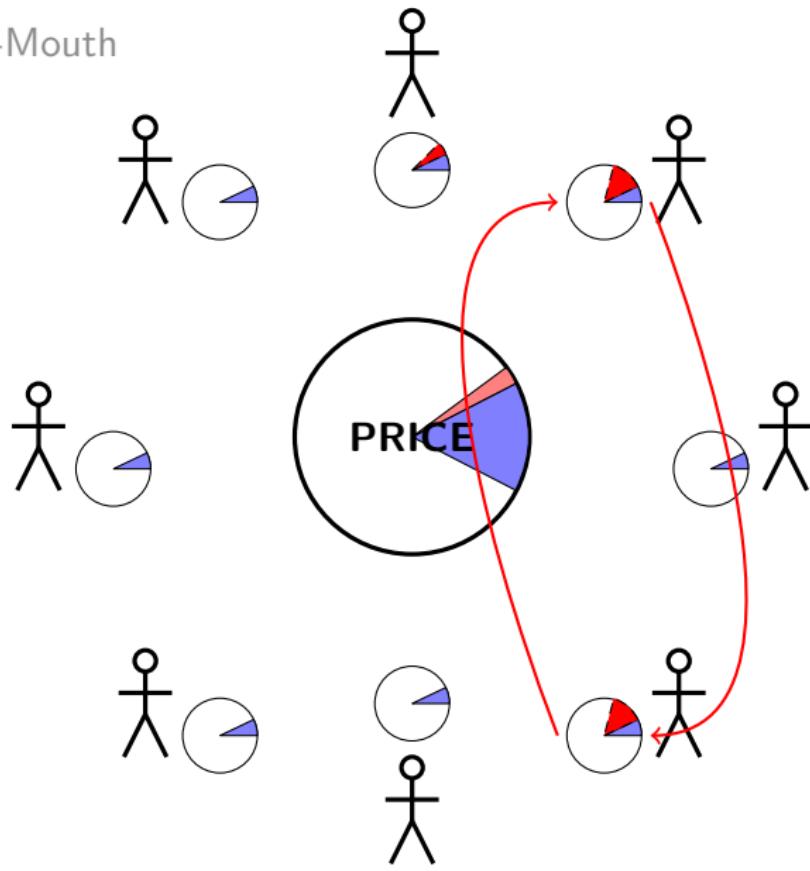
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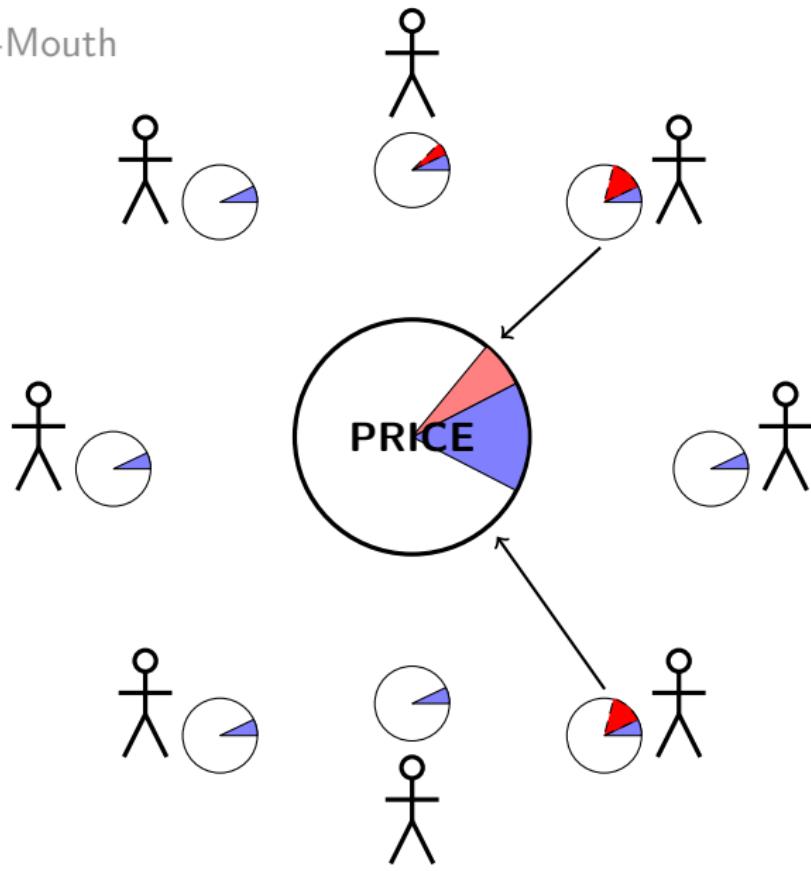
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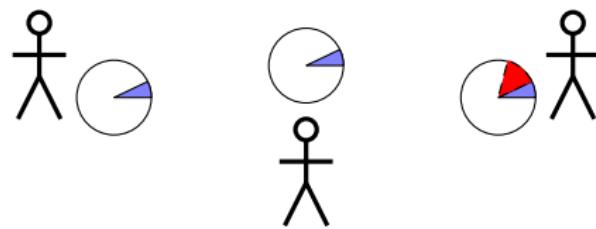
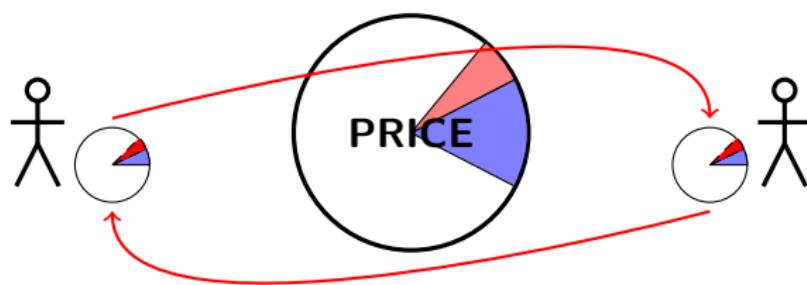
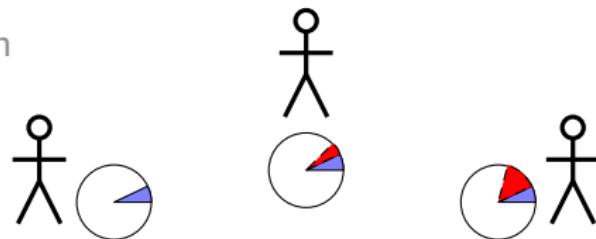
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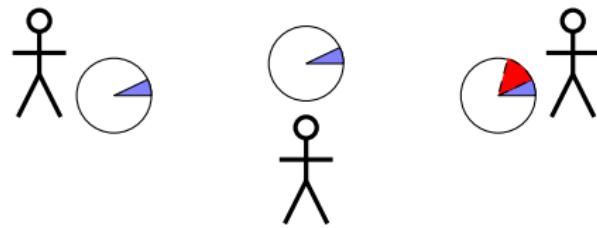
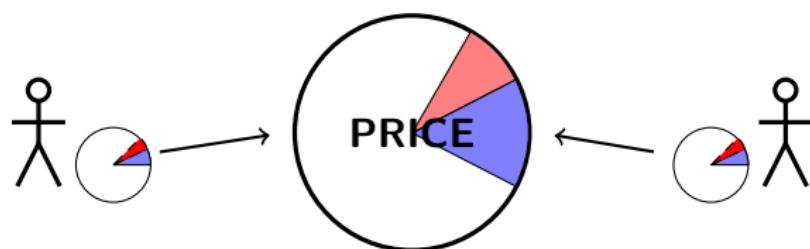
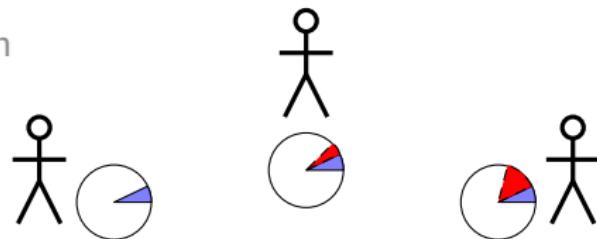
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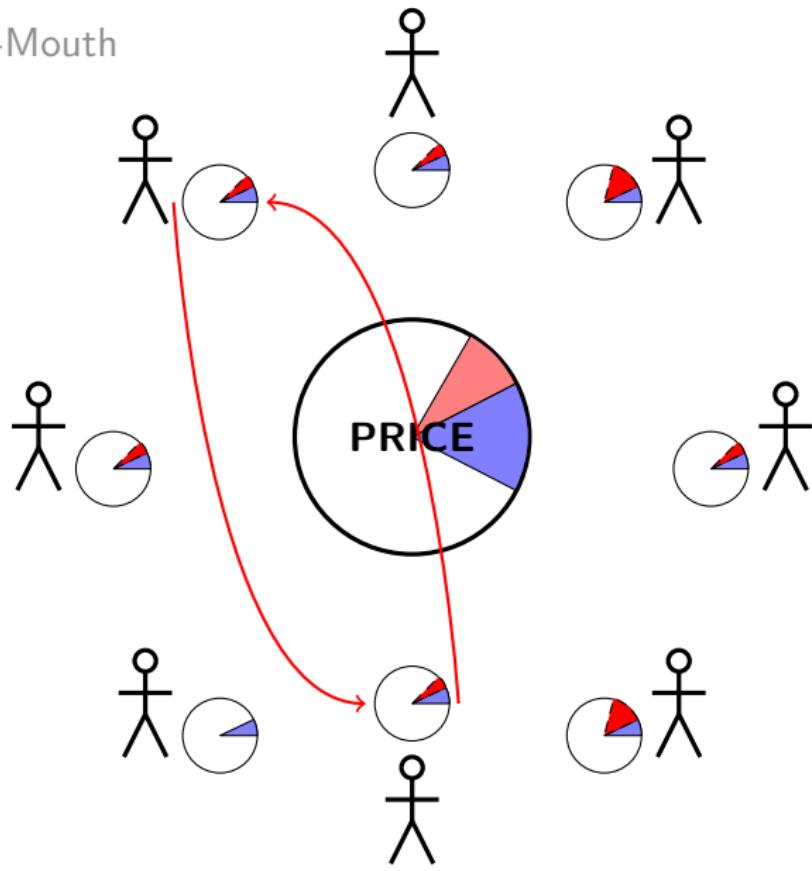
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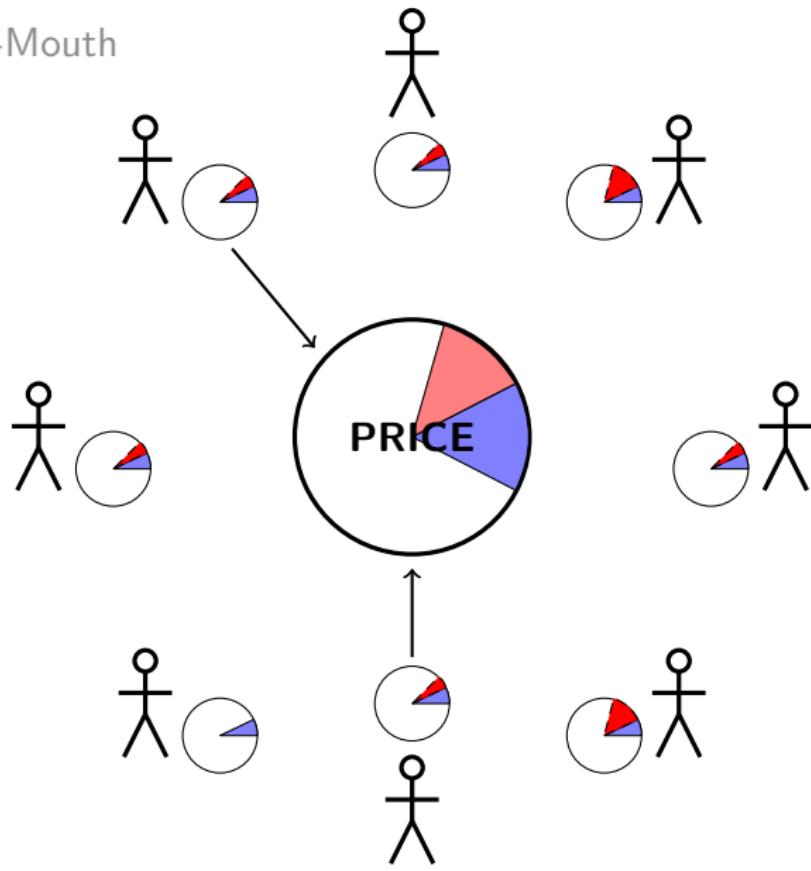
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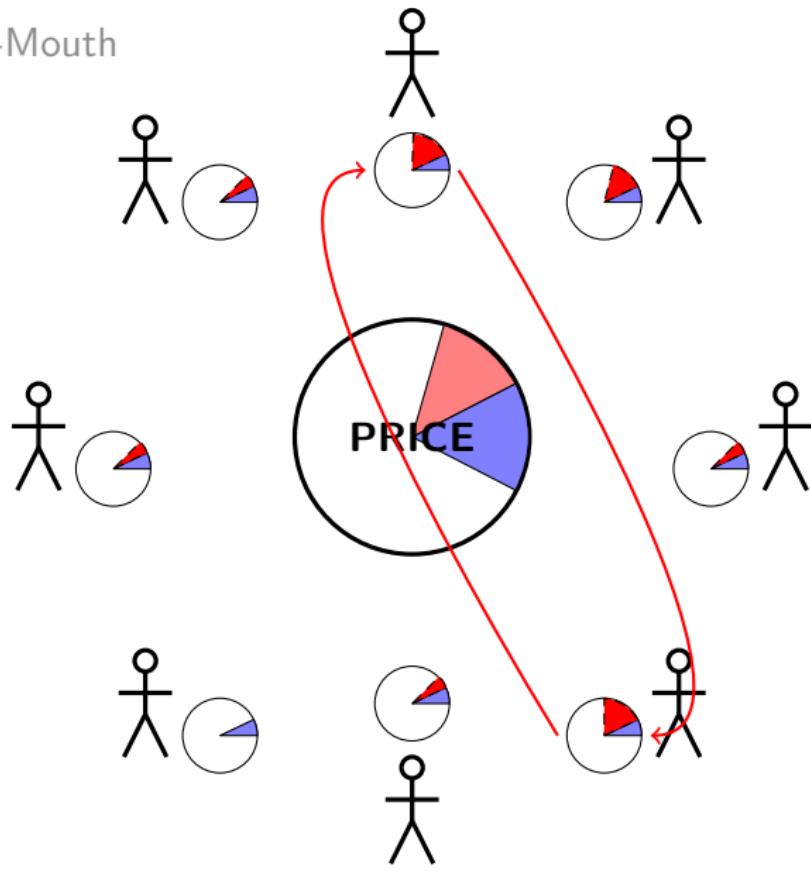
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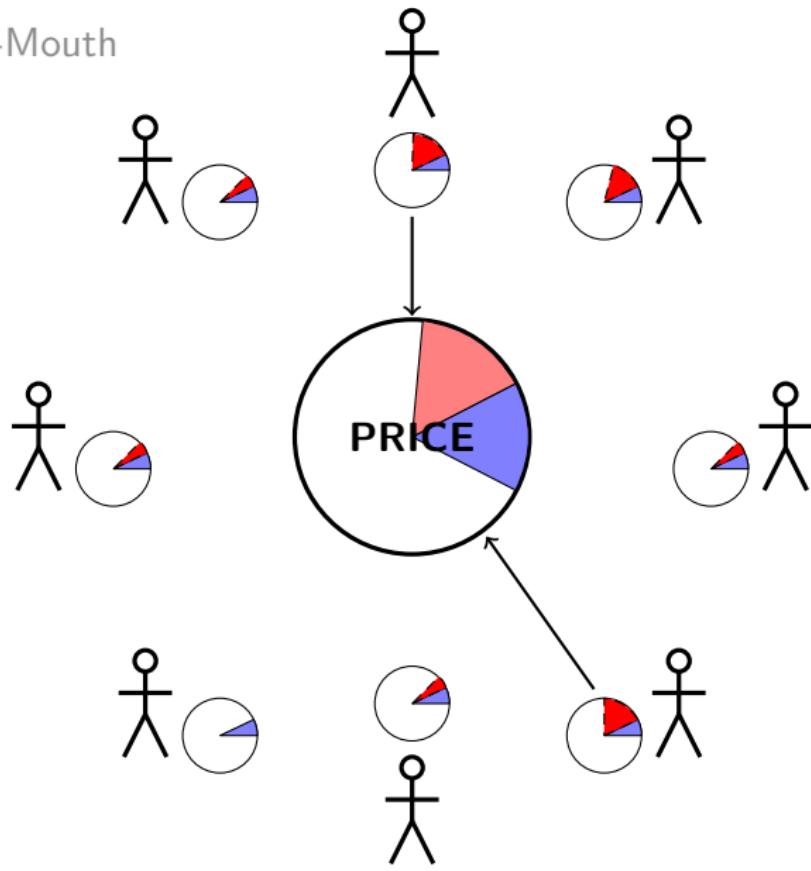
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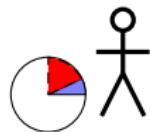
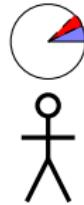
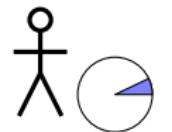
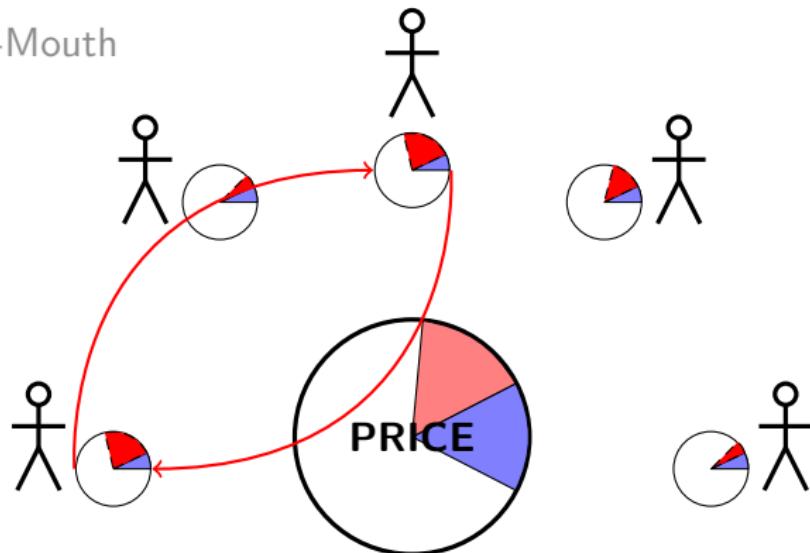
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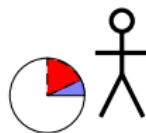
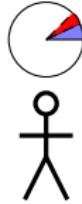
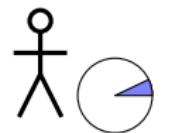
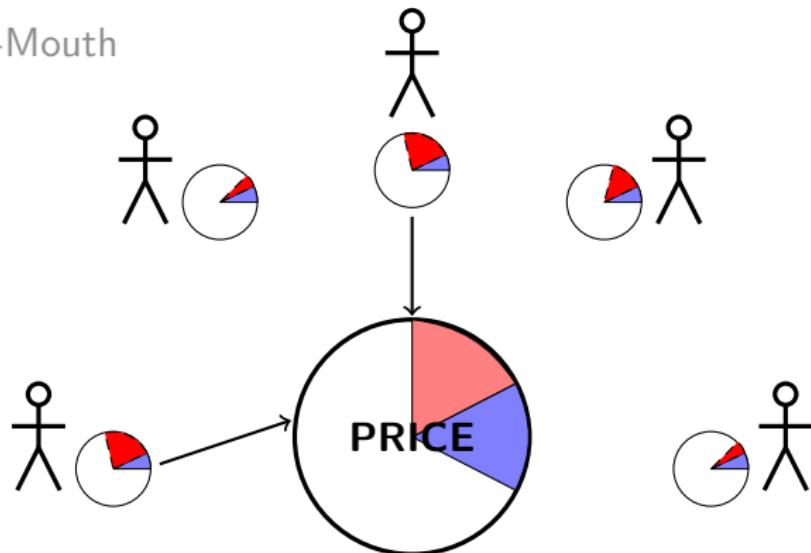
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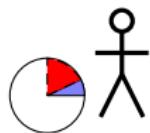
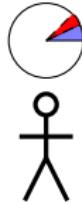
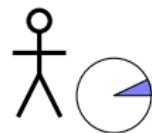
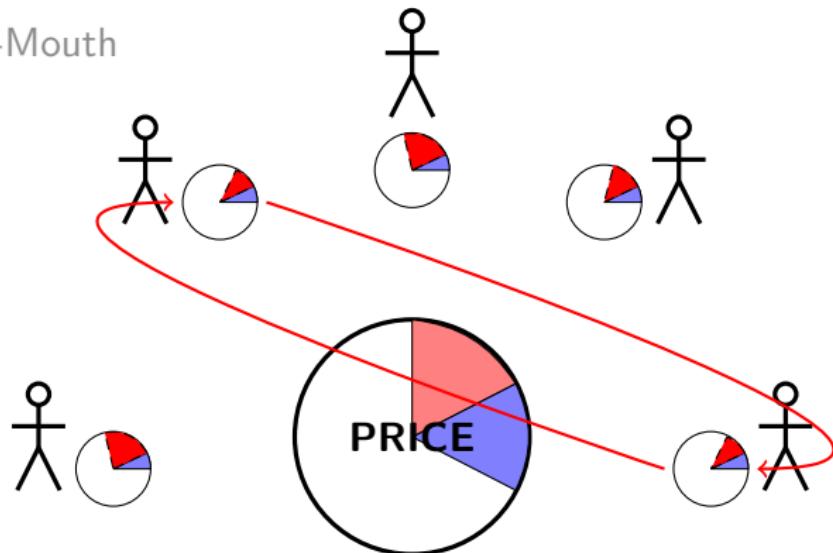
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Word-of-Mouth



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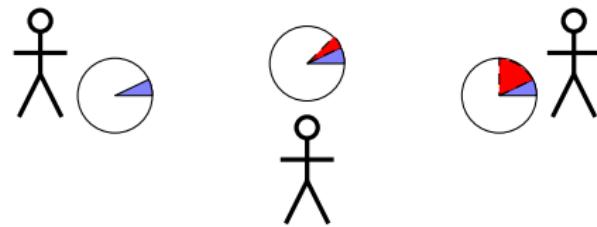
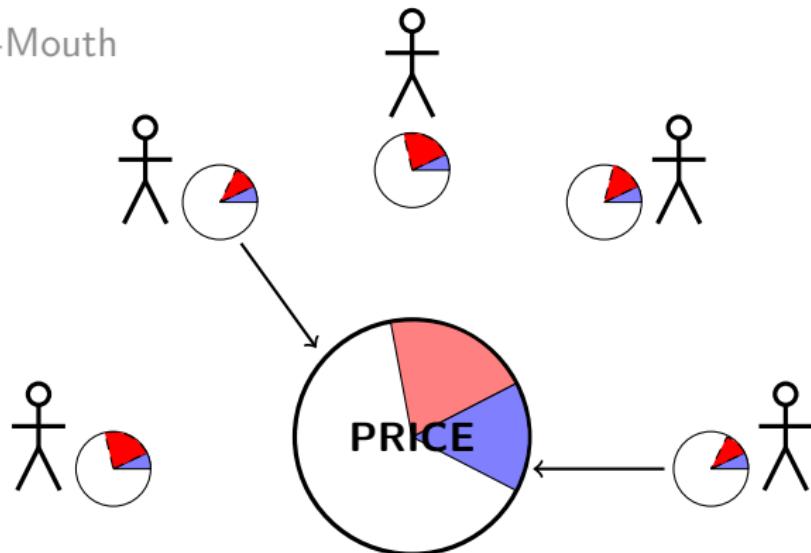
Word-of-Mouth



J. Cujean

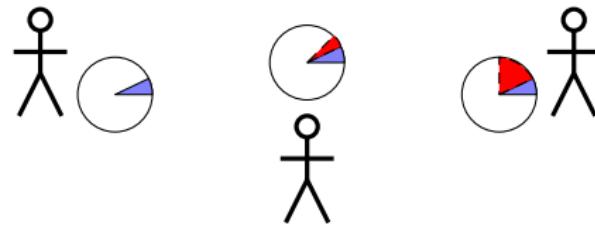
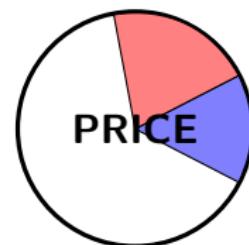
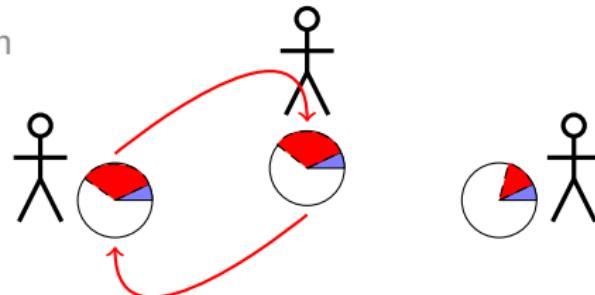
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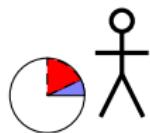
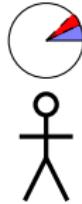
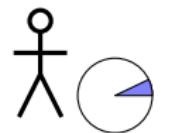
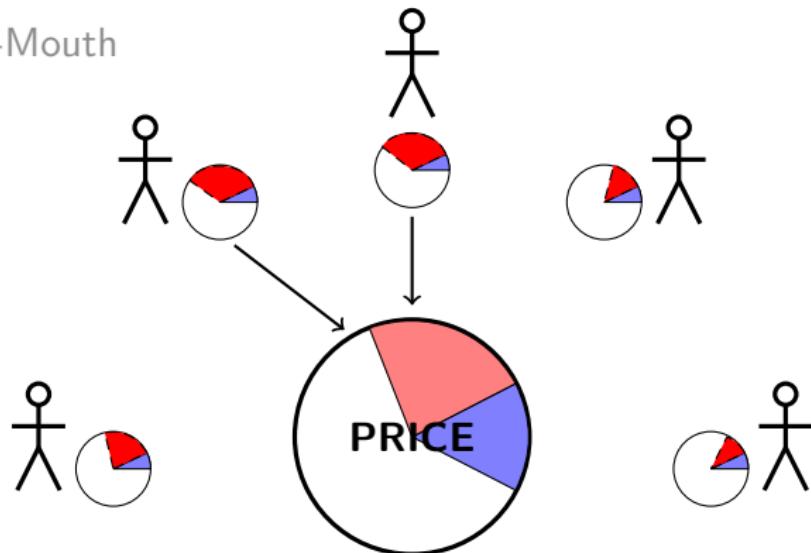
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Word-of-Mouth



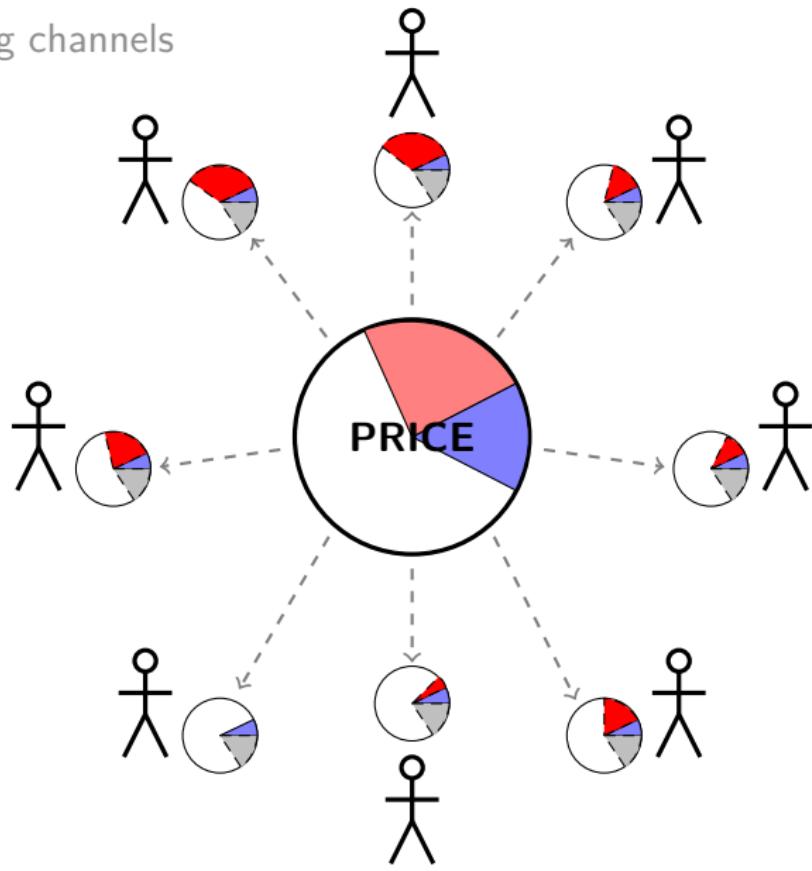
# My Contribution

Word-of-Mouth



# My Contribution

Two learning channels



# Outline

I Model

II Solution Method

III Results

# Outline

I Model

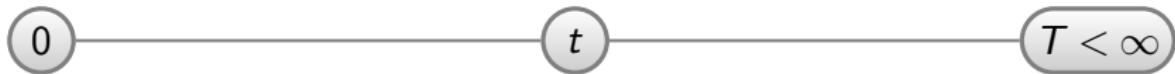
II Solution Method

III Results

# Setup

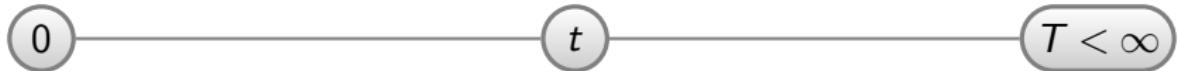
# Setup

Continuous trading on  $[0, T)$



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Announcement  
 $\Pi \sim \mathcal{N}(0, \sigma_\Pi^2)$

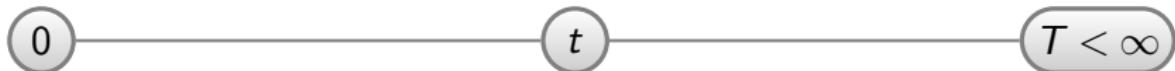
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1) Risky Claim  $P$  to

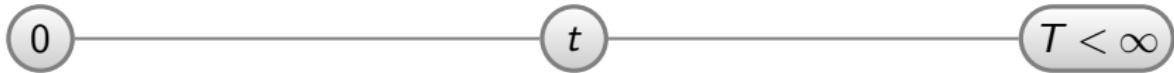
Announcement

$$\Pi \sim \mathcal{N}(0, \sigma_{\Pi}^2)$$

2) Bond ( $r = 0$ )

# Setup

Continuous trading on  $[0, T)$



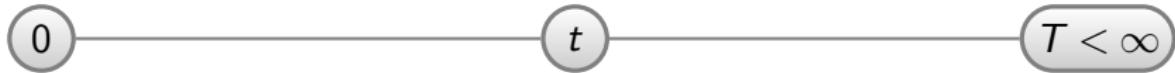
Crowd of managers  
 $i \in [0, 1]$  with CARA =  $\gamma$   
utility

- 1) Risky Claim  $P$  to
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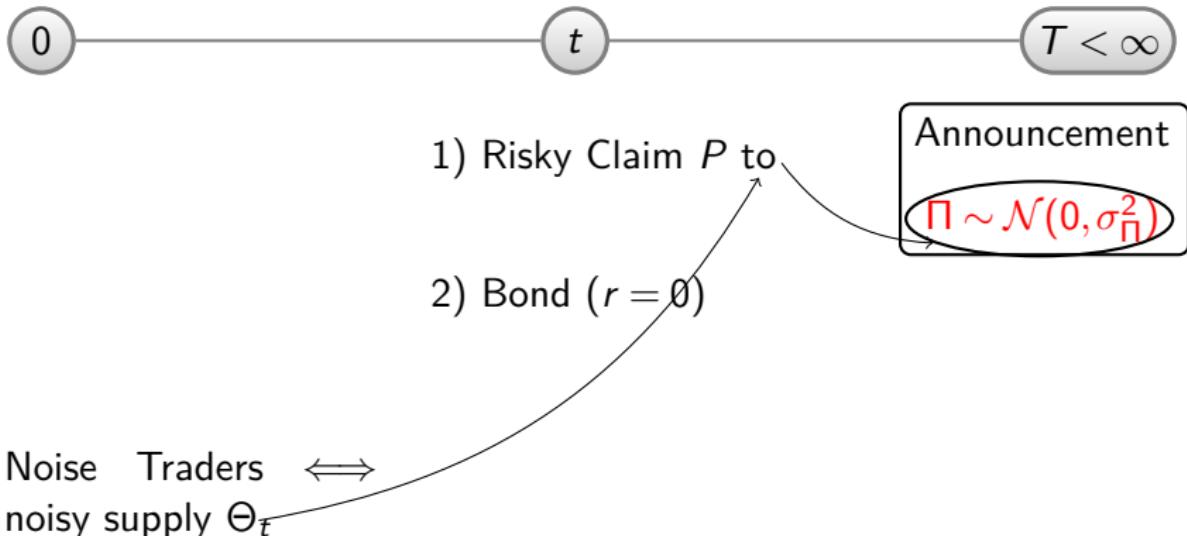
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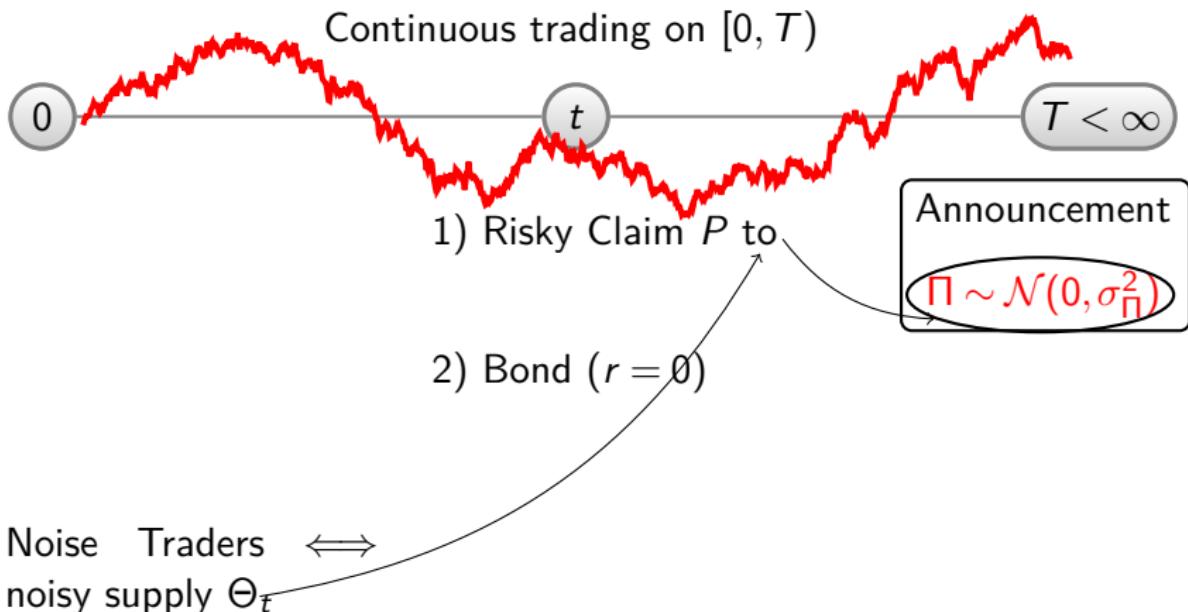
No fund flows: managers maximize  $E[U[W_T^i] | \mathcal{F}_t^i]$ ,  $i \in [0, 1]$

# Setup

Continuous trading on  $[0, T)$

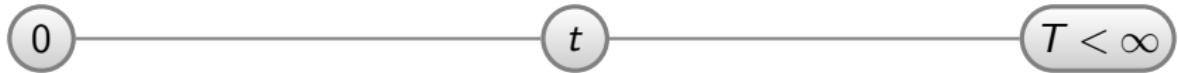


# Setup

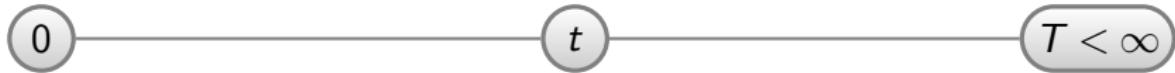


$$d\Theta_t = -a_\Theta \Theta_t dt + \sigma_\Theta dB_t^\Theta$$

# Introducing Social Dynamics



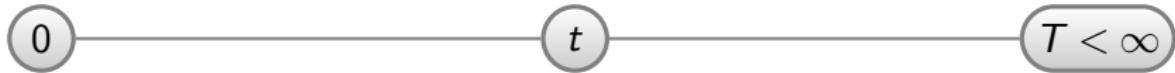
# Introducing Social Dynamics



Each manager  $i$   
starts with an  
idea  $S_1^i = \Pi + \epsilon_1^i$

$$\{S_j^i\}_{j=1}^1 = \{S_1^i\}$$

# Introducing Social Dynamics

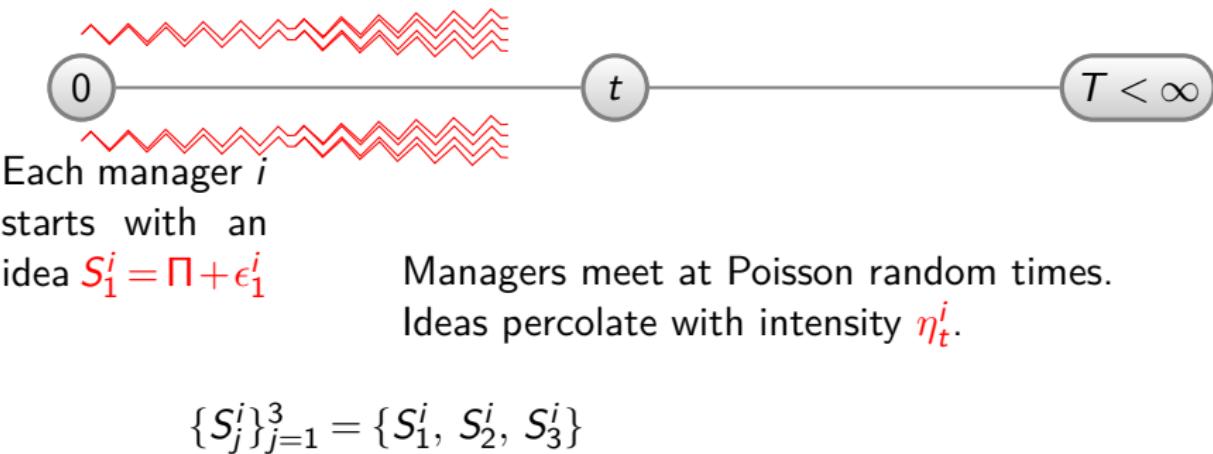


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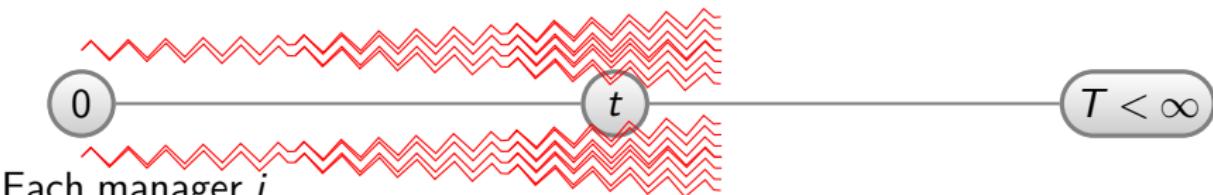
Managers meet at Poisson random times.  
Ideas percolate with intensity  $\eta_t^i$ .

$$\{S_j^i\}_{j=1}^1 = \{S_1^i\}$$

# Introducing Social Dynamics



# Introducing Social Dynamics

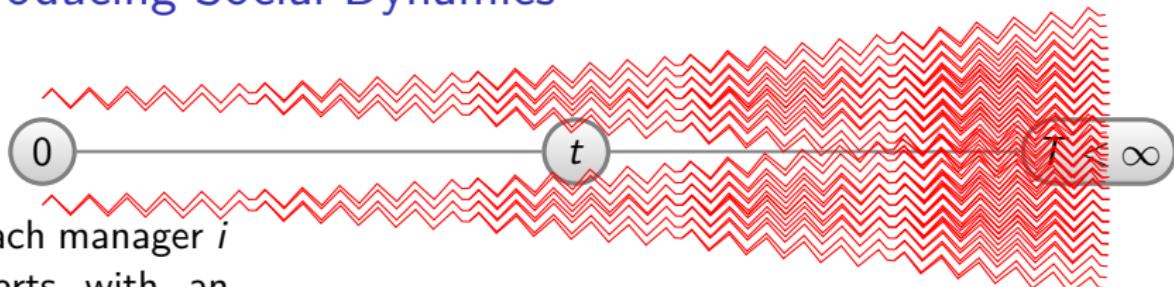


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Managers meet at Poisson random times.  
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$$\{S_j^i\}_{j=1}^6 = \{S_1^i, S_2^i, S_3^i, S_4^i, S_5^i, S_6^i\}$$

# Introducing Social Dynamics

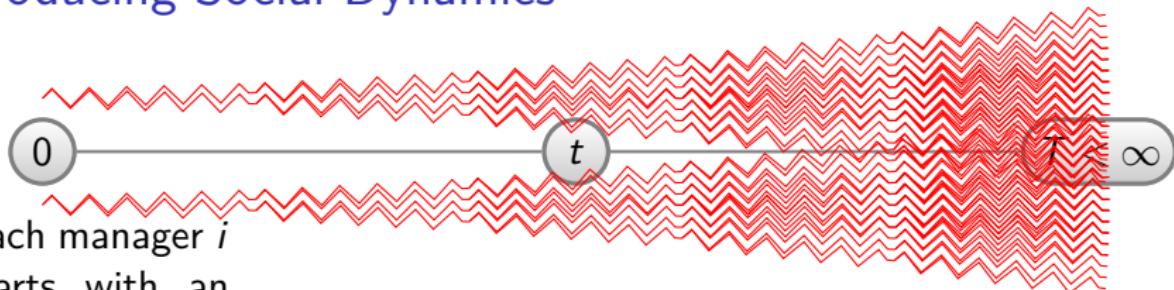


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Managers meet at Poisson random times.  
Ideas percolate with intensity  $\eta_t^i$ .

$$\{S_j^i\}_{j=1}^{n_t} = \{S_1^i, S_2^i, \dots, S_{n_t}^i\}$$

# Introducing Social Dynamics



Each manager  $i$   
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idea  $S_1^i = \Pi + \epsilon_1^i$

Managers meet at Poisson random times.  
Ideas percolate with intensity  $\eta_t^i$ .

$$\{S_j^i\}_{j=1}^{n_t} \iff Y_t^i \equiv \Pi + \frac{1}{n_t} \sum_{j=1}^{n_t} \epsilon_j^i$$

# Introducing Social Dynamics

*"Relatively underdeveloped ideas can travel long distances over the network. More valuable ideas, by contrast, tend to remain localized among small groups of agents."*

Stein (2008)

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ideas  $n \geq N$

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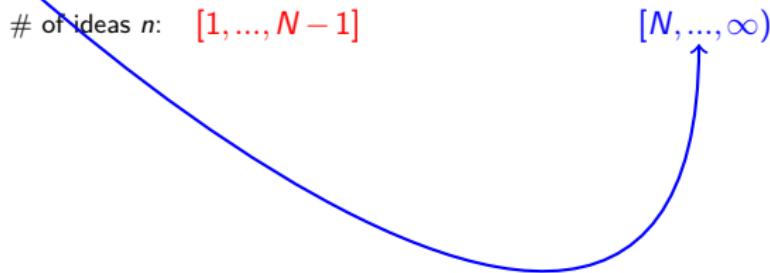
# of ideas  $n$ : [1, ...,  $N - 1$ ]

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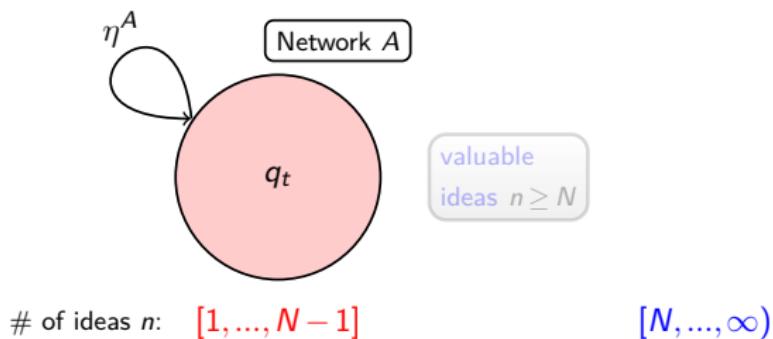
Stein (2008)

valuable

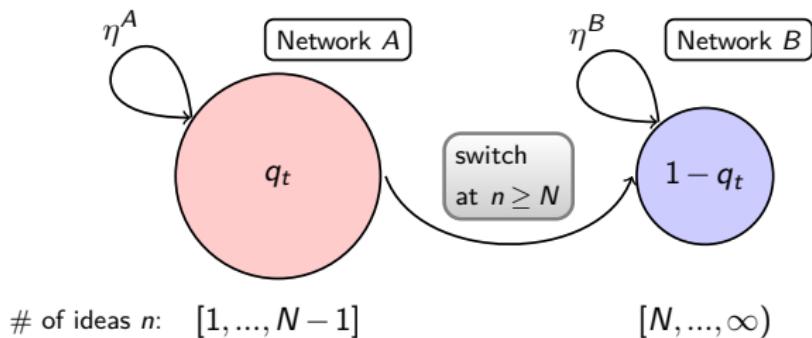
ideas  $n \geq N$

# of ideas  $n$ : [1, ...,  $N - 1$ ] [  $N, \dots, \infty$  )

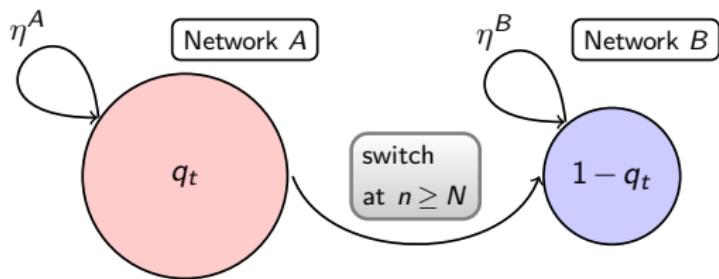
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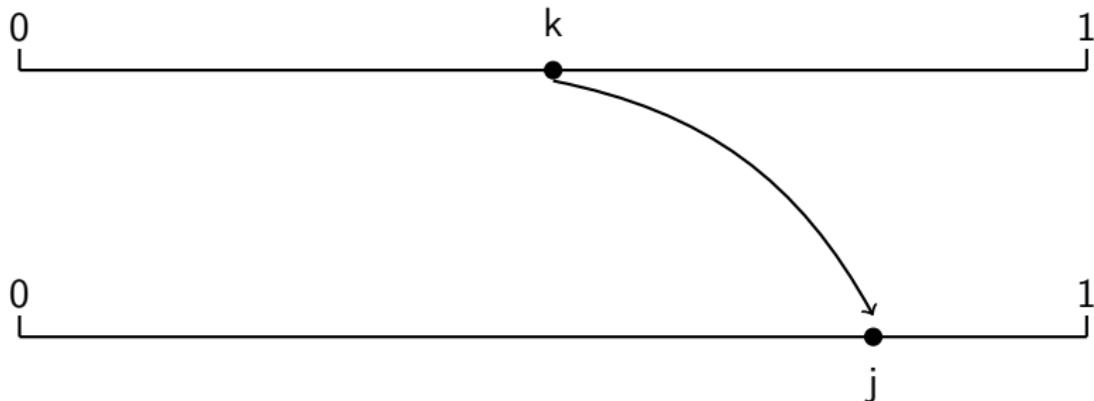
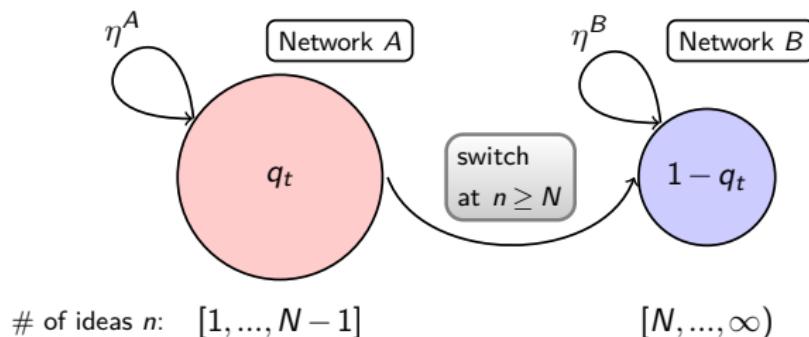


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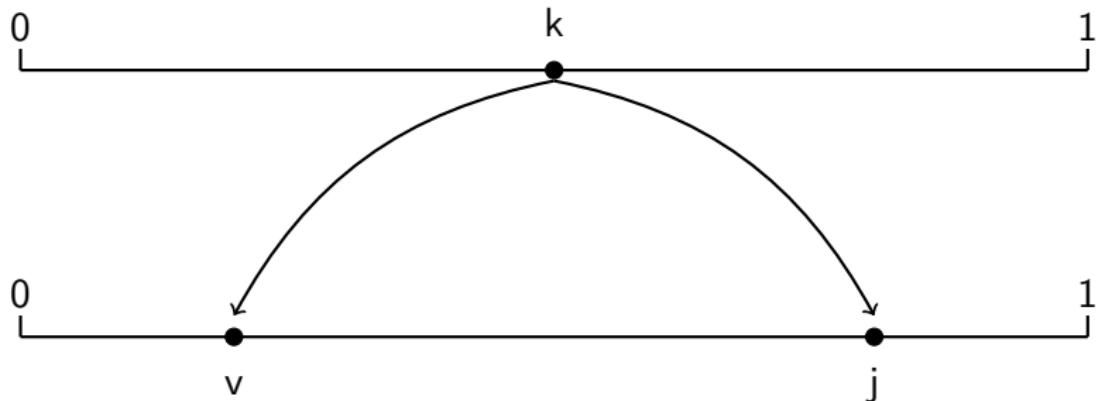
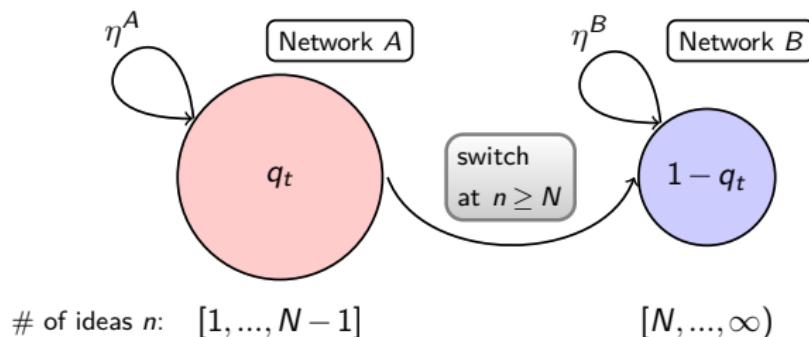
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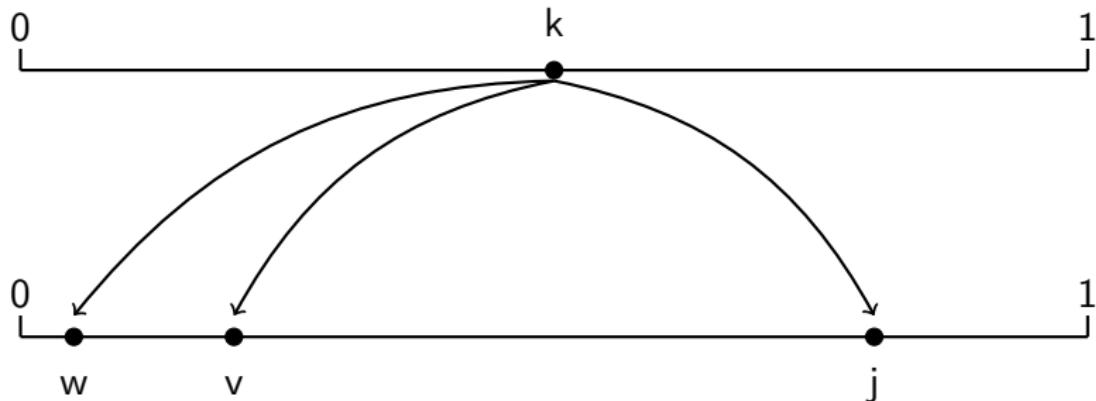
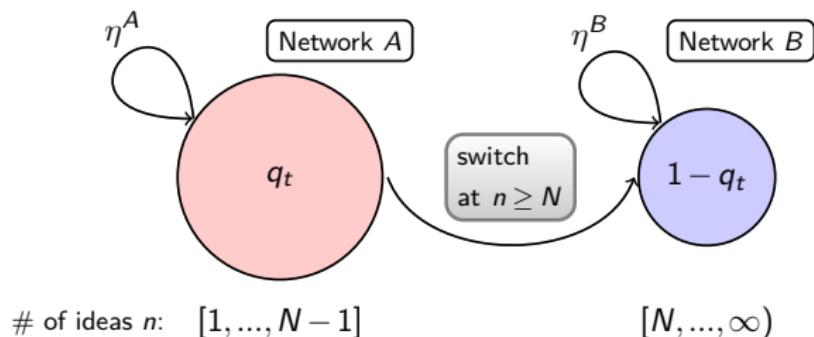
# Introducing Social Dynamics



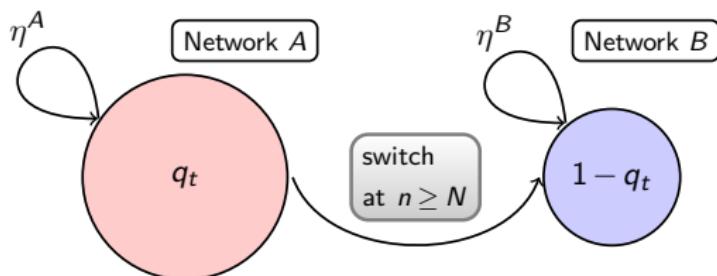
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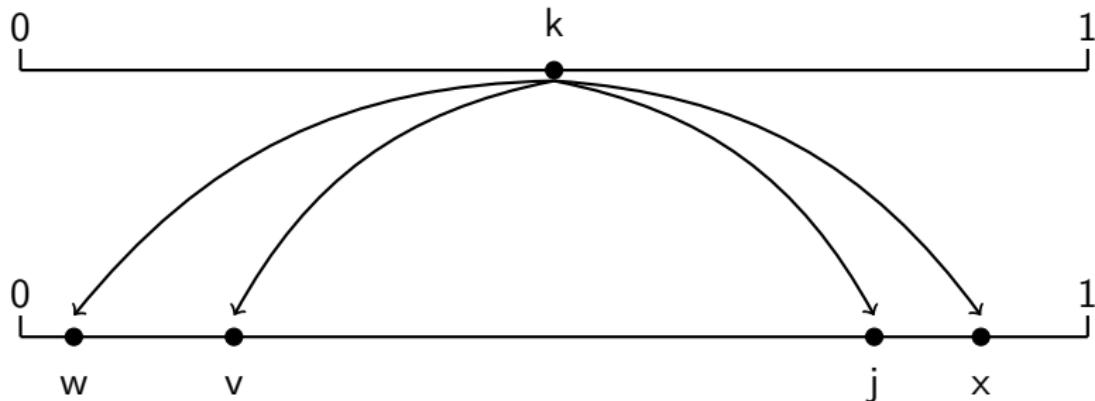


# Introducing Social Dynamics

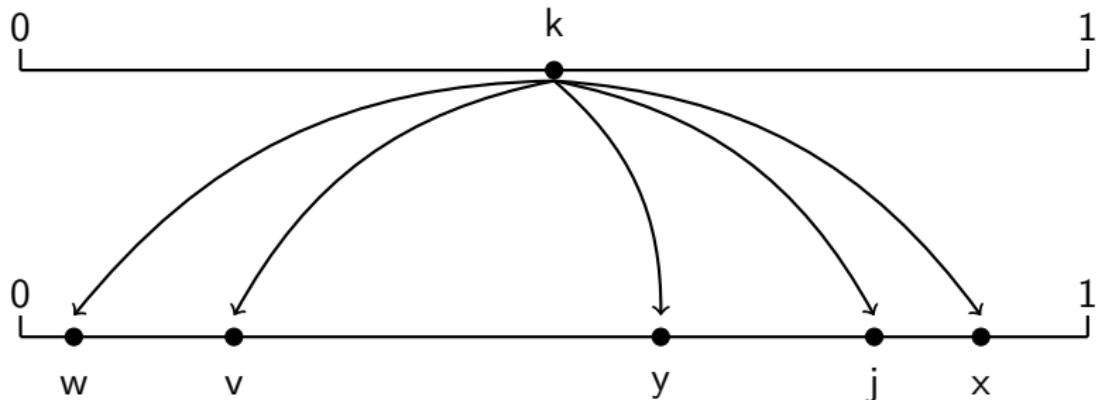
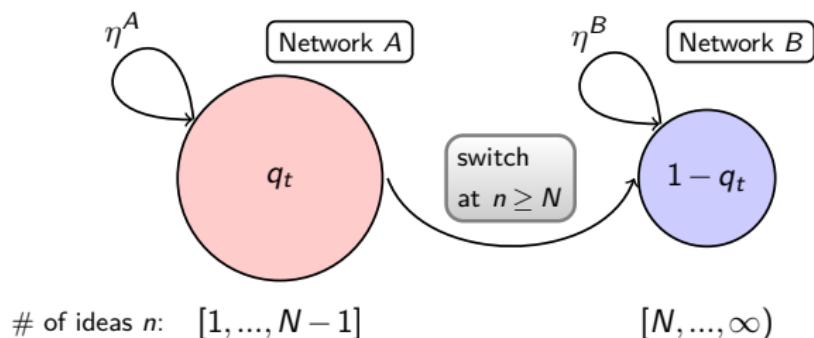


# of ideas  $n$ :  $[1, \dots, N-1]$

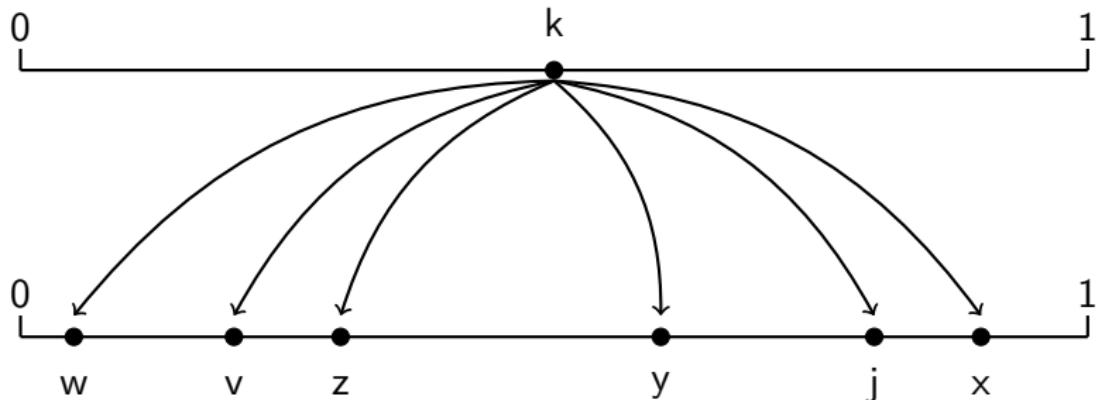
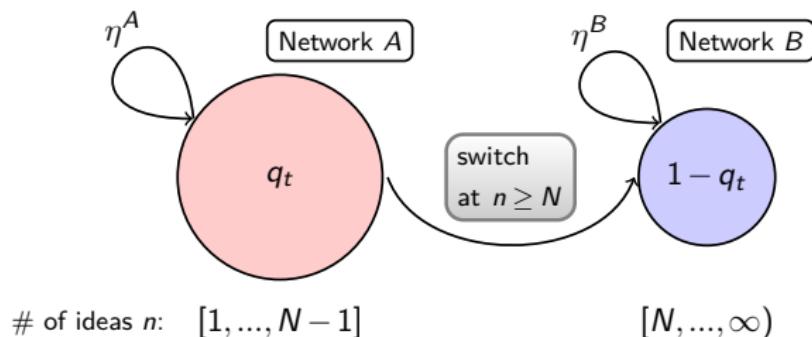
$[N, \dots, \infty)$



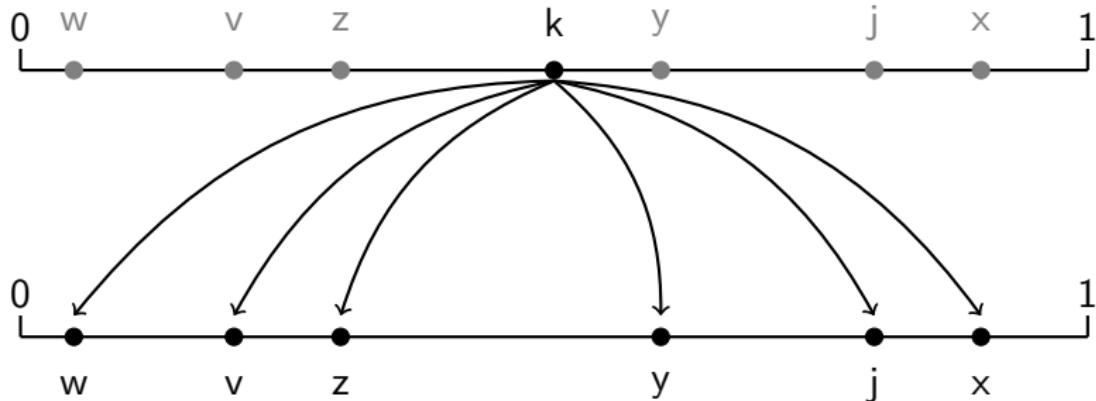
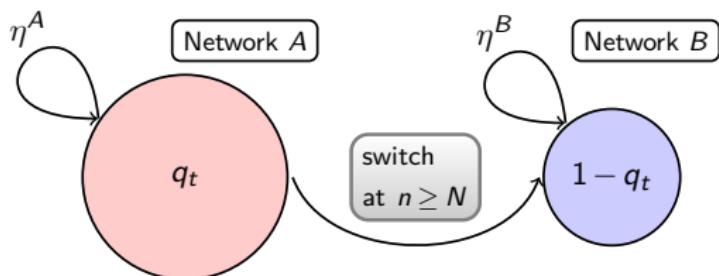
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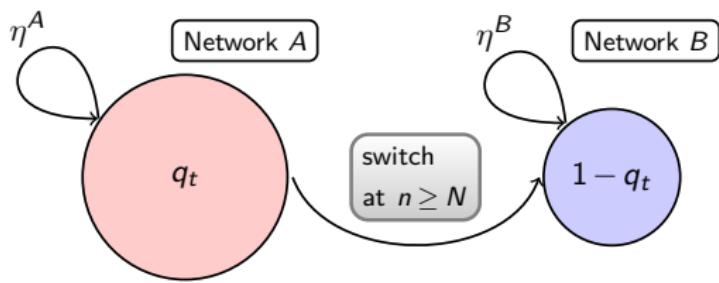
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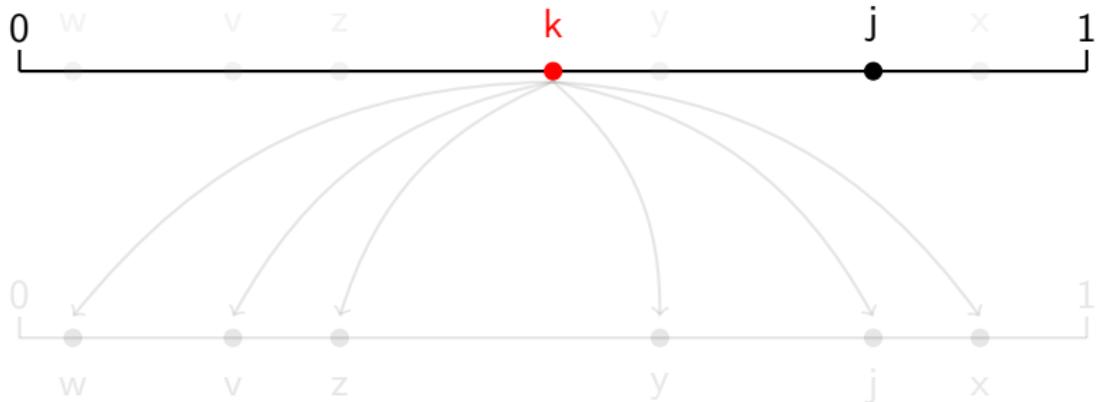


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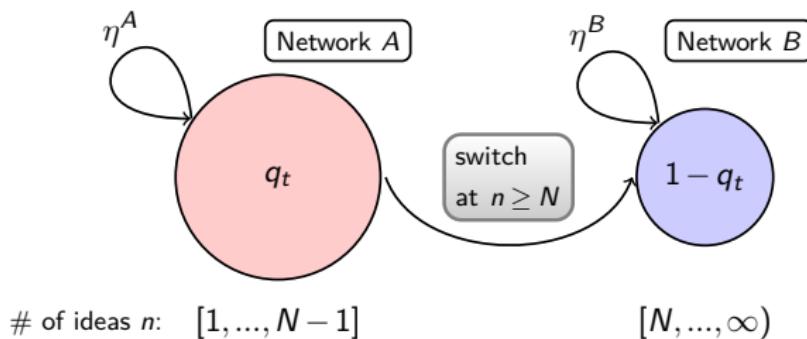


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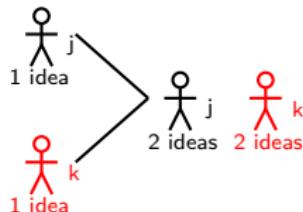
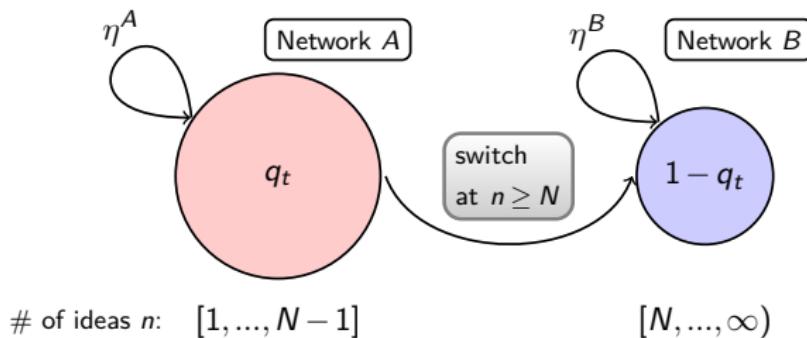
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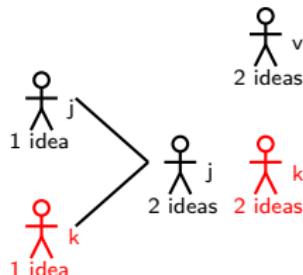
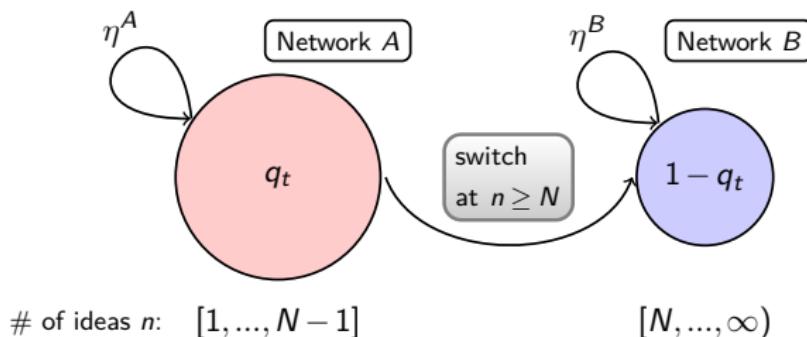
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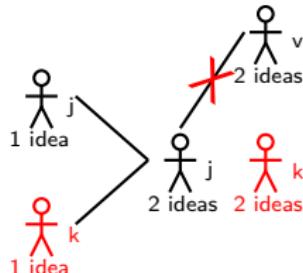
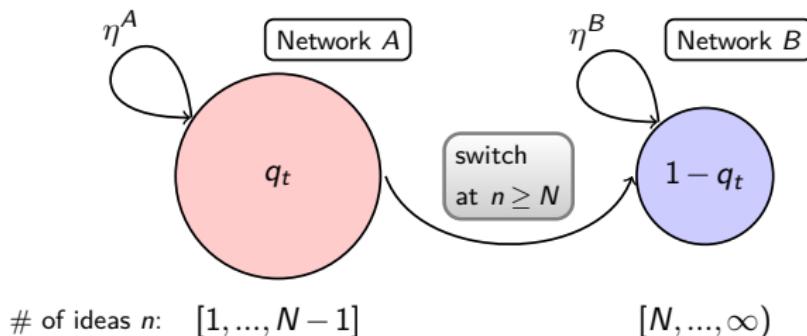
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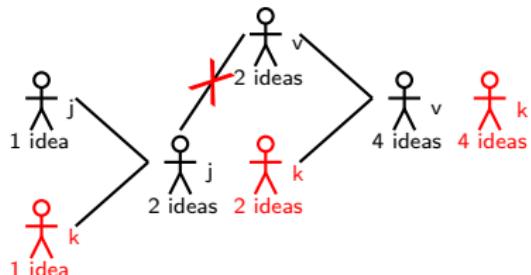
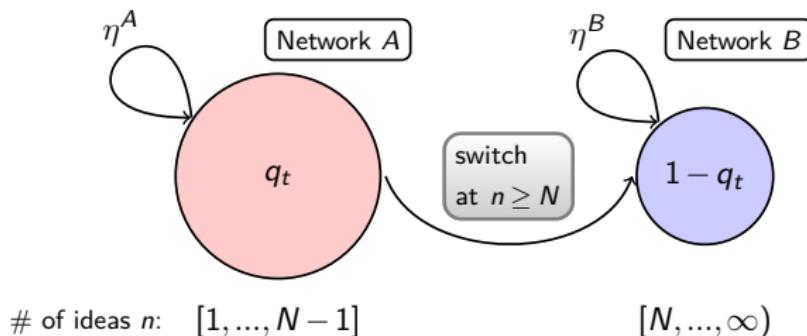
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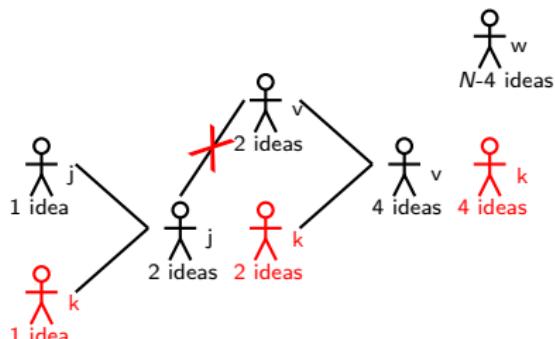
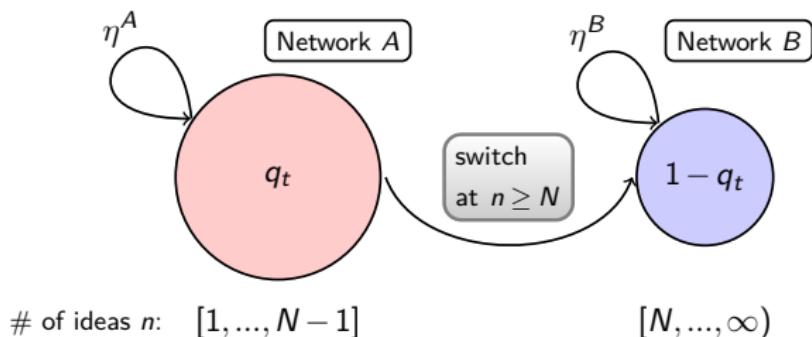
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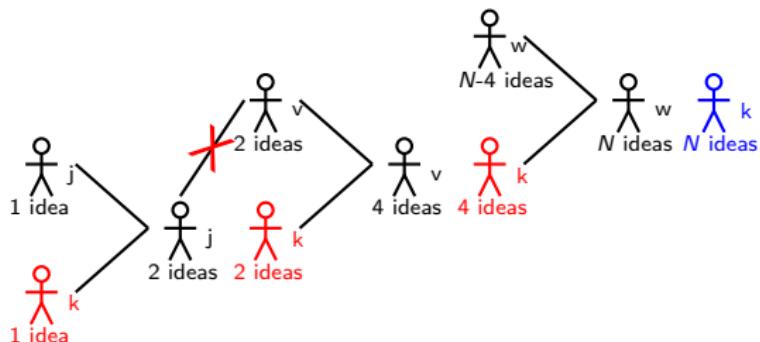
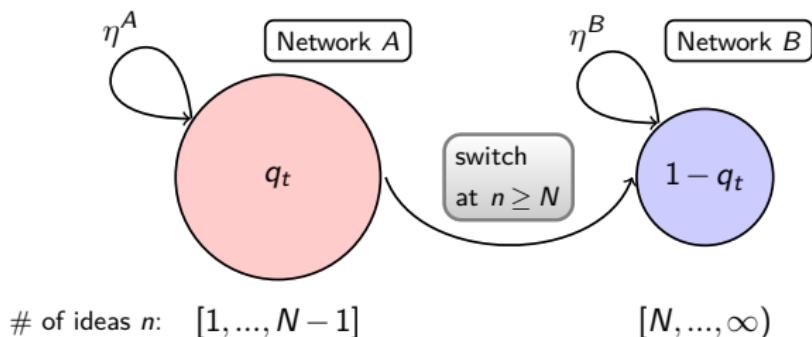
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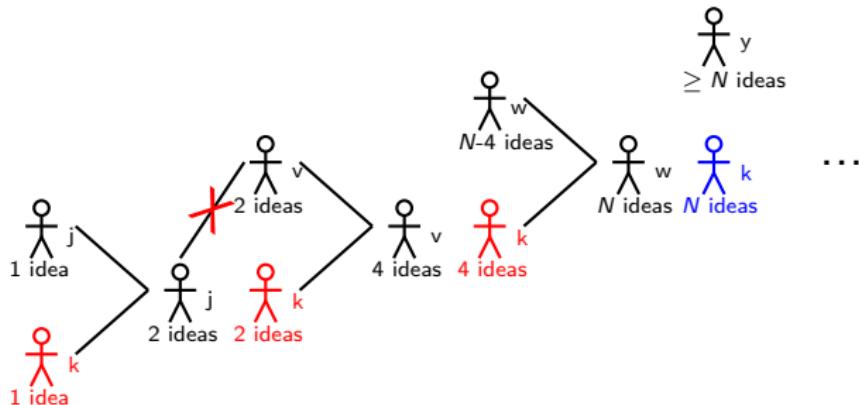
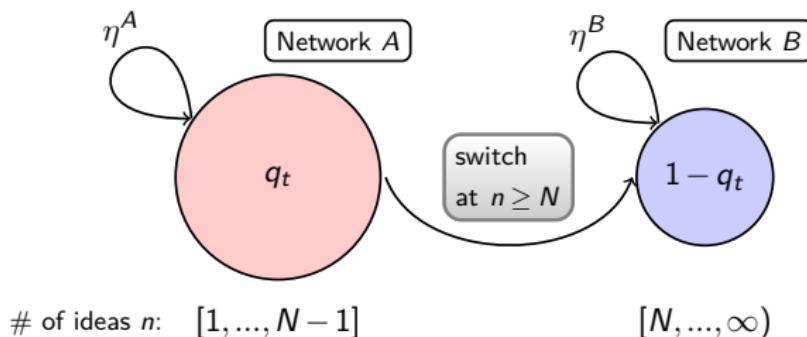
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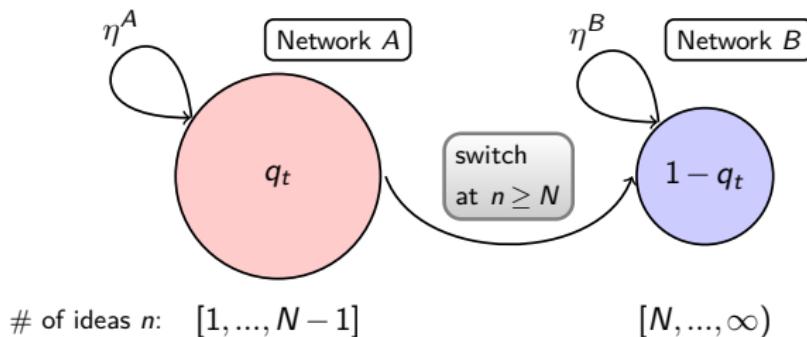
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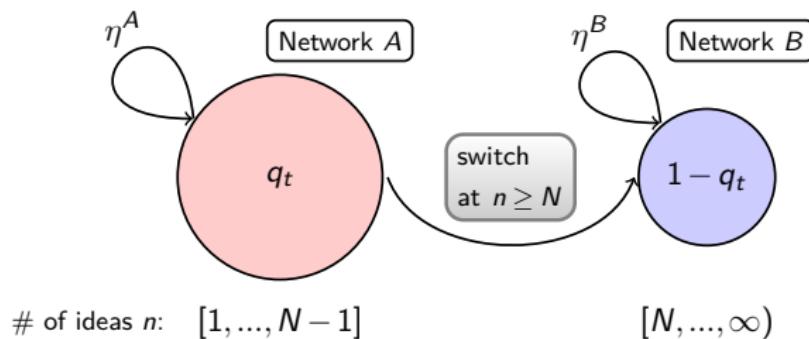
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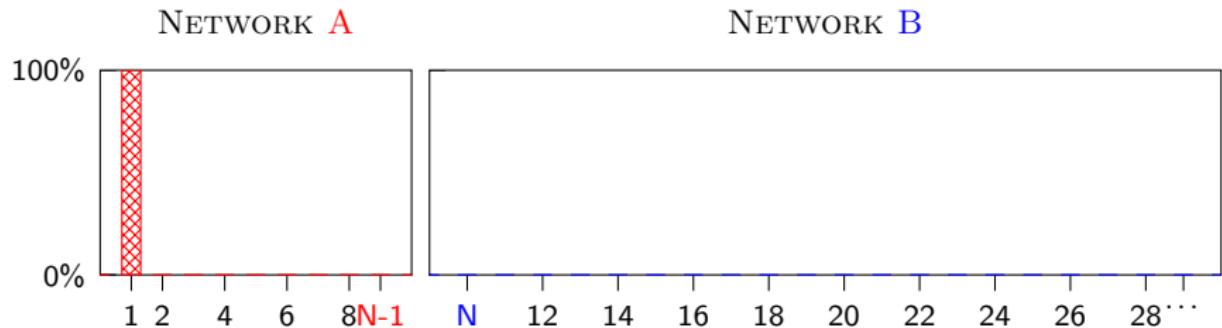
## Cross-Sectional Distribution of ideas

$$\frac{d}{dt} \mu_t(n) = \underbrace{\eta^A \mathbf{1}_{n < 2N} \sum_{m=1 \vee (n-N+1)}^{(n-1) \wedge (N-1)} \mu_t(m) \mu_t(n-m) + \eta^B \mathbf{1}_{n \geq 2N} \sum_{m=N}^{n-N} \mu_t(m) \mu_t(n-m)}_{\text{Meetings}} - \underbrace{\eta^A \mathbf{1}_{n \in A} q_t \mu_t(n) - \eta^B \mathbf{1}_{n \in B} (1 - q_t) \mu_t(n)}_{\text{Replacements}}$$

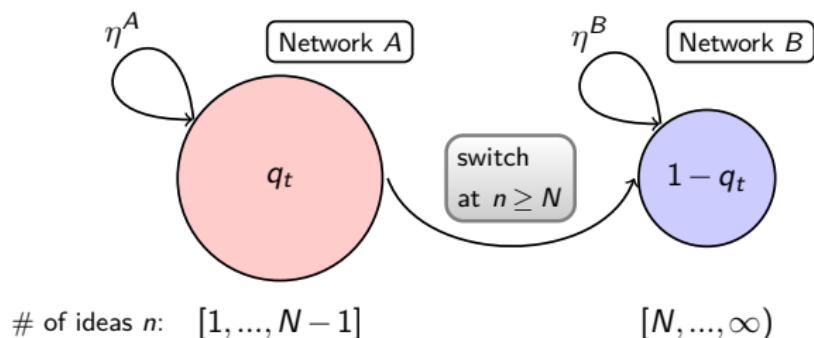
## Introducing Social Dynamics



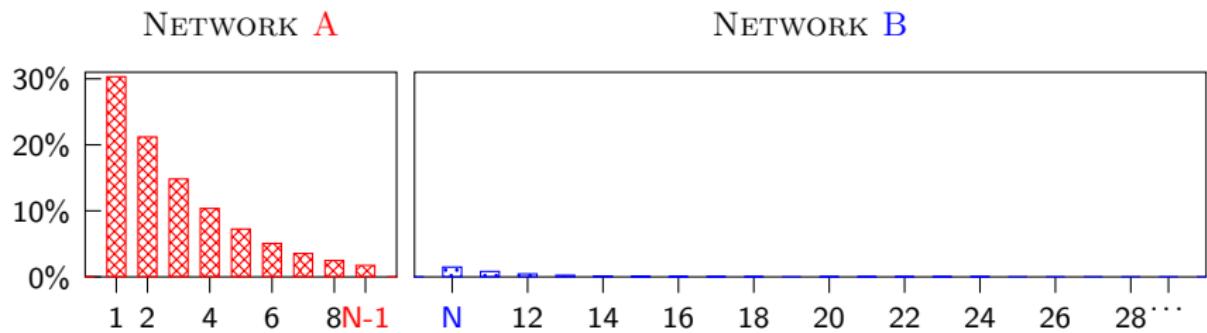
## Cross-Sectional Distribution at time $t = 0$



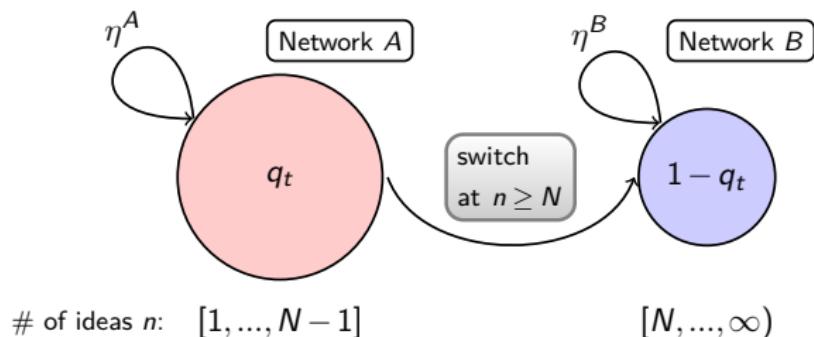
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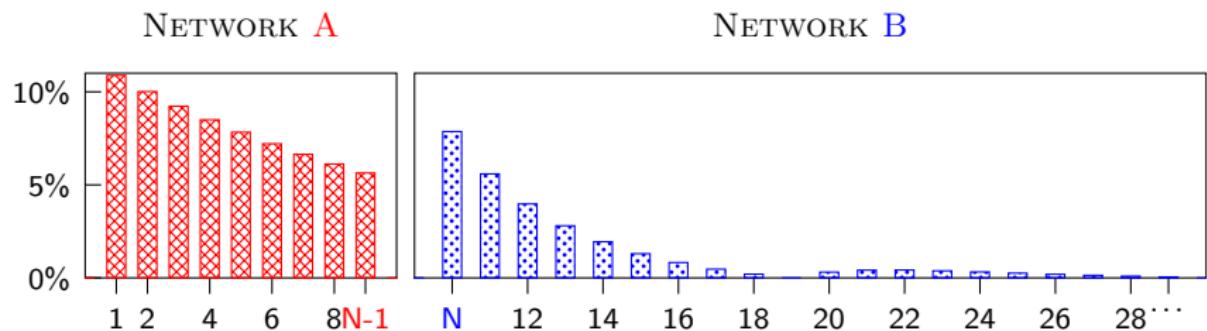
Cross-Sectional Distribution at time  $t = 0.2$



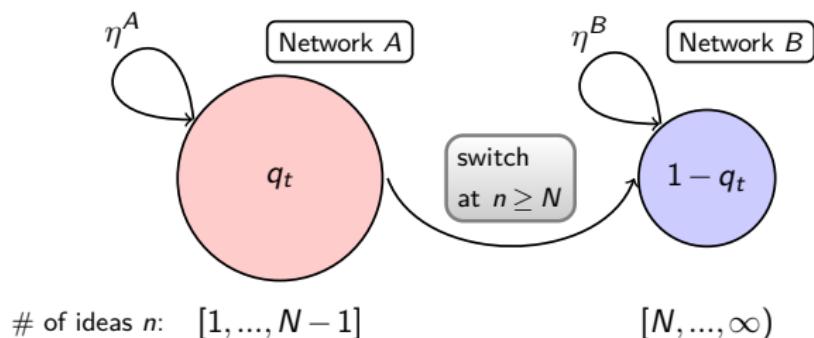
# Introducing Social Dynamics



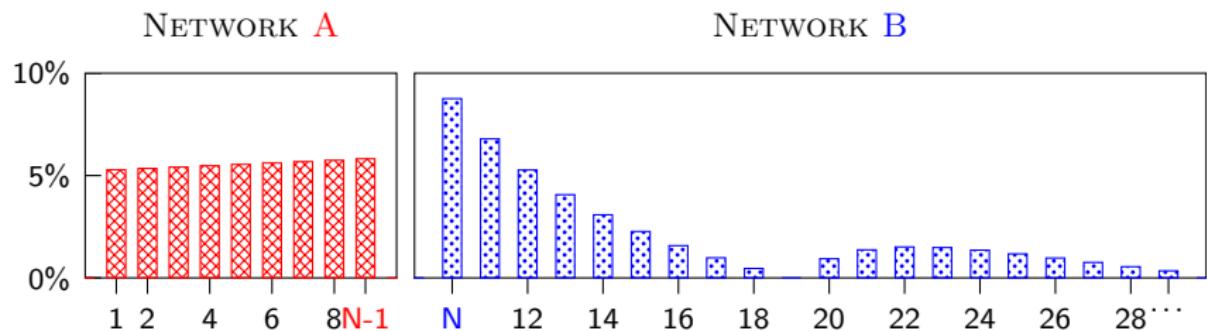
Cross-Sectional Distribution at time  $t = 0.4$



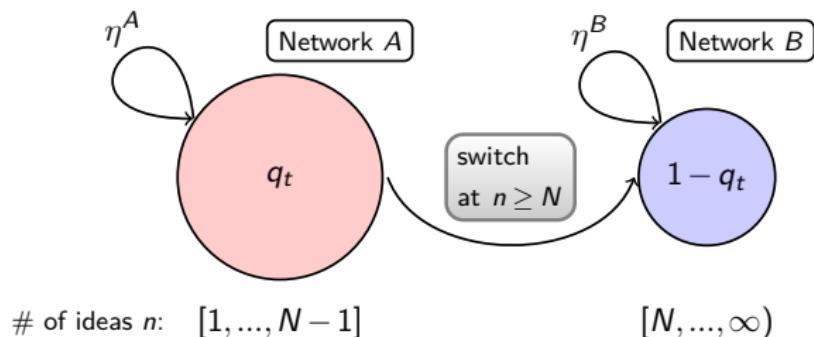
# Introducing Social Dynamics



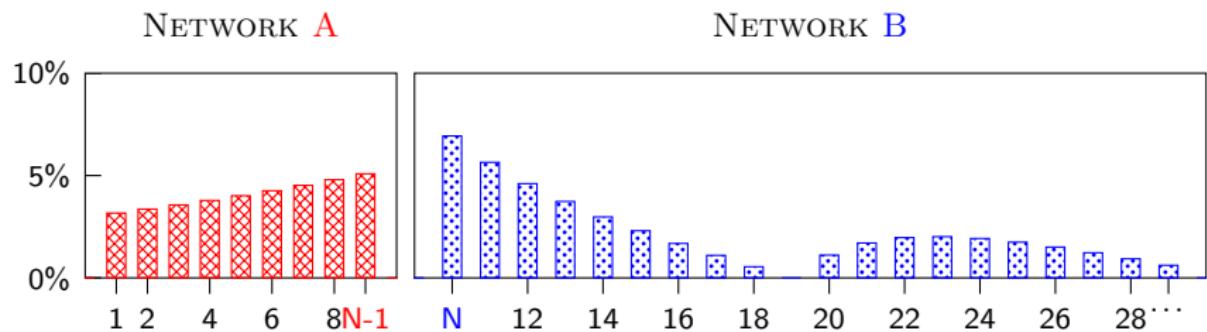
Cross-Sectional Distribution at time  $t = 0.6$



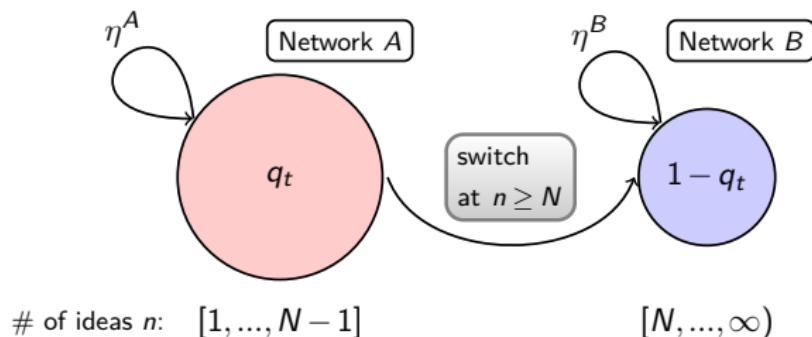
# Introducing Social Dynamics



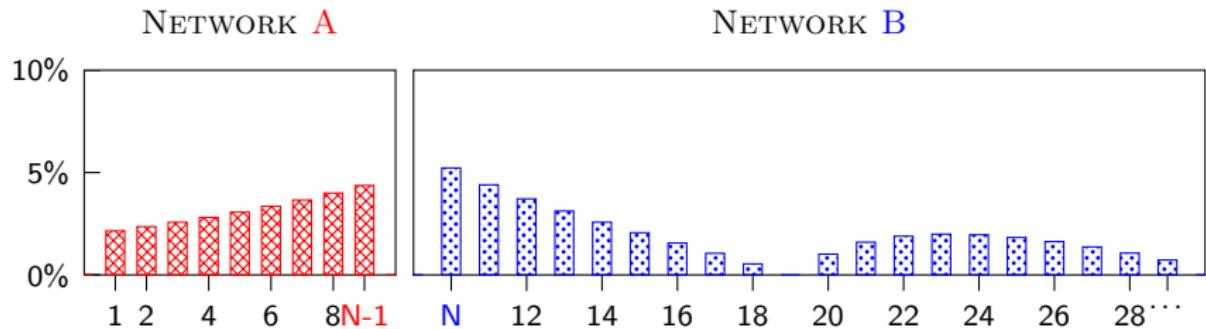
Cross-Sectional Distribution at time  $t = 0.8$



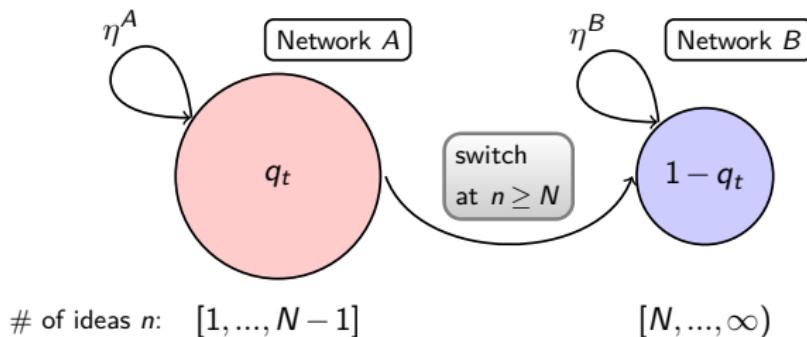
# Introducing Social Dynamics



Cross-Sectional Distribution at time  $t = 1$



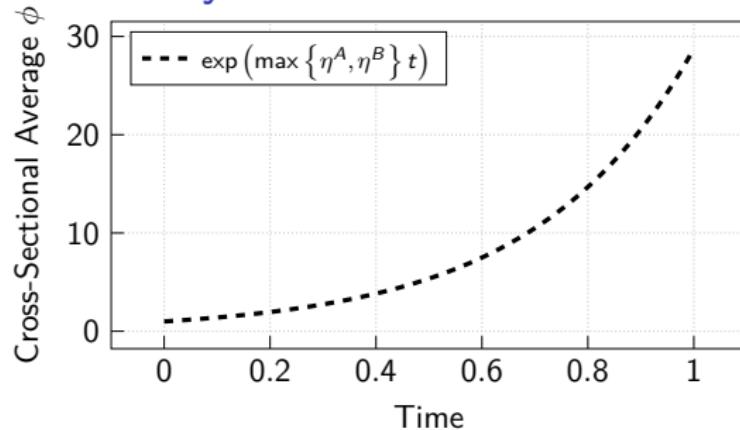
# Introducing Social Dynamics



## Cross-Sectional Average Number of ideas

$$\begin{aligned} \phi_t &= b_t^{-\frac{\eta^B}{\eta^A}} e^{\eta^B t} \left( 1 + \int_0^t e^{-\eta^B s} b_s^{\frac{\eta^B}{\eta^A}-1} \left( (\eta^A + \eta^B) q_s - \eta^B \right) \frac{1 + a_s^{N-1} (a_s(N-1) - N)}{(1-a_s)^2} ds \right) \\ &\leq \exp \left( \max \left\{ \eta^A, \eta^B \right\} t \right) \end{aligned}$$

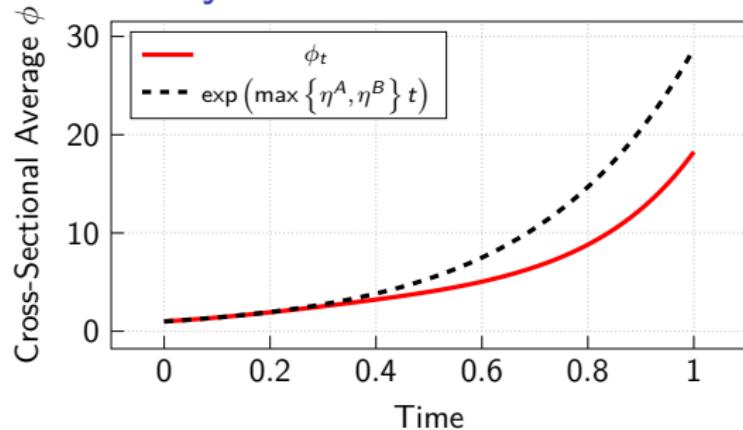
# Introducing Social Dynamics



## Cross-Sectional Average Number of ideas

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# Introducing Social Dynamics



## Cross-Sectional Average Number of ideas

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# Outline

I Model

II Solution Method

III Results

# Steps (He and Wang (1995))

1. Price Conjecture

4. Market Clearing

2. Learning Process

3. Individual Optimization

# Steps (He and Wang (1995))

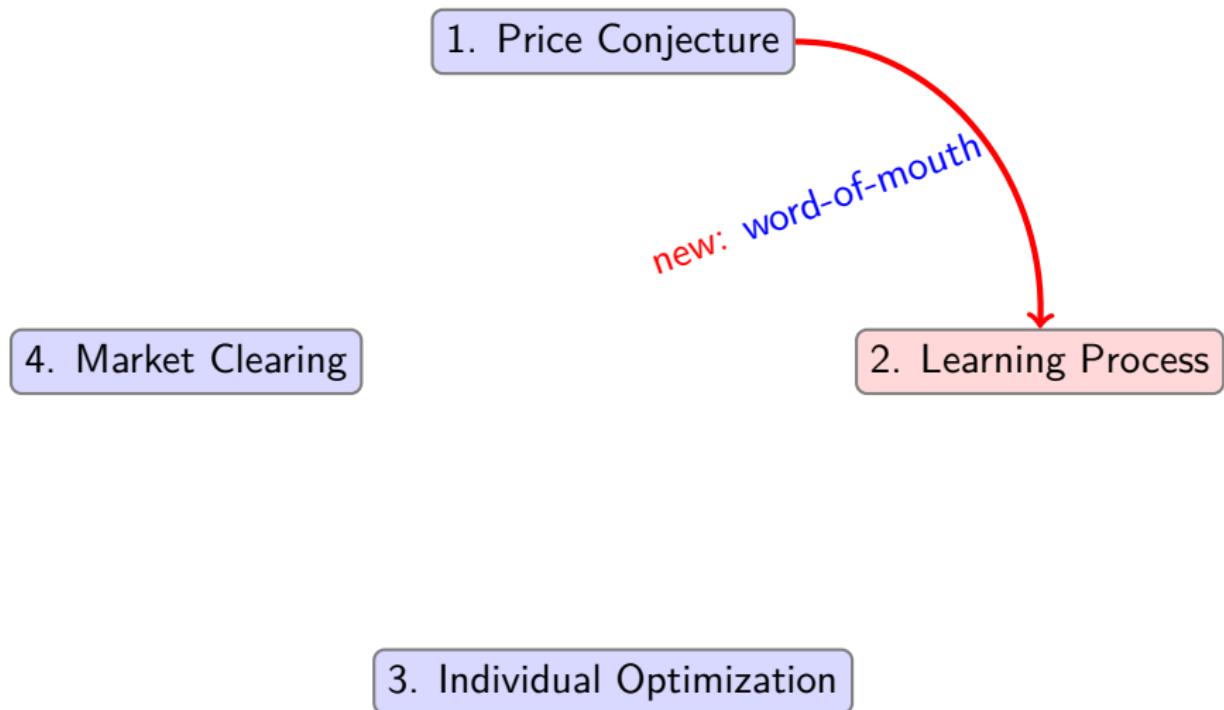
1. Price Conjecture

4. Market Clearing

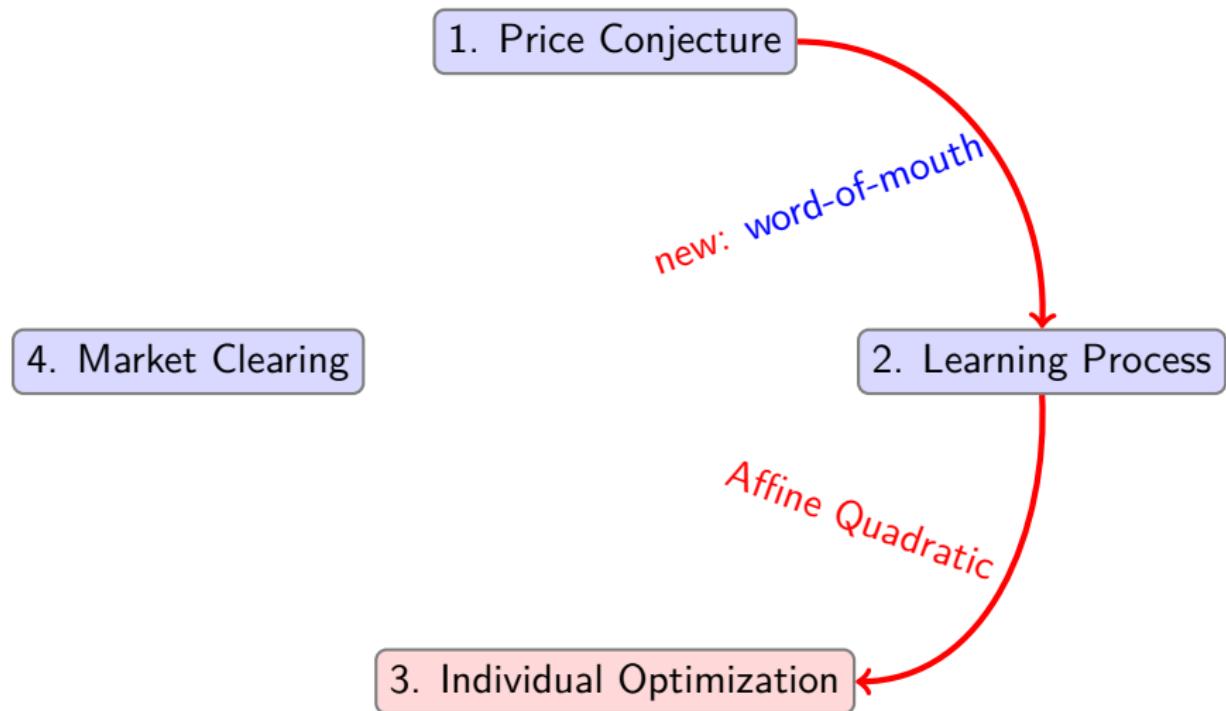
2. Learning Process

3. Individual Optimization

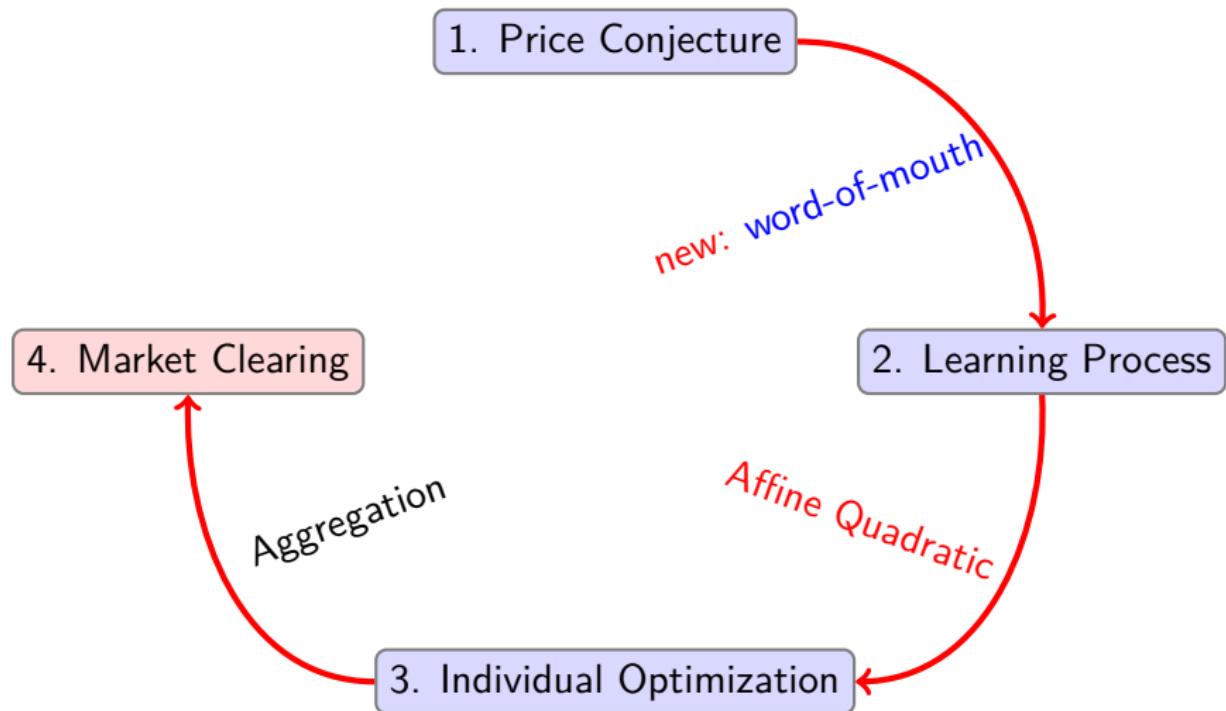
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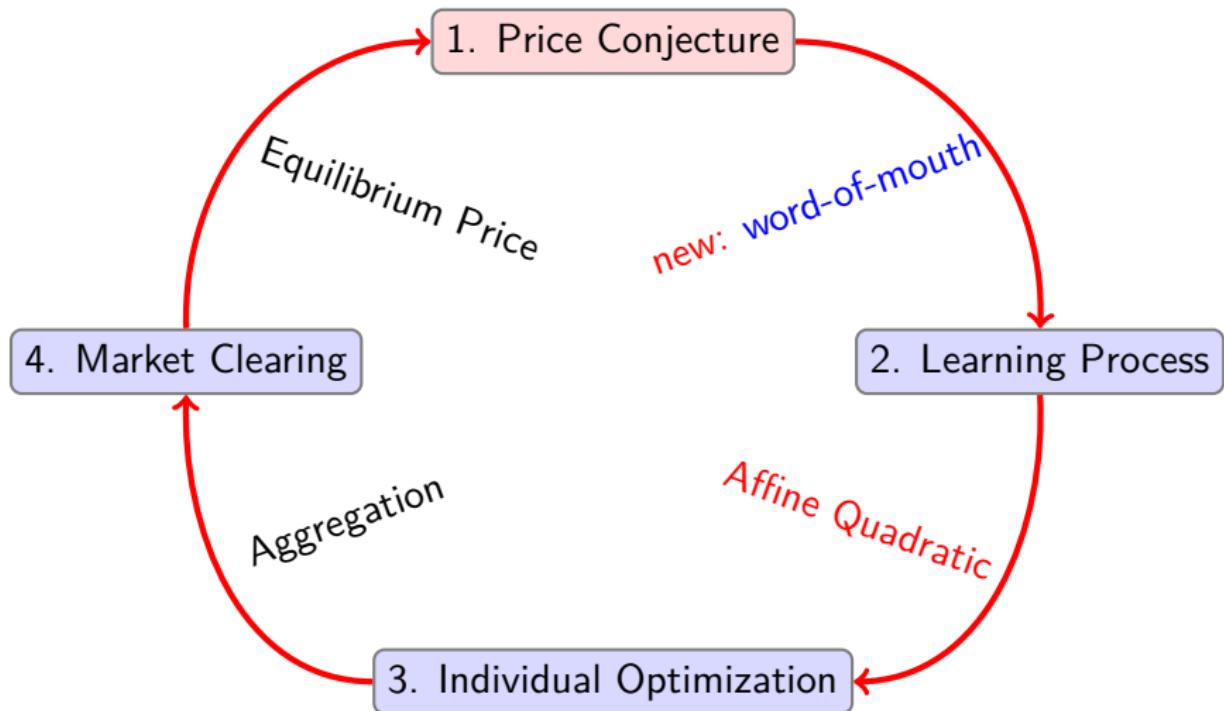
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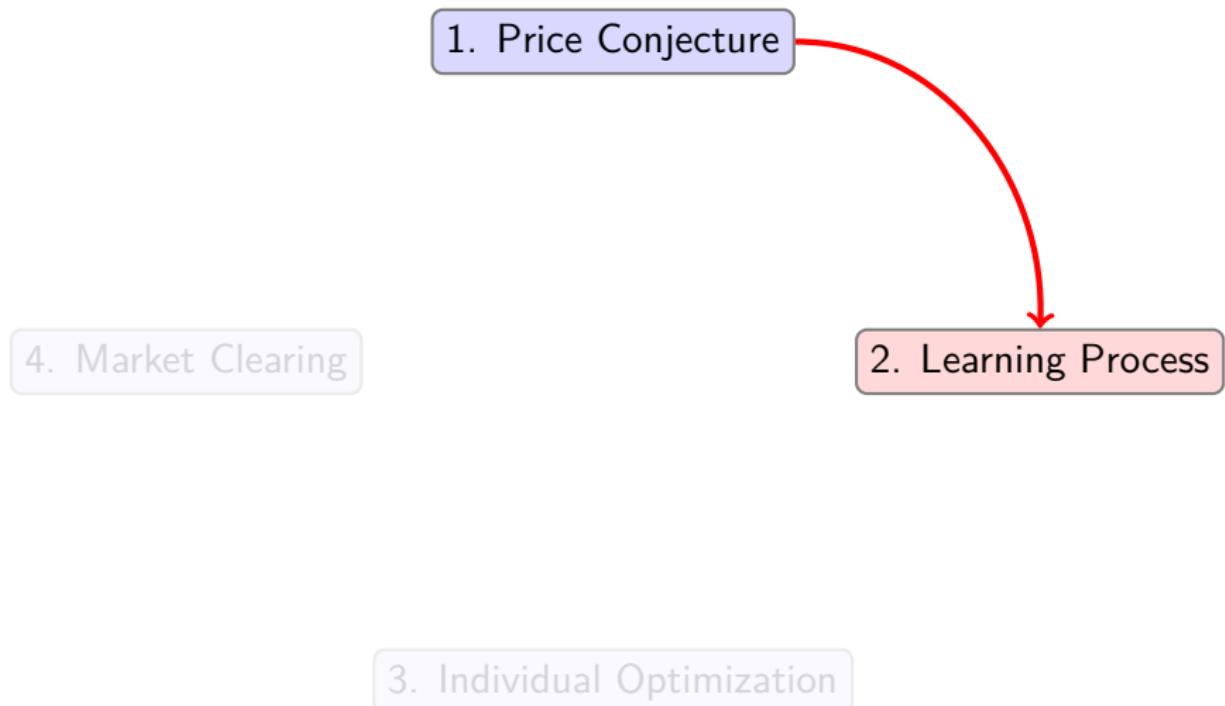
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# Steps (He and Wang (1995))



# Word-of-Mouth Learning

A manager  $i$ 's expectations evolve as

$$d\hat{\Pi}_t^i = o_{t-}^i k_t d\hat{B}_t^i + o_t^i \frac{\Delta n_t^i}{\sigma_S^2} \hat{Y}_t^i dN_t^i,$$

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WATCHING THE TAPE

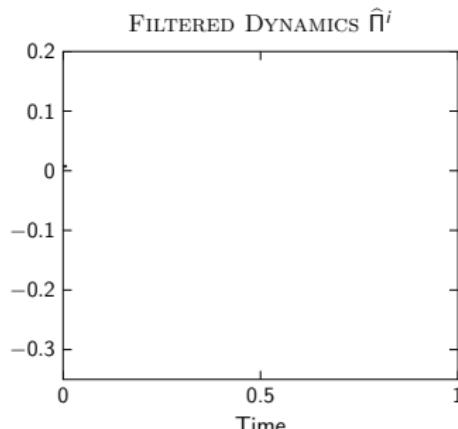
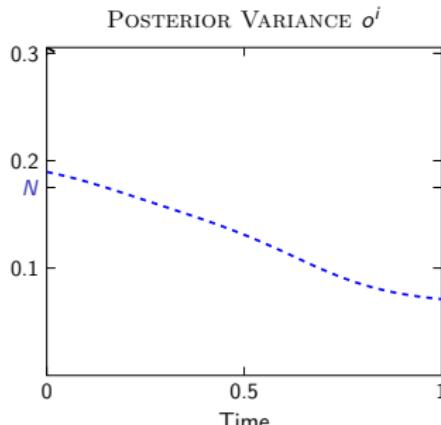
MEETINGS

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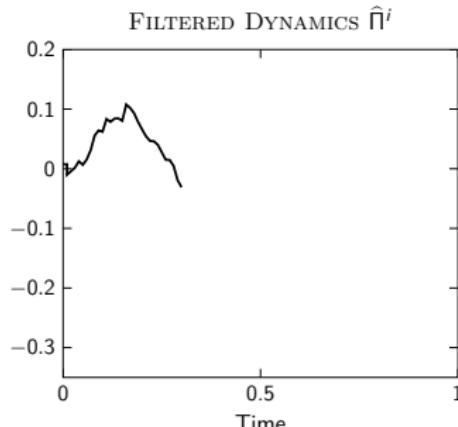
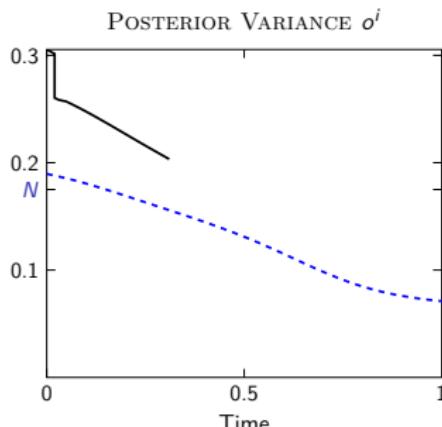


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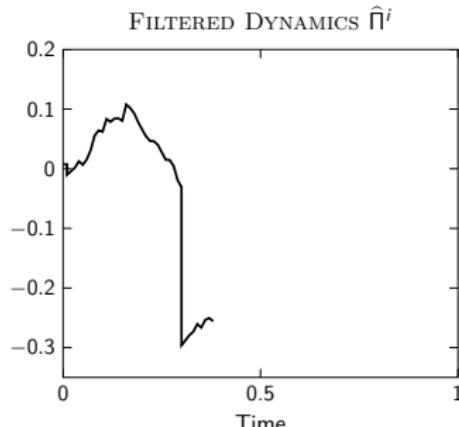
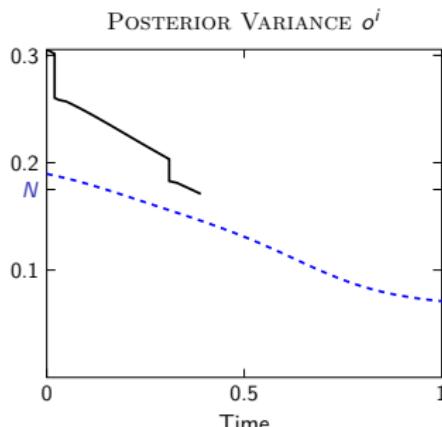


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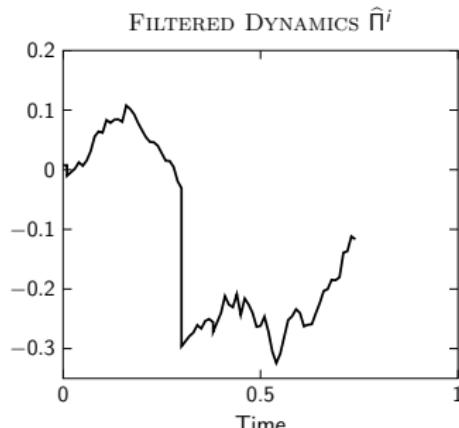
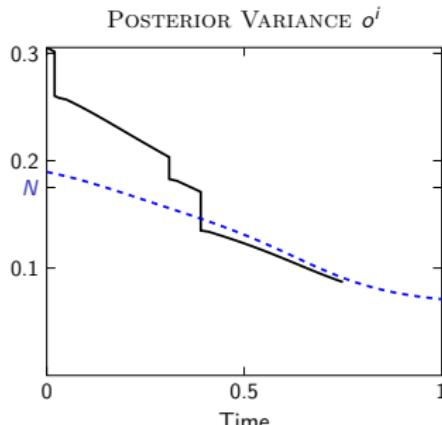


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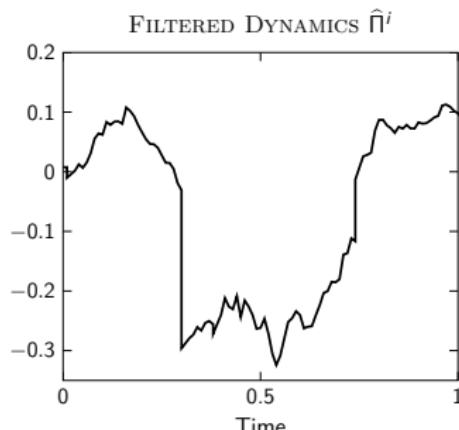
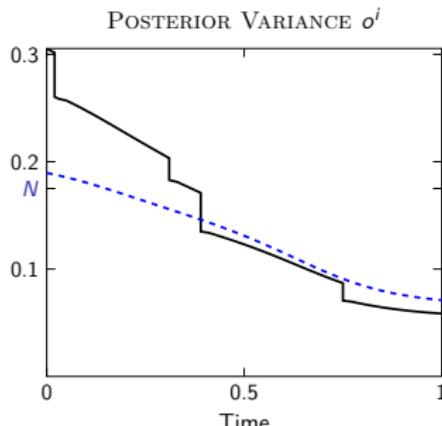


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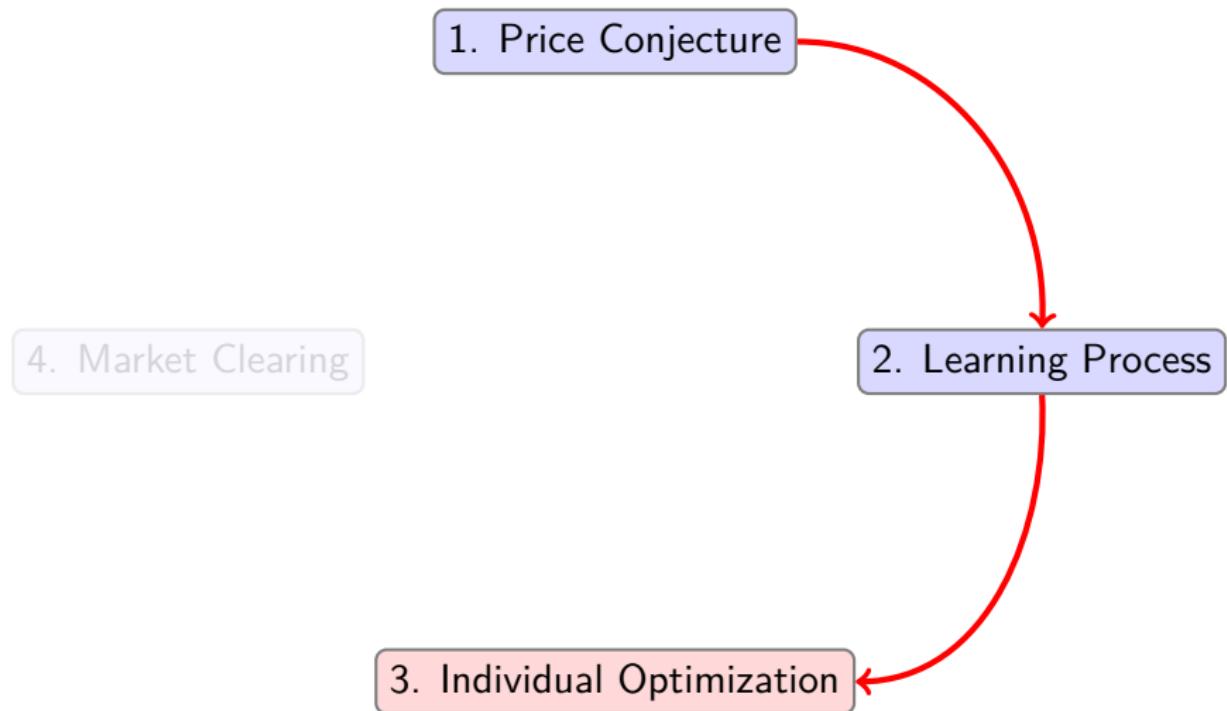
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# Steps (He and Wang (1995))



# Social Interactions Increase Trading Aggressiveness

A manager  $i$  builds an optimal portfolio  $\theta^i$ :

$$\theta_t^i = \frac{A_{Q,t} - B_{Q,t} \left( o_t(n^i) \right)^{-1}}{\gamma B_{Q,t}^2} B_{\Psi,t}(n^i)^\top \Lambda_t \begin{bmatrix} \hat{\Pi}^i - \hat{\Pi}^c \\ \hat{\Theta}^i \end{bmatrix}$$

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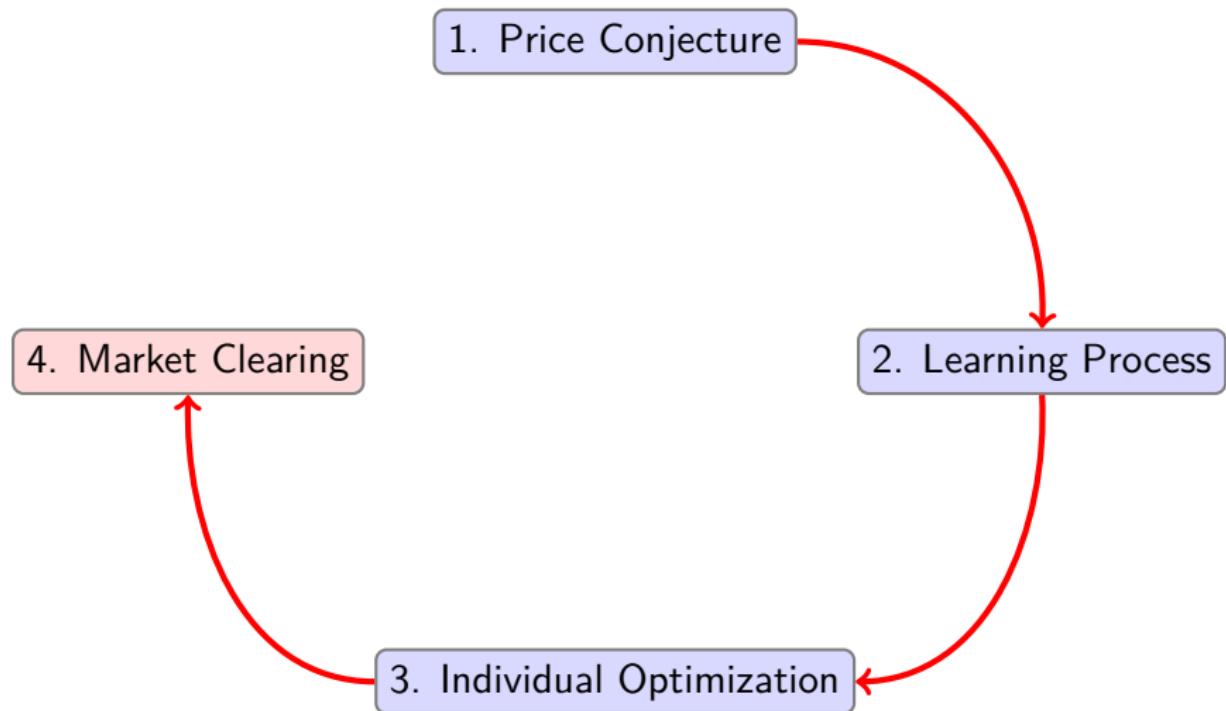
$$\theta_t^i = \frac{\overbrace{A_{Q,t} - B_{Q,t} \left( o_t(n^i) \right)^{-1}}^{\text{Precision}} B_{\Psi,t}(n^i)^\top \Lambda_t}{\gamma B_{Q,t}^2} \begin{bmatrix} \hat{\Pi}^i - \hat{\Pi}^c \\ \hat{\Theta}^i \end{bmatrix}$$

# Informational Holdings

A manager  $i$  builds an optimal portfolio  $\theta^i$ :

$$\theta_t^i = \underbrace{\Theta_t}_{\text{Market Making}} + \underbrace{\hat{\theta}_t^i}_{\text{Informational Position}}$$

# Steps (He and Wang (1995))



# A Zero-Sum Game Among Managers

The market-clearing condition implies:

$$\int_{i \in I} \theta_t^i di = \underbrace{\Theta_t}_{\text{Market Making}} + \underbrace{\int_{i \in I} \widehat{\theta}_t^i di}_{\substack{\text{Informational Trading} \\ \equiv 0}} = \Theta_t.$$

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In equilibrium, the informational position satisfies

$$\widehat{\theta}_t^i = \underbrace{\frac{\sqrt{n^i}}{\gamma \sigma_S} \epsilon_t^i}_{\text{Noise (Luck)}} + \underbrace{\frac{n^i - \phi_t}{\gamma \sigma_S^2 |k_t|} \times \text{Sharpe Ratio}}_{\text{Information}}$$

# A Zero-Sum Game Among Managers

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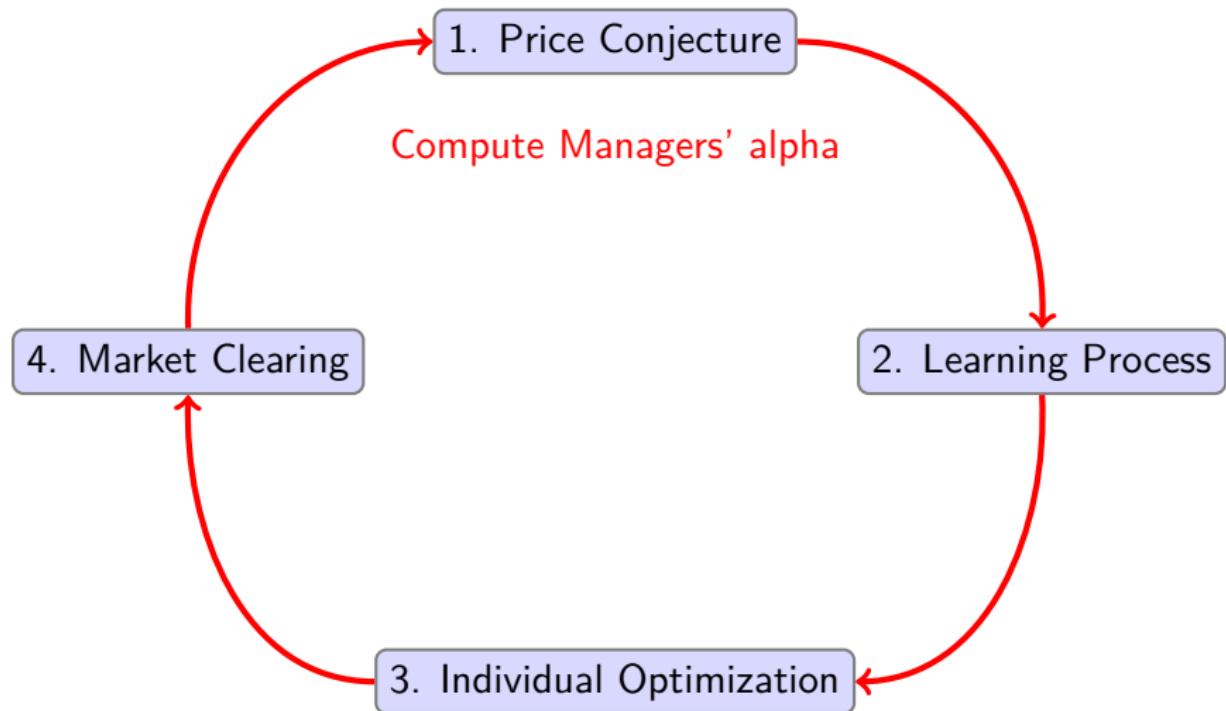
In equilibrium, the informational position satisfies

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$\underbrace{\gamma \sigma_S}_{\text{Noise (Luck)}}$

**the average manager ( $n^i \equiv \phi$ ) trades on noise only**

# Steps (He and Wang (1995))



# Informational Alpha Estimation

The econometrician observes managers' returns  $(\theta_t^i dP_t)_{t \geq 0}$  and past prices  $(P_t)_{t \geq 0}$  and thus managers' informational position  $(\widehat{\theta}_t^i)_{t \geq 0}$ :

$$\mathcal{F}_t = \sigma \left( \left( \Theta_s, \widehat{\theta}_s^i \right) : 0 \leq s \leq t, i \in [0, 1] \right) \bigvee \sigma(\Pi), \quad 0 \leq t < T.$$

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# Informational Alpha is a Noisy Measure of Skill

The unconditional estimate,  $\hat{\alpha}_t^i$ , of manager  $i$ 's alpha is

$$\hat{\alpha}_t(n^i) = \underbrace{(n^i - \phi_t)}_{\text{Skill}} \frac{\sigma_S^4 k_t (k_t - \gamma \sigma_\Theta) o_t^c}{\gamma (\sigma_S^2 + o_t^c \phi_t)^3} \underbrace{\mathbb{E}[(\Delta_t^2 + \gamma o_t^c \Delta_t \Theta_t)^2]}_{\text{Market-Timing Gains (MTG)}}$$

Its unconditional  $t$ -statistic,  $\hat{t}_{\alpha,t}$ , is given by

$$\hat{t}_{\alpha,t}(n^i) = \sqrt{\frac{2}{\pi}} \frac{\text{Sharpe Ratio}(\Delta_t^2 + \gamma o_t^c \Delta_t \Theta_t)}{\text{MTG} + \left( \frac{\sigma_S}{\sigma_S^2 + o_t^c \phi_t} \frac{n^i - \phi_t}{\sqrt{n^i}} \right)^2}$$

sign( $n^i - \phi_t$ )

Skill-to-Luck Ratio

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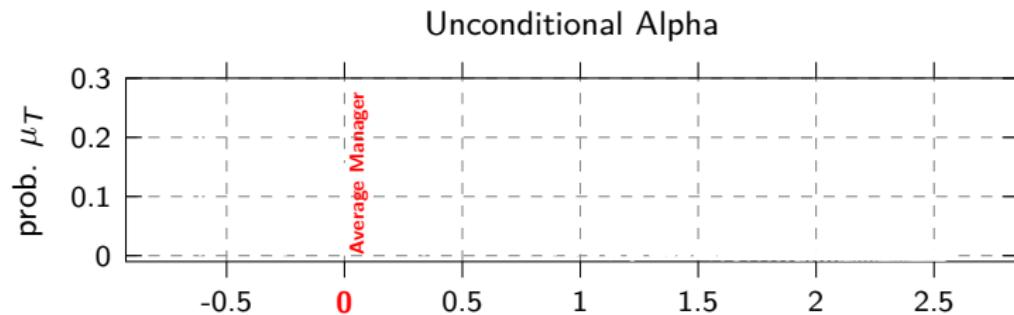
# Outline

I Model

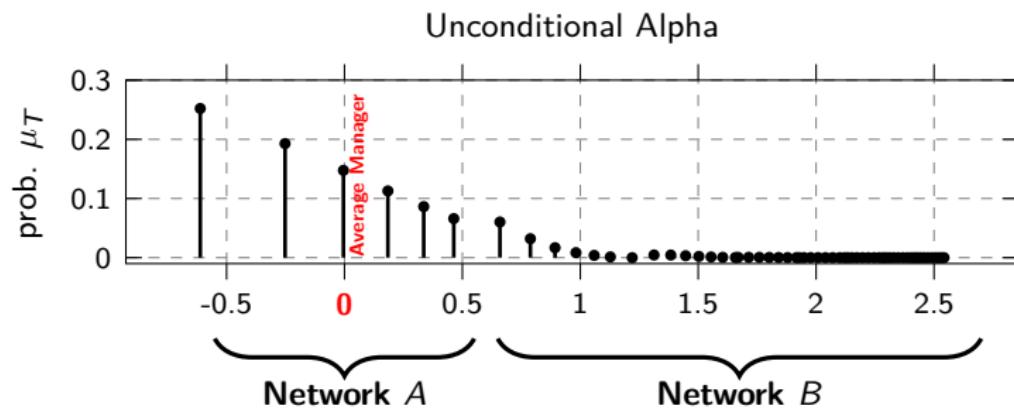
II Solution Method

III Results

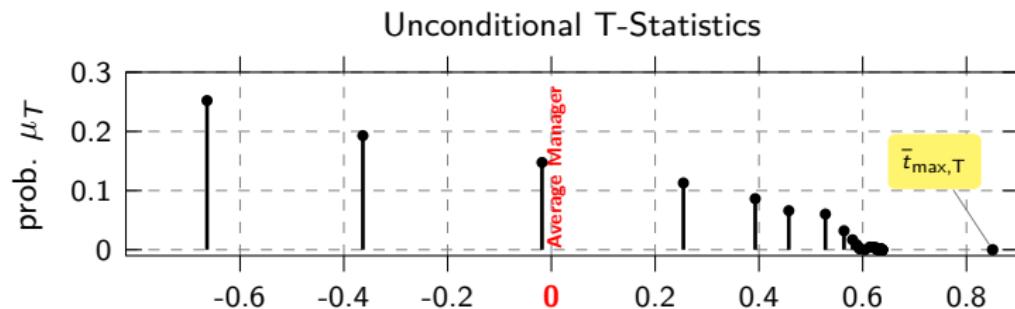
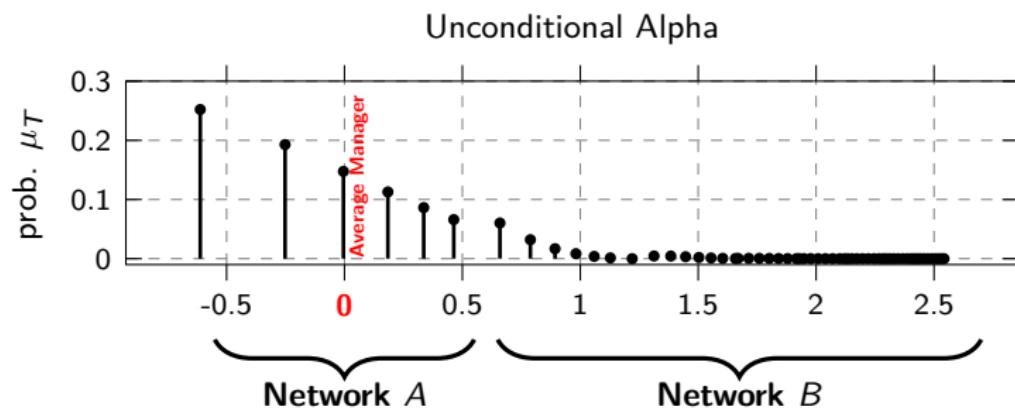
# Cross-Sectional Performance



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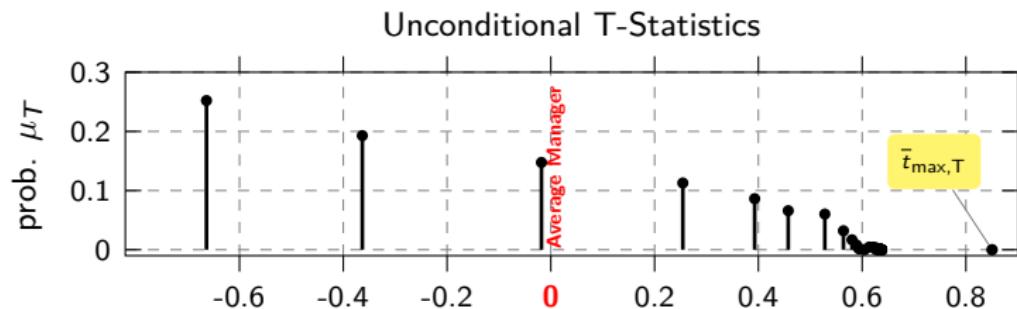
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A manager's  $t$ -statistic depends on her **skill-to-luck ratio**:

$$t_{\alpha,t}(n^i) = (-1)^{\mathbf{1}_{n^i < \phi_t}} \sqrt{\frac{2}{\pi}} \frac{\sigma_S^2 |k_t|}{\sigma_S^2 + o_t^c \phi_t} \frac{\text{Timing Gains}}{\text{Timing Gains} + \underbrace{\frac{\sigma_S}{\sigma_S^2 + \phi_t o_t^c} \frac{n^i - \phi_t}{\sqrt{n^i}}}_{\text{Skill-to-Luck Ratio}}}$$



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- ① In a standard REE with a Brownian flow of information (He and Wang (1995) in continuous time), the average rate of information arrival is  $\phi_t = t$  and

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In this model with Network A only, the average rate of information arrival is  $\phi_t = \exp(\eta^A t)$  and

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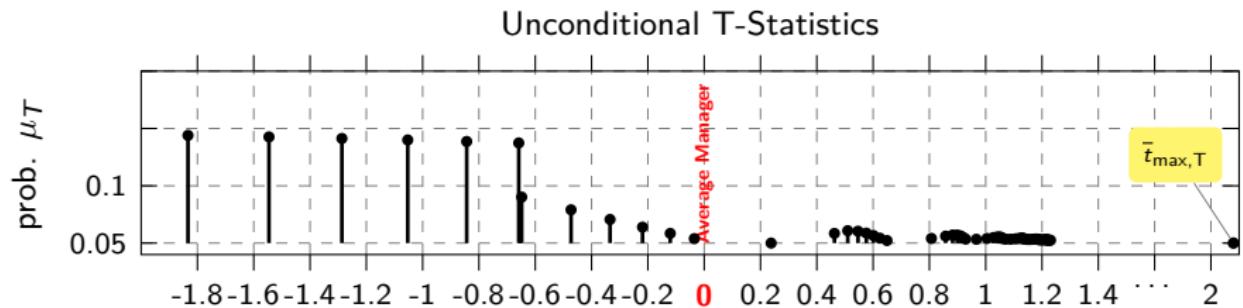
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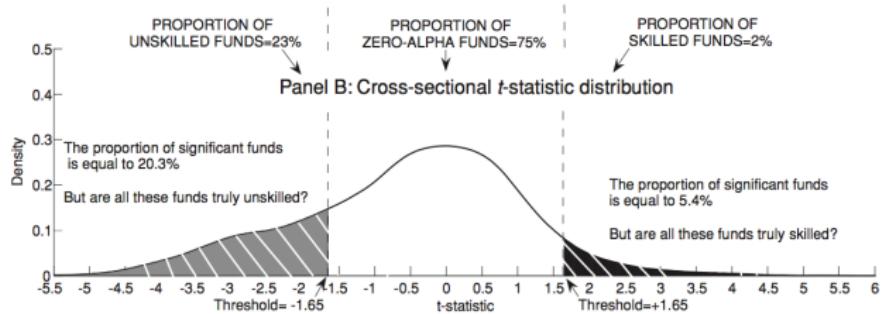
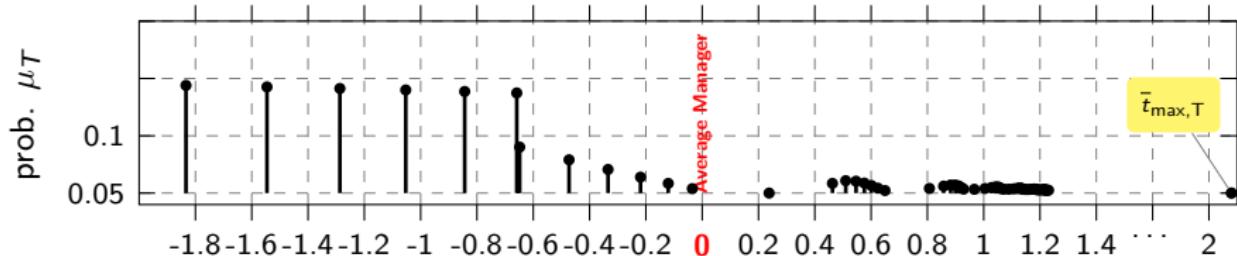
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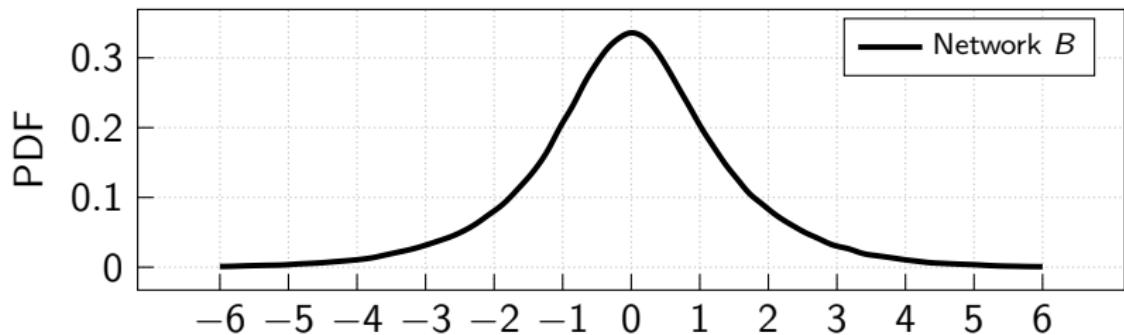
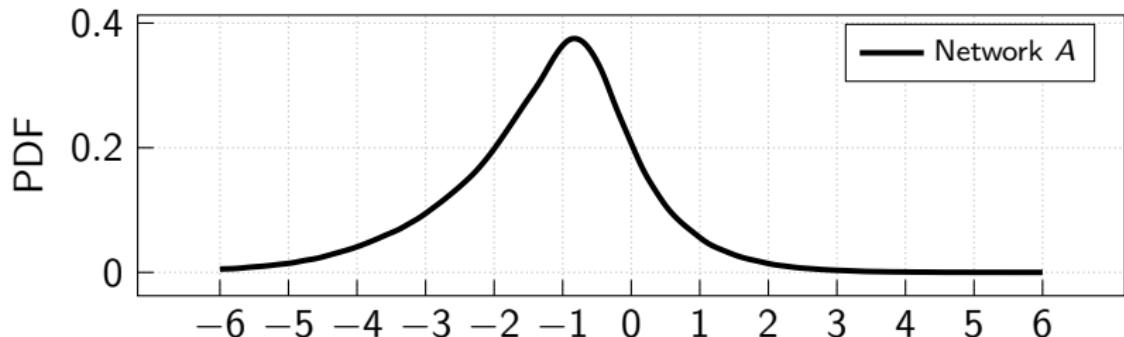
# Cross-Sectional Performance

Unconditional T-Statistics

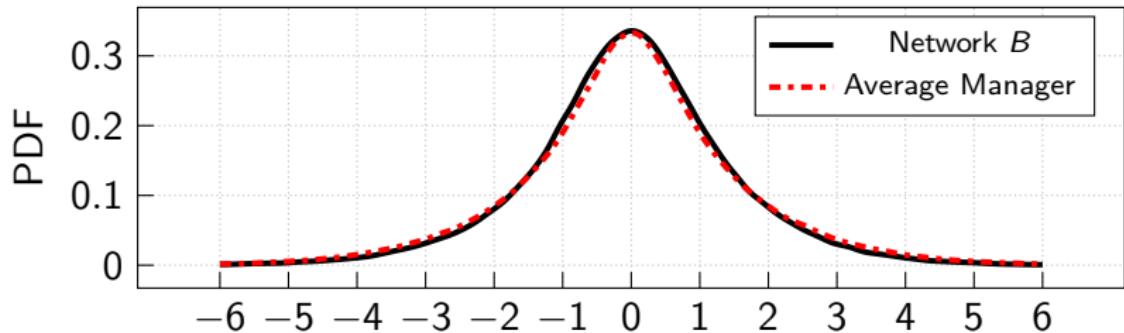
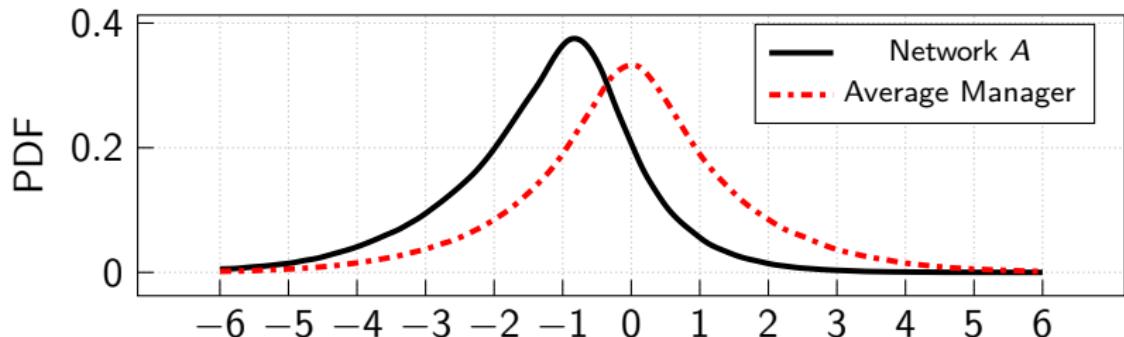


SOURCE: BARRAS, SCAILLET & WERMERS (JF, 2010)

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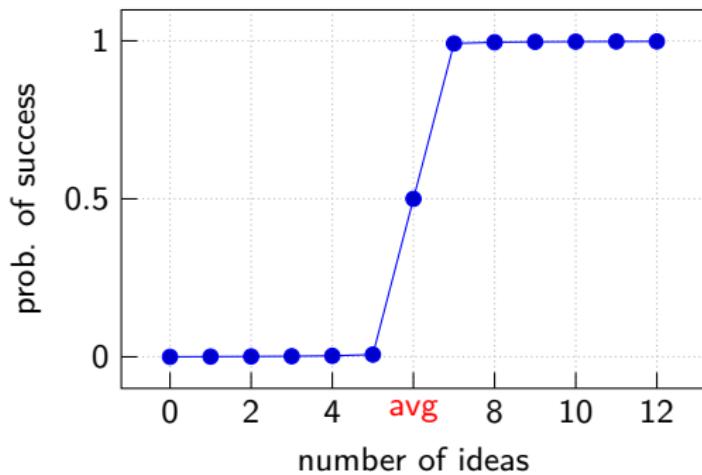
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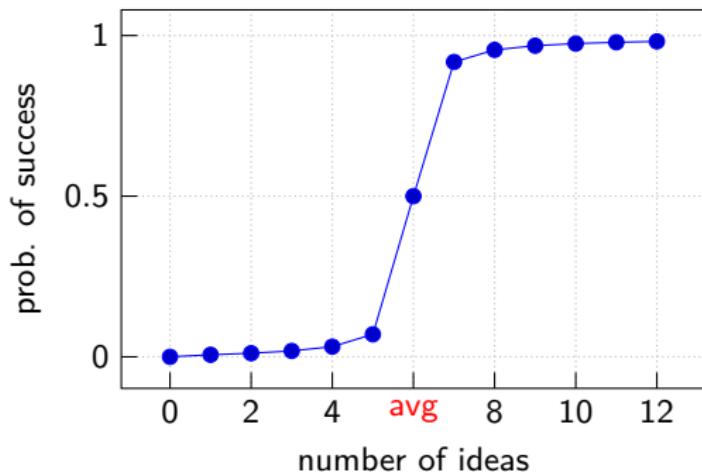
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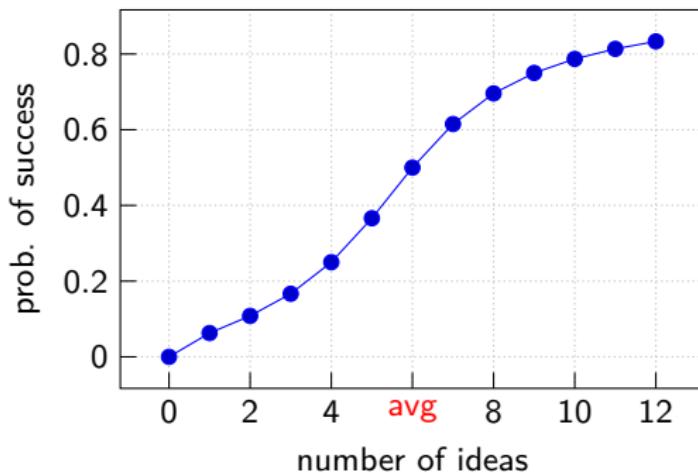
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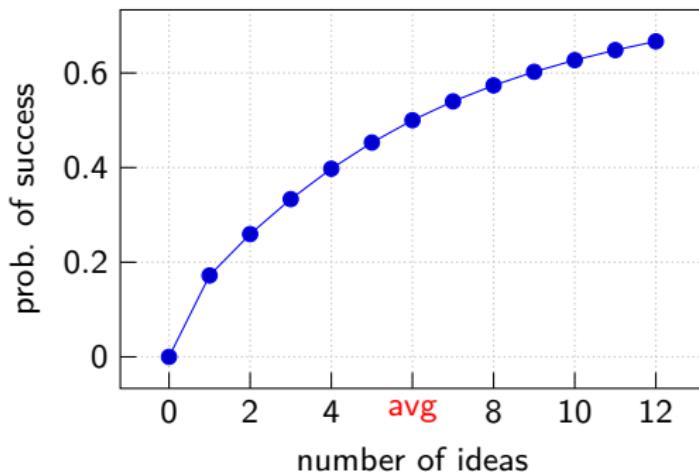
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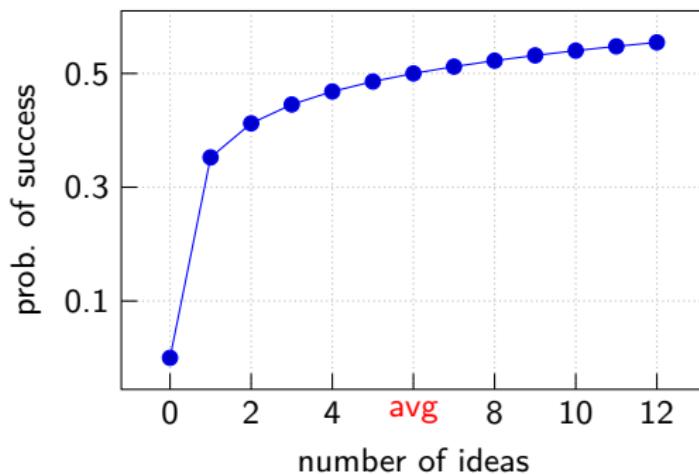
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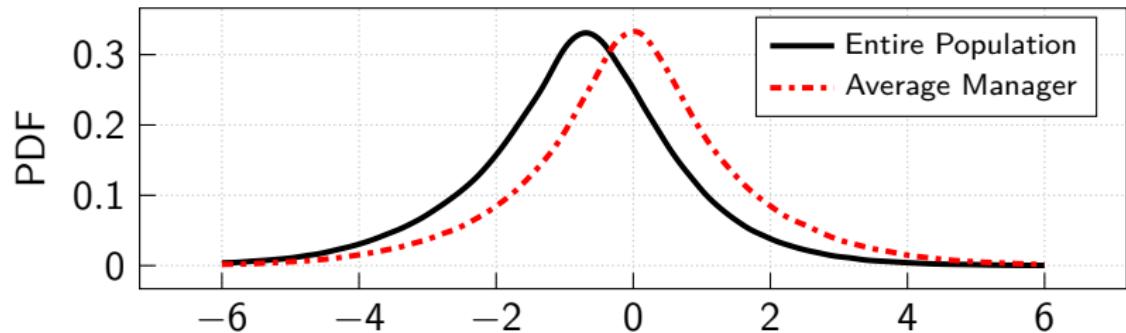
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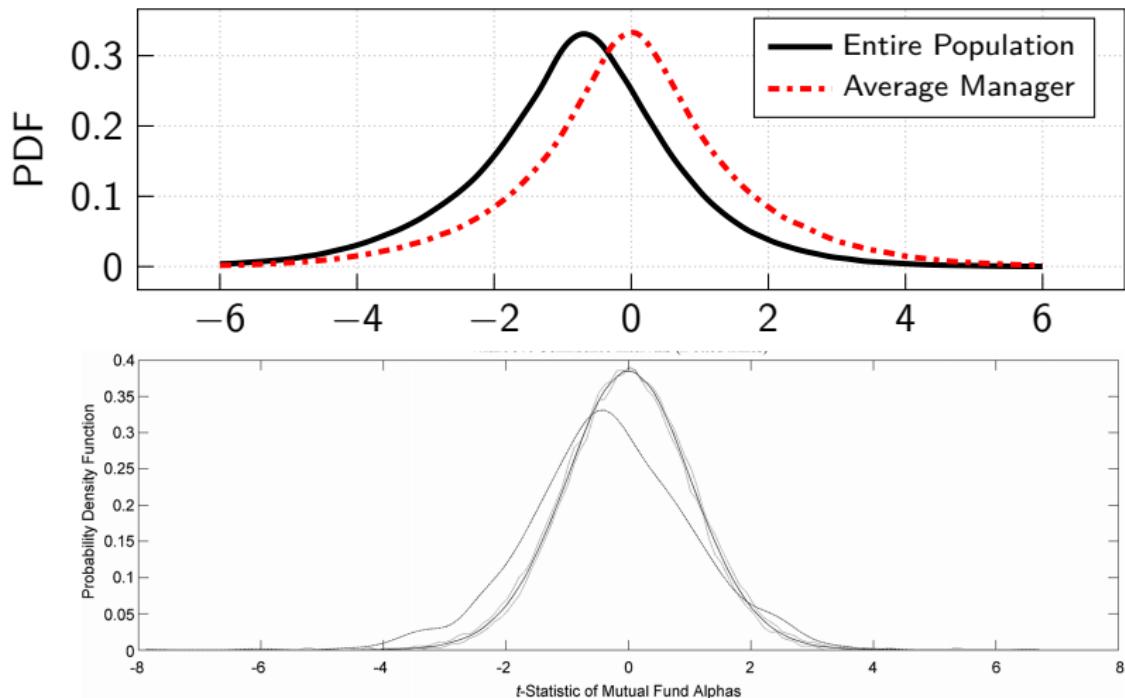
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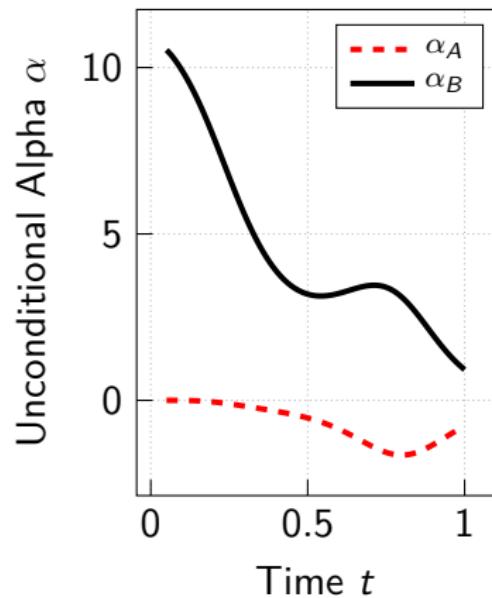


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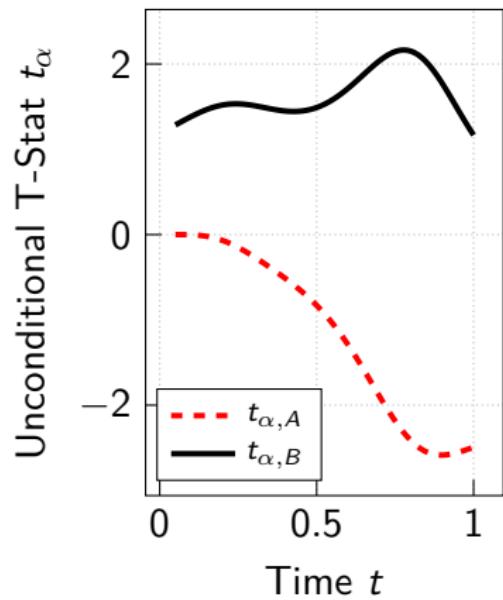
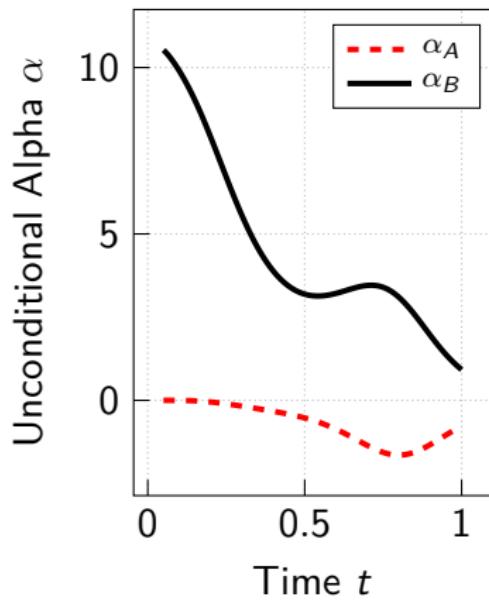


SOURCE: KOSOWSKI, TIMMERMANN, WERMERS & WHITE (JF, 2006)

# Persistence



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