The Social Dynamics of Performance*

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Abstract

A pervasive empirical finding is that mutual fund managers do not maintain their performance. In this paper, I show that social interactions can explain this fact. To do so, I allow a "crowd" of managers to meet at random times and exchange ideas within a rational-expectations equilibrium model. I show that social interactions simultaneously allow prices to become more efficient and better-informed managers to reap larger profits. Yet, social interactions cause managers' alpha to become insignificant. The main implication is that increased efficiency causes managers to implement passive investment strategies for which they should not be rewarded. In addition, by increasing price informativeness, social interactions produce momentum in stock returns and induce most managers to become momentum traders, consistent with empirical findings.

Keywords. rational-expectations equilibrium, information percolation, social interactions, performance measure, performance persistence, momentum.

JEL Classification. D53, D82, D83, D85, G14, G23.

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1 Introduction

There is ample evidence that the performance of mutual fund managers does not persist.[1] The vast majority of managers are unable to generate abnormal returns and a significant fraction even underperforms passive benchmarks. The handful of managers who outperform seldom maintain their performance.

That performance does not persist is usually taken to imply that managers do not possess superior information (e.g., Carhart (1997)). In this paper, I show that a manager who possesses superior information does not necessarily maintain her performance if she interacts socially with other managers. To do so, I develop a rational-expectations equilibrium *à la* Grossman and Stiglitz (1980) and Wang (1993), in which agents gather information from both prices and social interactions. In the "noisy rational-expectations" literature, the typical setting involves a population of agents endowed with private information, who attempt to infer the information of other agents from prices. Other learning channels are absent. In my model, social interactions account for an alternative channel whereby information gets passed from one agent to another through private conversations, naturally complementing the price-learning channel. Social interactions simultaneously allow prices to become more efficient and better-informed managers to reap larger profits, and yet cause their performance, as traditionally measured (e.g., alpha of Jensen (1968)), not to persist. The main implication is that increased efficiency induces managers to implement passive investment strategies for which they should not be rewarded.

My model builds on a large literature arguing that social interactions play an essential role in the marketplace.[2] For instance, a survey by Shiller and Pound (1989) shows that interpersonal communications are an important component of institutional investors' decisions. To the question "*Was the fact that someone (whom you know or may know of) bought stock in the company influential in getting you to buy the stock ?*", 44% answered "yes". While the empirical prevalence of word-of-mouth communication is increasingly acknowledged, theory has made little headway in this direction.

In this paper, I show that social interactions allow to explain, within a unified model, a number of stylized facts in the mutual fund literature. To do so, I consider a

[1] Barras, Scaillet, and Wermers (2010) find that the fraction of skilled managers has dropped from 14% in the early 90s to a mere 2% in the last decade. Jensen (1968), Malkiel (1995), Gruber (1996) and Carhart (1997) find that most mutual funds fail to outperform, and often underperform, passive benchmarks, even before transactions costs. See Wermers (2000) for a literature review.

[2] See Hong, Kubik, and Stein (2005) and Cohen, Frazzini, and Malloy (2008) for references related to the mutual fund industry. Massa and Simonov (2011) show that college-based interactions influence portfolio decisions. Hong, Kubik, and Stein (2004) show that investors find the market more attractive when more of her peers participate. Brown, Ivkovic, Smith, and Weisbenner (2008) reach similar conclusions. Other evidence includes Grinblatt and Keloharju (2001) or Ivkovic and Weisbenner (2005).

rational-expectations equilibrium model in which a crowd of fund managers attempt to forecast some fundamental, while noise traders continuously blur the price signal about this fundamental. As in prior contributions, managers initially develop a working idea regarding the fundamental. In contrast to prior contributions, however, managers interact socially and refine their initial idea through both prices and word-of-mouth communication. Specifically, managers meet at random times and share their ideas until the fundamental is revealed. When two managers meet, information flows in both directions as managers bounce ideas off of one another. After the meeting, managers wind up with new pieces of information and post an order. By clearing the market, prices centralize the outcome of conversations, albeit imperfectly due to noise trading.

This process leads to a novel form of learning, which includes a common and a private channel. What makes learning unique in my model is the way in which the private channel fits into the learning process and how the private channel affects the common channel. The pattern of information arrival is as follows. When a manager is socially inactive, she analyzes the price—the common channel—in an effort to infer private conversations among other managers. Information aggregated through the price flows tick-by-tick, which induces continuous updates in a manager's views. By contrast, conversations—the private channel—take place every once in a while. But upon meeting someone, a manager obtains large pieces of information, which she incorporates by updating her expectations in a discontinuous way. Private conversations ultimately feed back into the price and thereby make the public channel increasingly effective. If social interactions are sufficiently intense, the market learning process improves at great speed; a manager becomes progressively unable to preserve her informational advantage before the market catches up with her (that is, her performance does not persist).

Consistent with empirical findings, this mechanism applies to most managers in my model except a small group of top performers.[3] This result is perhaps surprising considering the rich information heterogeneity triggered by social interactions. While managers hold one idea initially, some may entertain many meetings but others may be less successful. Therefore, only a subset of managers eventually get a "good" idea—many pieces of information—about the fundamental. Building on Stein (2008), good ideas do not travel all over the marketplace. Good ideas rather remain localized. In my model, managers with good ideas do not talk with those who hold inferior ideas. This segmentation of information allows managers with good ideas to preserve their informational advantage over a longer time period. Yet, even if a manager gets a good idea, market efficiency eventually offsets her informational advantage so that her

[3] Kosowski, Timmermann, Wermers, and White (2006), Barras, Scaillet, and Wermers (2010) and Fama and French (2010) find no evidence of skill, except in the upper tail of the cross-sectional distribution of managers.

alpha rapidly becomes insignificant. Only exceptionally good ideas allow a manager to consistently generate abnormal returns.

This result, however, does not imply that managers with good ideas do not extract large rents. Before accelerating the information flow through prices, private conversations first allow managers to improve their market-timing ability. As social interactions intensify, managers with good ideas speculate more aggressively and rapidly accumulate large profits in dollar value. Hence, even if price informativeness eventually weakens their timing ability, these managers pocket substantial profits in the meantime. In contrast, managers with inferior ideas immediately rally to the market consensus and lose their timing ability. Information heterogeneity therefore creates large and persistent spreads in trading gains across managers.

Overall, social interactions do not prevent managers with good ideas from accumulating large trading gains, but cause their performance, as measured by their alpha, not to persist. The reason is that social interactions induce most managers—even those with good ideas—to progressively implement passive investment strategies for which they should not be rewarded. As a result, not only does their performance become non-persistent, but their alpha becomes even negative. This mechanism may be powerful enough to make it literally impossible to distinguish alphas generated by managers with good ideas from alphas generated by managers with inferior ideas.

The model offers two additional empirical implications. First, social interactions produce momentum in stock returns, one of the most pervasive facts in Financial Economics.[4] That my model generates momentum is intriguing for two reasons. First, in a rational-expectations equilibrium, returns usually exhibit reversal (e.g., Wang (1993)). Second, in my model, momentum is a rational phenomenon that is not simply driven by a risk factor (noise trading). Indeed, momentum hinges upon the diffusion of information through private conversations.[5] Carhart (1997) argues that the weak evidence of performance persistence in the mutual fund industry is the sole result of the one-year momentum effect. By providing a new theory for momentum based on social interactions, my model suggests that momentum profits are rather modest and that managers pursuing momentum strategies are unlikely to maintain their performance.

Second, social interactions give rise to trading patterns that are consistent with those identified in the mutual fund industry. By increasing price informativeness, social

[4]See Poterba and Summers (1988), Jegadeesh and Titman (1993), Rouwenhorst (1998), Menzly and Ozbas (2010) and Moskowitz, Ooi, and Pedersen (2011), among others. This result is consistent with Wermers (1999) who argues that mutual fund managers are likely to play an important role in the momentum mechanism.

[5]See Daniel, Hirshleifer, and Subrahmanyam (1998), Barberis, Shleifer, and Vishny (1998) or Hong and Stein (1999) for behavioral explanations. See Berk, Green, and Naik (1999) and Johnson (2002) for rational risk-based accounts.

interactions encourage lesser-informed managers to chase the trend in an attempt to "free-ride" on the information of others. Instead, managers with superior information tend to break away from the herd to exploit their informational advantage, as in Coval and Moskowitz (2001).[6] As social interactions intensify, the market processes more and more information so that even well-informed managers eventually align with the market consensus. For most managers, momentum trading becomes sooner or later an optimal strategy. This result is in line with Grinblatt, Titman, and Wermers (1995) who find that 77% of mutual funds in their sample are momentum investors.

After discussing related works, the balance of the paper is as follows. Sections 2 and 3 respectively present and solve the model. Sections 4, 5 and 6 contain the results. Section 7 concludes. Mathematical derivations are collected in the technical appendix.

Review of the Literature

My model relates to three strands of literature. First, in the absence of social interactions, my model nests the "noisy rational-expectations" setup of He and Wang (1995), except for allowing trading to be continuous. This model serves as a natural benchmark against which to evaluate the effects of social interactions. Private information is dispersed among investors and learning is exclusively guided by prices. This model has two additional advantages: first, it involves an economy with a finite horizon, a convenient framework to study performance persistence. Second, the model involves long-lived information, a necessary feature for social interactions to impact prices. My paper differs from He and Wang (1995) in two respects, though. While He and Wang (1995) investigate the effect of dispersed information on trading volume, my main focus is on performance. But more important, unlike He and Wang (1995), my purpose is to let agents interact socially.

Second, in my model, social interactions are based on the recent developments in the "information-percolation" literature initiated by Duffie and Manso (2007). While information percolation is usually applied to decentralized markets, Andrei and Cujean (2011) show that information percolation is equally relevant when markets are centralized but information is not—agents meet to share their information. This approach to modeling social interactions is also advocated by Andrei (2012). Unlike Andrei and Cujean (2011) and Andrei (2012), agents in my model are free to trade whenever they receive new information. As a result, public information continuously flows from prices, while private information flows at discrete, random points in time. This reconciles the meeting and the trading frequency, changes the pattern of information arrival, and leads to a novel form of learning. As a technical contribution, the learning process requires a

[6] See also Brennan and Cao (1997) and Feng and Seasholes (2004).

new filtering technique. Furthermore, unlike Andrei (2012), information is inherently long-lived in my model, allowing me to study how long informational advantages subsist. Finally, unlike Andrei and Cujean (2011) and Andrei (2012), I adapt the percolation mechanism to embed the finding of Stein (2008) that good ideas remain localized. I show that even if good ideas remain localized, the market rapidly catches up. Therefore, good ideas only provide a short-lived informational advantage.

Finally, a well-accepted explanation for the lack of performance persistence in the mutual fund industry is that of Berk and Green (2004). If the provision of funds by investors is competitive and superior managerial ability is scarce, abnormal performance attracts flows of funds that compete away subsequent abnormal returns. The view expressed in this paper is distinct in several respects. First, the mechanism I highlight does not rely on fund flows: performance becomes non-persistent, even in the absence of flows into and out of the fund. Second, while Berk and Green (2004) build their explanation on the notion of "superior ability", the source of which they do not model, mine is based on the familiar notion of informational heterogeneity: some managers are endogenously better informed than others.[7] Lastly, the model of Berk and Green (2004) is partial equilibrium (trading does not affect benchmark returns). Instead, my results rely on an equilibrium mechanism whereby prices become gradually more efficient.

2 A Model of Socially-Interacting Fund Managers

The goal of this section is to introduce social interactions in an otherwise standard Grossman and Stiglitz (1980) framework. Subsections 2.1 and 2.2 present the economy and its information structure. Subsection 2.3 describes how fund managers interact socially and then derives the resulting distribution of information in the economy.

2.1 Assets, Trading, and the Economy

The economy has a finite horizon $T < \infty$ and trading continuously occurs on $[0, T)$. An economy with a finite horizon offers a convenient framework to study the persistence of performance. Moreover, because my purpose is to introduce social interactions whereby agents meet and share information on the time span $[0, T)$, it is important to let agents trade when they receive new information.

The market consists of two assets. The first asset is a riskless claim with perfectly elastic supply. Its rate of return is normalized to $r = 0$, as in He and Wang (1995).[8]

[7] See Chevalier and Ellison (1999a), Coval and Moskowitz (2001), Cohen, Frazzini, and Malloy (2008) and Christoffersen and Sarkissian (2009).

[8] See also Admati (1985), Wang (1993), He and Wang (1995) or Brennan and Cao (1996). However,

The second asset is a risky stock with equilibrium price P_t at time t. The stock pays a liquidating dividend $\Pi + \delta$ to be revealed at date T and which I regard as the stock's fundamental value.

The supply Θ of the stock is stochastic and follows an Ornstein-Uhlenbeck process:

$$\mathrm{d}\Theta_t = -a_\Theta \Theta_t \mathrm{d}t + \sigma_\Theta \mathrm{d}B_t^\Theta \tag{1}$$

where $(B_t^\Theta)_{t\geq 0}$ is a Brownian shock with amplitude $\sigma_\Theta > 0$ and mean-reversion speed $a_\Theta > 0$. As is customary, I interpret Θ as the supply of the stock available to the market, while noise traders—agents who trade for reasons unrelated to fundamental information—have inelastic demands of $1 - \Theta$ units of the stock (in total supply of 1).[9] Trades posted by noise traders make the supply Θ noisy through (1) and thereby prevent prices from fully revealing the fundamental.

The economy is populated with a continuum of fund managers. Each manager j in the crowd attempts to predict the fundamental value of the stock. Manager j does so by analyzing the tape, and by privately discussing with other managers. Information collected by manager j up to time t is contained in her information set $\mathcal{F}_t^j$.

I am interested in the heterogeneity of information $\mathbb{F}^j = (\mathcal{F}_t^j)_{t\geq 0}$ generated by social interactions among fund managers. Differences in risk aversion is not my main focus. I thus assume that managers share a common absolute risk aversion γ and, for simplicity, exhibit CARA utility. The problem of a manager j is to find a predictable portfolio strategy θ^j maximizing her expected utility over terminal wealth

$$E\left[-e^{-\gamma W_T^j}\middle|\,\mathcal{F}_t^j\right]$$

subject to the constraint

$$W_T^j = W_0^j + \int_{[0,T)} \theta_t^j \mathrm{d}P_t + \theta_T^j \Delta P_T. \tag{2}$$

I make two modeling choices regarding (2). First, I neglect fund flows. One may think of funds in my model as closed-end funds.[10] Second, for simplicity, I follow Koijen (2012) and do not impose short-sales or borrowing constraints, although managers may face certain trading restrictions.[11] Finally, notice that trading in (2) allows for a jump in prices of size ΔP_T at the announcement date. I elaborate on this price jump in the next subsection.

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see Loewenstein and Willard (2006) for the implications of endogenizing the riskfree rate.

[9] Performance will be measured not including the losses of noise traders.

[10] See Chevalier and Ellison (1997) and Sirri and Tufano (1998) for empirical treatments.

[11] See, for instance, Almazan, Brown, Carlson, and Chapman (2004).

2.2 The Information Structure

The fundamental value of the stock consists of two parts, Π and δ. Managers do not observe δ, nor do they get information about it. They view $\delta \sim \mathcal{N}(0, \sigma_\delta^2)$ as an independently-drawn normal variable. I refer to δ as *residual uncertainty*, since managers cannot possibly learn about it.[12] I need this feature to keep managers with highly accurate information from posting massive orders at the announcement date, i.e., an equilibrium would not exist without residual uncertainty.[13] Overall, residual uncertainty accounts for a surprise at the announcement date, which entails a price jump ΔP_T in (2).

Managers only get information about Π, the other part of the stock's fundamental value. This information is organized as follows. Consider the crowd $(I, \mathcal{I}, \iota)$ of managers and split it into two groups $I = I_1 \cup I_2$. Managers in the first group I_1 observe Π and constitute an initial fraction $1 - \omega_0$.[14] These managers are *informed* and are therefore labelled by i (that is, $j = i$). Managers in the second group I_2 represent the remaining fraction ω_0. They do not observe Π and thus need to learn about it. As *learning* individuals, they are indexed by l (that is, $j = l$). Managers l assume that Π and the initial supply Θ_0 are independent and start with prior beliefs $\Pi \sim \mathcal{N}(0, \sigma_\Pi^2)$ and $\Theta_0 \sim \mathcal{N}(0, \frac{\sigma_\Theta^2}{2a_\Theta})$.[15] In addition, each manager l is endowed with an initial signal about Π:

$$S_1^l = \Pi + \epsilon^l, \tag{3}$$

which I view as a *working idea* that managers l have privately developed. Working ideas are imperfect, as they contain an agent-specific error $\epsilon^l \sim \mathcal{N}(0, \sigma_S^2)$.[16]

Now comes the central feature of my model. From the initial date onwards, managers l interact through private meetings and *socially refine their idea in* (3) *they initially developed on their own.* In particular, a manager l builds a collection $\{S_k^l\}_{k=1}^{n_t^l}$ of ideas, where the number n_t^l of ideas she collected increases through the vagaries of her meetings. This meeting process constitutes a central component of my model, which I describe in the next subsection.

[12] See Grundy and McNichols (1989), Hirshleifer, Subrahmanyam, and Titman (1994), He and Wang (1995) or Manela (2011) for other references making use of residual uncertainty.

[13] One can regard residual uncertainty as capturing a failure of information to completely flow through prices, e.g., the post-earning announcement drift of Bernard and Thomas (1989). Alternatively, δ may account for a shortfall in forecasting a firm's default. The price of bonds or equities often drops precipitously at or around the time of default, as in Duffie and Lando (2001).

[14] $(I, \mathcal{I}, \iota)$ is a nonatomic space with $\iota(I) = 1$, i.e. its size is 1. See Duffie (2011) for further details. Managers i represent a fraction $\iota(\{j : j_0 \in I_1\}) = 1 - \omega_0$ with $\omega_0 \in [0, 1]$.

[15] As in He and Wang (1995), the prior variance of the noisy supply is set to its stationary level.

[16] Errors are initially independent among the crowd; $\epsilon^k \perp \epsilon^j$, $\forall k \neq j$ $\forall j \in I_2$, $\epsilon^l \perp (B_t^\Theta)_{t \geq 0}$ and $\epsilon^l \perp \Pi$. Their magnitude is controlled by managers' precision $\frac{1}{\sigma_S^2} > 0$.

2.3 Modeling the Social Network of Fund Managers

Mutual fund managers interact socially in several ways. Evidence indicates that managers keep in touch with CEOs or chairmen who were former classmates (Cohen, Frazzini, and Malloy (2008)) and conduct conversations with other managers who operate in the same city (Hong, Kubik, and Stein (2005)). But all types of social interactions do not result in the same information quality. For instance, ideas exchanged on large-access investment forums, such as Yahoo groups or Motley Fool, are presumably less valuable than those exchanged on highly-selective on-line communities of top investors, such as The Value Investor Club or The Alburn Village. In the same manner, direct interactions with firm insiders and so-called "expert networks", another example of social interactions, deliver highly accurate information.[17]

To reproduce this kind of information heterogeneity, I acknowledge that "good" ideas do not travel all over the marketplace. Building on Stein (2008), I assume that good ideas stay confined to a small group of managers. I also allow some managers to acquire "perfect" information, which they do not circulate. This segmentation of information allows better-informed managers to keep their informational advantage over a longer time period. I use this property to highlight the impact of social interactions on price informativeness—even if a manager gets good ideas, market efficiency eventually offsets her informational advantage.

To formalize this idea, I adapt the "information-percolation" mechanism developed by Duffie, Malamud, and Manso (2009). This approach to modeling word-of-mouth communication circumvents many issues associated with alternative formulations in the literature.[18] In particular, in my model, information percolation accommodates both Bayesian learning and learning from prices. Moreover, information percolation is tractable and leads to strong information heterogeneity.

[17]Expert networks are controversial firms that link managers with firm insiders. During the trial of one such firm, a former hedge-fund executive testified that the information provided was "absolutely perfect". Source: "Linking Expert Mouths with Eager Ears", *The Economist*, June 16th, 2011.

[18]Some researchers use graph theory to model the information structure—agents are "nodes" of a network. The resulting network is often admittedly complex and the results are sensitive to its specification (see Ozsoylev and Walden (2011)). See Colla and Mele (2010) for a cyclical network, Acemoglu, Bimpikis, and Ozdaglar (2010) for an endogenous network structure and DeMarzo, Vayanos, and Zwiebel (2003) for results sensitivity regarding the network specification. Another strand of the literature resorts to epidemiological models—agents typically progress from "susceptible" to "infected" and back to "recovered". Because learning is tantamount to the spread of a disease, this approach does not accommodate Bayesian updating nor learning from prices. Additionally, generating a wide heterogeneity among agents is challenging. See, Hong, Hong, and Ungureanu (2010) who use such models to explain the dynamics of trading volume and Burnside, Eichenbaum, and Rebelo (2010) who model the booms in the housing market.

2.3.1 Information Percolation and Information Segmentation

I first describe the segmentation of information and the meeting process. I then derive the resulting distribution of information. At any date t, a fraction $1 - \omega_t$ of managers are perfectly informed and do not communicate. Communication exclusively takes place among imperfectly informed managers l. The population of managers l is further divided into two networks: Networks A and B. Managers of Network B do not talk to those of Network A (and vice-versa). The fraction of managers positioned within each network is given by q_t^A and q_t^B, respectively. Each manager l is initially part of Network A. Network B arises through the meeting process, which I now present.

A manager l meets other managers at Poisson arrival times with intensity η_t. When two managers meet, they are randomly selected from the crowd. Since managers belong to a continuum, they cannot meet more than once, nor can they individually influence prices. Therefore, they have no incentives to lie and, as they run into each other, I assume that they truthfully exchange their entire set of ideas.[19]

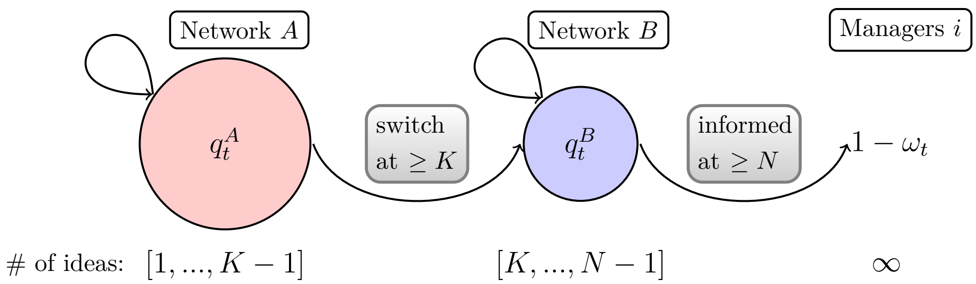
This matching procedure implies that information sharing is *additive-in-types*. If two managers respectively have $n - m$ and m ideas, then they both wind up with n ideas after they meet. Therefore, one does not need to keep track of the whole history of their meetings and only their current number n_t^l of ideas is relevant. Moreover, by Gaussian theory, their set of ideas $\{S_k^l\}_{k=1}^{n^l}$ can be conveniently summarized by their average idea $\bar{S}_n^l \equiv \frac{1}{n^l} \sum_{k=1}^{n^l} S_k^l$, i.e., a single but more precise idea.

I now introduce the notion of "good" ideas. I define good ideas as those composed of $K(\geq 2)$ or more pieces of information. To prevent good ideas from traveling over the whole network of managers, I assume that managers who gather good ideas leave Network A and enter Network B. Managers with good ideas keep talking but only to those of Network B, thus causing good ideas to remain localized. Furthermore, managers of Network B can potentially hold any number of ideas between K and infinity, which precludes a tractable solution to the model. To limit the number of types within Network B, I assume that managers eventually become perfectly informed after collecting a large number $N(\geq 2K)$ of ideas.

Overall, managers of Network A have at least their working idea, but do not hold good enough ideas to be part of Network B. As a result, the set of ideas exchanged in this network is given by $A = \{1, ..., K - 1\}$. Managers of Network B only exchange

[19]Agents do not attempt to hide part of their information, nor to add noise to their set of ideas. Strategic considerations only enter in networks comprised of a finite number of agents. Heuristically, one can think of truth-telling as follows. Kosowski, Timmermann, Wermers, and White (2006) show that active management skills not only generate higher performance but also superior cost efficiencies. This evidence can be combined with the result of Stein (2008)—competitors choose to share their information if more information generates lower costs, which induces strategic complementarity. Acemoglu, Bimpikis, and Ozdaglar (2010) show that truth-telling can be an equilibrium outcome.

Figure 1: Configuration of Social Dynamics



Network Configuration. Figure 1 shows the pattern of information transmission along with the segmentation of information.

valuable ideas, but, once they become perfectly informed, they stop communicating with other managers and leave Network B. As a result, the set of ideas exchanged in this network is given by $B = \{K, ..., N-1\}$. Finally, a manager l evolves through each network according to a Poisson counter $(N_t^l)_{t\geq 0}$; it records her number of meetings (up to time t), which take place with a network-specific intensity $\eta_t \equiv \eta(q_t^A \mathbf{1}_{\{n_t^l \in A\}} + q_t^B \mathbf{1}_{\{n_t^l \in B\}})$. This pattern of information transmission is summarized in Figure 1.

Having described the meeting process, the goal now is to determine the resulting distribution of information in Networks A and B. Consider first the distribution μ^A of ideas within Network A. To determine how μ^A behaves over time, two observations are needed: first, Network A has its own endogenous size q^A and, second, managers in this network meet with intensity ηq_t^A. The reason is that managers of Network A only share information with others of their own network. I adapt Boltzmann's equation for μ^A in Duffie and Manso (2007) accordingly and highlight the result in the proposition below.

Proposition 1. *Let $\mu_t^A(\mathbb{A}) = \iota(\{j : j_t \in \mathbb{A}\})$ be the cross-sectional distribution of types in Network A where $\mathbb{A} \subseteq A$. Let $q_t^A = \sum_{n=1}^{K-1} \mu_t^A(n)$ be the mass of managers in Network A. For any $n \in A$, $\mu^A(\cdot)$ obeys*

$$\frac{d}{dt}\mu_t^A(n) = -\eta q_t^A \mu_t^A(n) + \eta \sum_{m=1}^{n-1} \mu_t^A(n-m)\mu_t^A(m) \tag{4}$$

with initial conditions $\mu_0^A(n) = \delta_1$ where δ_1 is a Dirac mass at $n = 1$.

Proof. See Appendix A. Q.E.D.

Meetings have two effects on the evolution of the distribution of ideas in Network A. First, meetings cause managers to change their type, the first term in (4). Specifically, if a manager holding n ideas meets someone, she ends up with $n+m$ ideas and leaves type n. Her new number $n+m$ of ideas can be in either A or B. Second, by delivering m new ideas, meetings cause managers formerly holding $n-m$ ideas to enter type n. The second term in (4) accounts for all the meetings that result in gathering n ideas.

Finally, consider the distribution μ^B of ideas in Network B. This distribution evolves according to the dynamics that I report in Proposition 2 below.

Proposition 2. *Let $\mu_t^B(\mathbb{B}) = \iota(\{j : j_t \in \mathbb{B}\})$ be the cross-sectional distribution of types in Network B where $\mathbb{B} \subseteq B$. Let $q_t^B = \sum_{n=K}^{N-1} \mu_t^B(n)$ be the mass of managers in Network B. For any $n \in B$, $\mu^B(\cdot)$ obeys*

$$\frac{d}{dt}\mu_t^B(n) = -\eta q_t^B \mu_t^B(n) + \eta \begin{pmatrix} \mathbf{1}_{\{n\in[K,2(K-1)]\}} \sum_{m=n-(K-1)}^{K-1} \mu_t^A(n-m)\mu_t^A(m) \\ +\mathbf{1}_{\{n\in[2K,N-1]\}} \sum_{m=K}^{n-K} \mu_t^B(n-m)\mu_t^B(m) \end{pmatrix} \quad (5)$$

with initial conditions $\mu_0^B(n) = 0 \ \forall n \in B$. The mass of managers i is then given by $1-\omega_t = 1 - q_t^A - q_t^B$.

Proof. See Appendix A. Q.E.D.

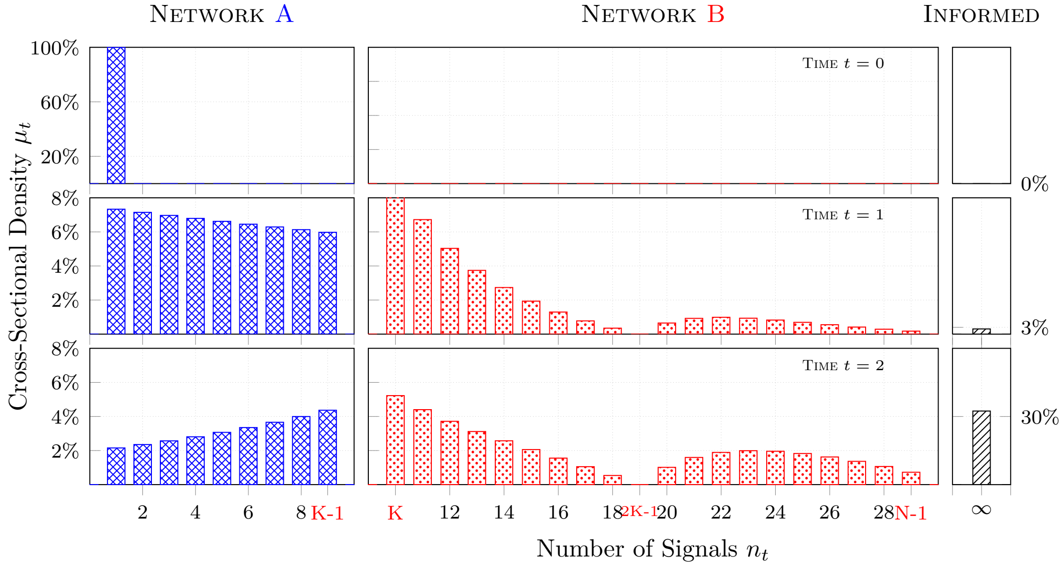
The distribution μ^B behaves similarly to μ^A. In particular, Network B also has its own endogenous size q^B and managers in this network meet with intensity ηq_t^B. In addition, μ^B accommodates the migration of managers who develop good ideas, the first term in the bracket in (5).

2.3.2 Illustration of the Social Dynamics

Figure 2 illustrates the evolution of the distribution μ^A of ideas in Network A. Network A is initially populated with managers who only hold one idea. These managers progressively become a minority as more and more managers eventually meet someone. In turn, the size q^A of Network A decreases over time, as successive meetings cause some managers to gather sufficiently many ideas to enter Network B (see Figure 3).

Interestingly, Figure 3 shows that the size q^B of Network B moves non-monotonically. Network B is initially empty, as managers are either part of Network A or perfectly informed. As information starts to percolate, Network B is fed with migrating managers who no longer talk to managers of Network A. After a while, managers who collect N ideas become perfectly informed and leave Network B (insider information does not circulate), ultimately causing the size of Network B to fall and the fraction of informed managers to rise. The distribution μ^B follows the same pattern. Figure 2 shows that μ^B has two parts that represent respectively migrating managers and incumbents to Network B. The fraction of migrating managers progressively decreases

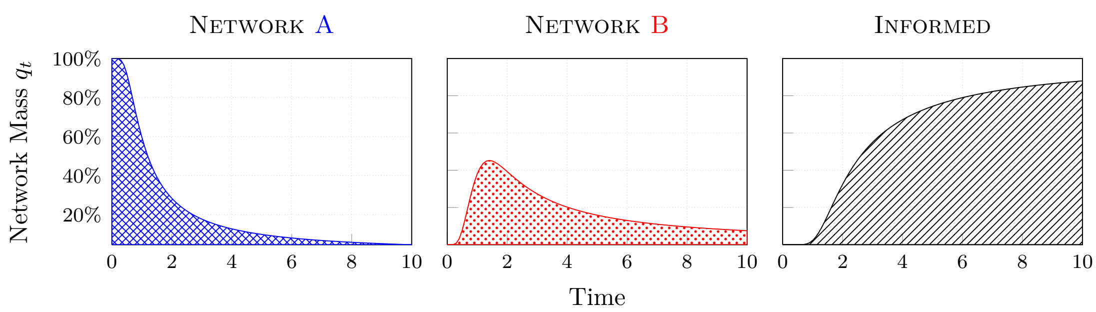
Figure 2: Evolution of Cross-Sectional Densities



Evolution of the Social Network (Part I). Figure 2 depicts the cross-sectional distribution μ_t of the number n_t of ideas at times $t = 0, 1, 2$, each corresponding to a given row. The leftmost and center columns correspond, respectively, to the distribution of ideas within Network A (▨) and Network B (▨). The rightmost column represents the mass of managers i (▨). The social intensity is $\eta = 3$ and the switching thresholds are $K = 10$ and $N = 30$ and $\omega_0 = 0$.

as its source—Network A—dries up, while the fraction of incumbents first increases and then decreases.

Figure 3: Evolution of Network Masses



Evolution of the Social Network (Part II). Figure 3 depicts the evolution of the mass q_t of each network from time $t = 0$ to $t = 10$. The leftmost and center figures show the mass of Network A (▨) and B (▨), respectively. The rightmost figure shows the mass of managers i (▨). The social intensity is $\eta = 3$ and the switching thresholds are $K = 10$ and $N = 30$.

3 Equilibrium with Word-of-Mouth Learning

To construct the equilibrium of the economy, I use the usual guess and verify method (e.g., He and Wang (1995)). Specifically, I first conjecture a price function, which I use to derive the filtering process of managers (Subsection 3.1). Second, using these filters, I solve the optimization problem of managers (Subsection 3.2). Finally, I impose market clearing, which allows me to validate the price conjecture (Subsection 3.3).

The main result of this section is that social interactions lead to a new pattern of information arrival that induces managers to trade more aggressively, eventually causing prices to become more informative.

3.1 Word-of-Mouth Learning

Social interactions give rise to a novel form of learning. This subsection shows that learning can be neatly split into what managers learn from the tape—the public channel—and what they learn from conversations—the private channel. The pattern of information arrival is as follows. Information extracted from the price flows tick-by-tick, producing small and continuous updates in managers' estimates. By contrast, private conversations seldom take place. But upon meeting someone, managers obtain large pieces of information, which they incorporate by updating their expectations in a discontinuous way.

3.1.1 Price Conjecture

A common practice in the "noisy rational-expectations" literature is to conjecture a price that is a linear function of the state variables of the economy. In my setup, these variables include the (second part of the) fundamental Π, the supply Θ and common expectations $\widehat{\Pi}_t^c \equiv E\left[\Pi \middle| \mathcal{F}_t^c\right]$. Common expectations represent econometrician's estimates, which are made on the basis of past prices only, i.e., $\mathcal{F}_t^c = \sigma(P_s : 0 \leq s \leq t)$.

Definition 1. *In a linear equilibrium, the stock price function has the following form:*

$$P_t = \lambda_{1,t}\Pi + (1 - \lambda_{1,t})\widehat{\Pi}_t^c + \lambda_{2,t}\Theta_t \tag{6}$$

over $[0, T)$*, where* $\lambda_{1,t}$ *and* $\lambda_{2,t}$ *are deterministic functions and* $P_T = \Pi + \delta$*.*

Linear prices in (6) have two parts. The first part, $\lambda_1\Pi + (1 - \lambda_1)\widehat{\Pi}^c$, represents average market expectations about the present value of future dividends $\Pi + \delta$.[20] It corresponds to the price that would obtain if managers were risk-neutral. Because

[20] See, for instance, Campbell and Kyle (1993) or Hong and Wang (2000) and Appendix E.

managers are risk-averse, they need to be compensated for the aggregate risk Θ they have to bear. In equilibrium, λ_2 turns out to be negative and the second term $\lambda_2\Theta$ in (6) therefore acts as a discount on the price that increases expected stock returns.

At the announcement date, everyone discovers the fundamental value $\Pi + \delta$. In turn, a jump occurs in managers' information sets to include $\Pi + \delta$ as part of public information.[21] The equilibrium coefficients λ_1 and λ_2 will take this jump into account so that the following equality $P_T = \Pi + \delta$ holds when the stock pays out.

The linear equilibrium of Definition 1 does not directly obtain in my setting. In particular, the combination of continuous-time trading and word-of-mouth communication leads to intractable equilibria. Yet, in Subsection 3.2, I show that it only takes a simple and accurate approximation of managers' portfolio for Definition 1 to apply.

3.1.2 Filtering in the Presence of Word-of-Mouth Communication

The goal now is to determine how imperfectly informed managers l update their expectations about Π. In particular, managers l use their own information $\mathcal{F}_t^l$ to come up with an estimate $\widehat{\Pi}_t^l \equiv E\left[\Pi|\,\mathcal{F}_t^l\right]$ and a variance $o_t^l \equiv V\left[\Pi|\,\mathcal{F}_t^l\right]$ at any time t. This task is affected by the presence of noise traders through the supply of the stock, for which they also need an estimate $\widehat{\Theta}_t^l \equiv E\left[\Theta_t|\,\mathcal{F}_t^l\right]$.

Managers l build their information set $\mathcal{F}_t^l$ on two sources of information: first, they build a collection of ideas $\{S_k^l\}_{k=1}^{n_t^l}$ through social interactions. Second, they analyze the history of prices $(P_t)_{t\geq 0}$ in an attempt to infer conversations among other managers. Combining these two sources of information, an arbitrary manager l observes

$$\mathcal{F}_t^l = \sigma\left(\left(P_s, S_k^l\right) : 0 \leq s \leq t, k \leq n_s^l\right), \quad 0 \leq t < T. \tag{7}$$

Perhaps needless to say, informed managers i do not learn about Π, since they observe it. Hence, their information is summarized by

$$\mathcal{F}_t^i = \sigma((P_s, \Theta_s) : 0 \leq s \leq t) \vee \sigma(\Pi), \quad 0 \leq t < T. \tag{8}$$

Inspecting (8), one may be surprised that managers i observe the supply history $(\Theta_t)_{t\geq 0}$. But since the price in (6) is linear, managers i can readily invert the price and observe the current realization of the supply.[22]

[21] At time T, the information set of an agent l is given by $\mathcal{F}_T^l = \mathcal{F}_{T_-}^l \vee \sigma(\Pi + \delta)$. The same goes with both $\mathcal{F}_T^c = \mathcal{F}_{T_-}^c \vee \sigma(\Pi + \delta)$ and $\mathcal{F}_T^i = \mathcal{F}_{T_-}^i \vee \sigma(\delta)$. I refer the reader to Appendix E.

[22] $\mathcal{F}_t$ represents the filtration generated by (P_t, Θ_t, Π) for agents i and by $(P_t, \{S_k^l\}_{k=1}^{n_t^l})$ for an agent l who holds n_t^l signals at time t. Also, since an infinite number of ideas is tantamount to perfect knowledge of Π, (8) is obtained from (7) by letting $n^l \to \infty$.

In Proposition 3 below, I show how managers l and the econometrician (with common information c) dynamically update their expectations $\widehat{\Pi}$ and $\widehat{\Theta}$ and their variance o. A proof may be found in Appendix B.

Proposition 3. *In a linear equilibrium, conditional expectations solve the SDEs:*

$$d\begin{bmatrix}\widehat{\Pi}^c_t\\ \widehat{\Theta}^c_t\end{bmatrix}=\begin{bmatrix}0&0\\0&-a_\Theta\end{bmatrix}\begin{bmatrix}\widehat{\Pi}^c_t\\ \widehat{\Theta}^c_t\end{bmatrix}dt+\begin{bmatrix}o^c_t k_t\\ \sigma_\Theta-\frac{\lambda_{1,t}}{\lambda_{2,t}}o^c_t k_t\end{bmatrix}d\widehat{B}^c_t, \tag{9}$$

$$d\begin{bmatrix}\widehat{\Pi}^l_t\\ \widehat{\Theta}^l_t\end{bmatrix}=\begin{bmatrix}0&0\\0&-a_\Theta\end{bmatrix}\begin{bmatrix}\widehat{\Pi}^l_{t_-}\\ \widehat{\Theta}^l_{t_-}\end{bmatrix}dt+\begin{bmatrix}o^l_t(n^l_{t_-})k_t\\ \sigma_\Theta-\frac{\lambda_{1,t}}{\lambda_{2,t}}o^l_t(n^l_{t_-})k_t\end{bmatrix}d\widehat{B}^l_t+\begin{bmatrix}1\\ -\frac{\lambda_{1,t}}{\lambda_{2,t}}\end{bmatrix}Z^l_{m,t}dN^l_t \tag{10}$$

with

$$k_t\equiv\frac{1}{\sigma_\Theta\lambda^2_{2,t}}(\lambda'_{1,t}\lambda_{2,t}-\lambda_{1,t}(\lambda'_{2,t}-a_\Theta\lambda_{2,t})), \tag{11}$$

where the processes $(\widehat{B}^c_t)_{t\geq 0}$ and $(\widehat{B}^l_t)_{t\geq 0}$ are one-dimensional Brownian motions with respect to $\mathcal{F}^c_t$ and $\mathcal{F}^l_t$, and $Z^l_{m,t}\equiv\frac{\bar{S}^l_{m,t}-\widehat{\Pi}^l_{t_-}}{\sigma^2_S/m_t}o^l_t\left(n^l_t\right)$ denotes the jump occurring in conditional expectations as information $\bar{S}^l_m$ collected from new conversations is processed. The variances, o^c_t and o^l_t, of the respective filters satisfy

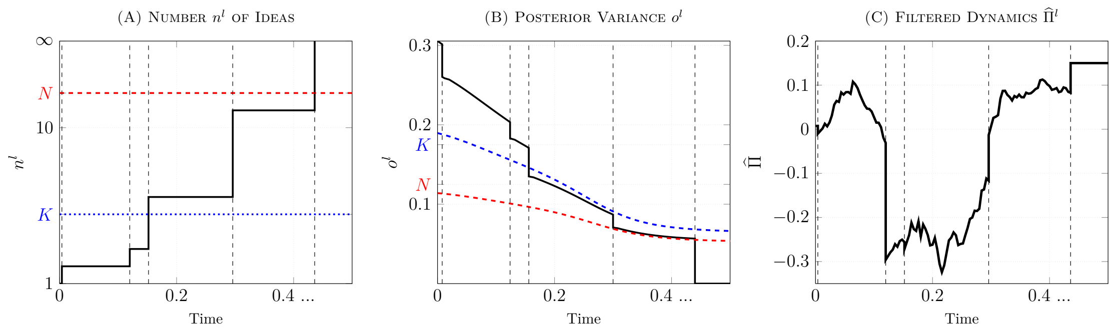
$$do^c_t=-k^2_t(o^c_t)^2dt,\quad and\quad \frac{1}{o^l_t(n^l_t)}=\frac{1}{o^c_t}+\frac{n^l_t}{\sigma^2_S}. \tag{12}$$

Proof. See Appendix B. Q.E.D.

The way managers update their expectations in (10) reflects the novel pattern of information arrival associated with my model. In particular, managers' expectations involve a continuous part pertaining to the observation of prices and a discontinuous part related to meetings. By contrast, common views in (9) are purely continuous. In Figure 4 below, I illustrate this unique feature of my model by simulating the path of a manager's number n^l of ideas (in Panel A), her variance o^l (in Panel B), and her expectations $\widehat{\Pi}^l$ (in Panel C).

Consider first the periods during which manager l does not meet anyone (the flat parts in Panel A). In such periods, manager l is socially inactive and therefore exclusively extracts new information from prices. Since this kind of information flows tick-by-tick, her views $\widehat{\Pi}^l$ are smooth, as seen in Panel C. Consider now the times at which manager l meets someone. These episodes are identified by sudden increases of her number of ideas (see Panel A). Each increase in n^l corresponds to new ideas collected from a conversation. This additional information comes in large pieces, which manager l processes in a discontinuous way. Therefore, her views jump (see Panel C). These jumps

Figure 4: A Simulated Path



Simulated Path of the Filter. Figure 4 plots, for an arbitrary agent l, a simulated path of her number n^l of ideas in Panel A, her individual posterior variance o^l (——) in Panel B, and her expectations $\widehat{\Pi}^l$ (——) in Panel C. The social intensity is set to one meeting every two months on average, $\eta = 6$.

are the result of conversations that suddenly augment her set $\bar{S}^l_n = \{S^l_k\}^n_{k=1}$ of ideas.

A manager l not only updates her views, she also becomes more precise. When she is socially inactive, her variance o^l decays deterministically according to (12). By contrast, every random meeting (in Panel A) causes a downward jump in her variance o^l (in Panel B), reflecting a sudden increase in her precision. Meetings substantially reduce her variance and, as she makes her way to Network B (when n^l crosses K in Panel A), her views (Panel C) become significantly less volatile. After two meetings within Network B, she eventually gains perfect knowledge of Π. As a result, o^l drops to zero (Panel B) and her expectations hit Π (Panel C).

3.2 Optimal Portfolio Strategy

Having now derived managers' learning process, I can determine how they trade on the basis of their ideas. I first provide an approximation that allows me to write managers' optimal portfolios as a linear function of the supply and their informational advantage. I can then show that managers have two trading motives: managers simultaneously make the market for noise traders and speculate on their informational advantage. Finally, I show that, as social interactions intensify, managers speculate more aggressively.

3.2.1 Approximation

To keep the tractability associated with the linear equilibrium of Definition 1, I need to approximate the optimal portfolio strategy of managers l.[23] The reason is that

[23] An approximation is needed due to continuous trading. It is one of the kind implemented in Duffie, Gârleanu, and Pedersen (2007). See also Appendix *E.2* in Vayanos and Weill (2008).

managers l cannot hedge the risk related to their random meetings. In particular, they cannot prevent their marginal utility from jumping when they meet someone. Hence, managers' value function jumps and the usual affine-quadratic setup does not apply to my model, precluding a tractable solution to managers' optimization problem.

Yet, it only takes a simple log-linearization of the jump size in managers' value function to recover the tractability associated with traditional models. Following Haugh, Kogan, and Wang (2006), I verify that this approximation is accurate.[24] Proposition 4 below contains the resulting portfolio strategies.

Proposition 4. *A fund manager l holding $n^l_{t_-}$ ideas at time t builds an approximately optimal portfolio strategy θ^l given by*

$$\theta^l_t \equiv \theta^l(\widehat{\Theta}^l_{t_-}, \widehat{\Pi}^l_{t_-} - \widehat{\Pi}^c_t, n^l_{t_-}, t) = d^l_{\Theta,t}(n^l_{t_-})\widehat{\Theta}^l_t + d^l_{\Delta,t}(n^l_{t_-})(\widehat{\Pi}^l_{t_-} - \widehat{\Pi}^c_t) \tag{13}$$

where $d^l_{\Delta,t}(n^l_{t_-})$ and $d^l_{\Theta,t}(n^l_{t_-})$ are deterministic, network-specific coefficients that depend on the number n^l_t of ideas held by agent l. Informed fund managers i build an optimal portfolio strategy θ^i given by

$$\theta^i_t \equiv \theta^i(\Theta_t, \Pi - \widehat{\Pi}^c_t, t) = d^i_{\Theta,t}\Theta_t + d^i_{\Delta,t}(\Pi - \widehat{\Pi}^c_t) \tag{14}$$

where $d^i_{\Delta,t}$ and $d^i_{\Theta,t}$ are deterministic coefficients.

Proof. See Appendix C. Q.E.D.

3.2.2 Market Making and Speculation

The portfolio of a manager l in (13) reflects a balance between her *market-making* activity, d^l_Θ, and her *speculative* activity, d^l_Δ, based on her informational advantage $\widehat{\Pi}^l - \widehat{\Pi}^c$. I make these two trading motives further apparent by rewriting (13) as:[25]

$$\theta^l_t \equiv \underbrace{d^l_{\Theta,t}\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}^l_{t_-})\right)}_{\text{Market-Making Position}} + \underbrace{d^l_{\Delta,t}\left(1 - \frac{o^l_t}{o^c_t}\right)(\bar{S}^l_{n,t} - \widehat{\Pi}^c_t)}_{\text{Speculative Position}}. \tag{15}$$

The expression for market-making in (15) carries a central insight—market making is obscured by the inference problem managers l face. To see this, notice that the expression in the first bracket represents the supply as perceived by managers l. Specifically,

[24] Appendix D contains a formal derivation as well as an upper bound on the approximation error. Simulations show that the upper and lower bounds on the initial value function are of the magnitude -0.43 and -0.41 for the range of parameters considered. This implies a relative error of less than 5%.

[25] See Appendix F.2 for a derivation.

managers l attempt to distinguish trades made by noise traders Θ from those made by better-informed managers (reflected in $\frac{\lambda_1}{\lambda_2}(\Pi - \widehat{\Pi}^l)$). The reason is that, to their mind, a change in prices has two possible interpretations. One possibility is that noise traders' demand varied; then, managers l provide the required liquidity by adjusting their market-making positions while keeping their speculative positions unchanged. Alternatively, a price change reflects fundamental information. If so, managers l first readjust their speculative positions, since they probably misevaluated Π. Second, they readjust their market-making positions (through $\frac{\lambda_1}{\lambda_2}(\Pi - \widehat{\Pi}^l)$) so as to minimize the chance that they end up on the wrong side of the trade. This intuition does not apply to informed managers who can perfectly interpret price changes. Their portfolio in (14) shows that they accommodate the supply and do not hedge against others' trades.

The second expression in (15) shows that managers l speculate on their set of ideas $\bar{S}^l_{n,t}$. In particular, they speculate based on their relative precision with respect to common uncertainty o^c. If a manager has few ideas, the ratio $\frac{o^l}{o^c}$ is close to 1 and she gives little weight to her ideas (and vice-versa). For informed managers, this ratio reduces to 0 and they fully exploit their informational advantage, $\Pi - \widehat{\Pi}^c$.

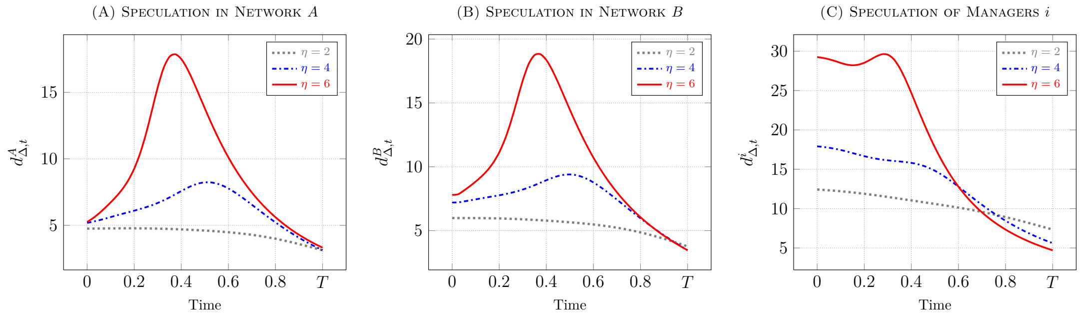
3.2.3 Aggressive Speculation

Social interactions, by increasing the average precision in the market, cause managers to speculate more aggressively. To show this, I plot the intensity $d^j_\Delta \equiv \sum_n \frac{\mu^j(n) d^l_\Delta(n)}{q^j}$ of speculative trading for the *average* manager in each group $j = A, B$. Figure 5 below depicts speculative intensities, as functions of time and the intensity of meetings η. To plot this figure, I anticipate the equilibrium behavior of prices, which will only be available when I impose market clearing (Subsection 3.3).

When social interactions are moderate (the dotted line in Panels A, B and C), speculative activity monotonically decreases until the announcement date. This pattern is typically associated with residual uncertainty (e.g., He and Wang (1995)). Managers are unwilling to speculate on residual uncertainty since they cannot learn about it.

Yet, as social interactions intensify, the speculative activity of managers l initially increases and keeps increasing over a long time period before it eventually decreases (see Panels A and B). This steep increase reflects managers' aggressive speculation. Speculation becomes more aggressive for two reasons. First, intense social interactions cause managers l to collect a larger number of ideas and, therefore, to become more precise. Second, intense social interactions cause managers l to rush to speculate before informed managers arrive. Instead, informed managers intensely speculate immediately after the initial date, taking their profits when information asymmetry is the strongest. They sharply reduce their speculative activity as the average precision rises (Panel C).

FIGURE 5: INTENSITY OF SPECULATIVE TRADING



Speculative Intensities and Social Interactions. Figure 5 plots speculative intensities d_Δ for the average manager in Network A and Network B and for manager i, in Panels A, B and C, respectively. Each is plotted as a function of time, for a social interaction of one meeting twice per year (⋯⋯), one meeting every quarter (-·-·-) and one meeting every two months (——). The calibration is reported in Table 1.

3.3 Equilibrium

The price function in Definition 1 is a conjecture that I now need to validate by imposing market clearing. This last step allows me to solve the fixed-point problem associated with my model and obtain the equilibrium dynamics for the coefficients λ_1 and λ_2. I do so in Proposition 5 below. I then show that *social interactions increase price informativeness*. Finally, I discuss the numerical implementation of the equilibrium.

Proposition 5. *Given the optimal portfolio strategies in* (13) *and* (14)*, the stock price has the linear form in* (6) *where the equilibrium price coefficients* λ_1 *and* λ_2 *satisfy the differential equations in* (72) *and* (73) *with boundary conditions in* (74) *and* (75).

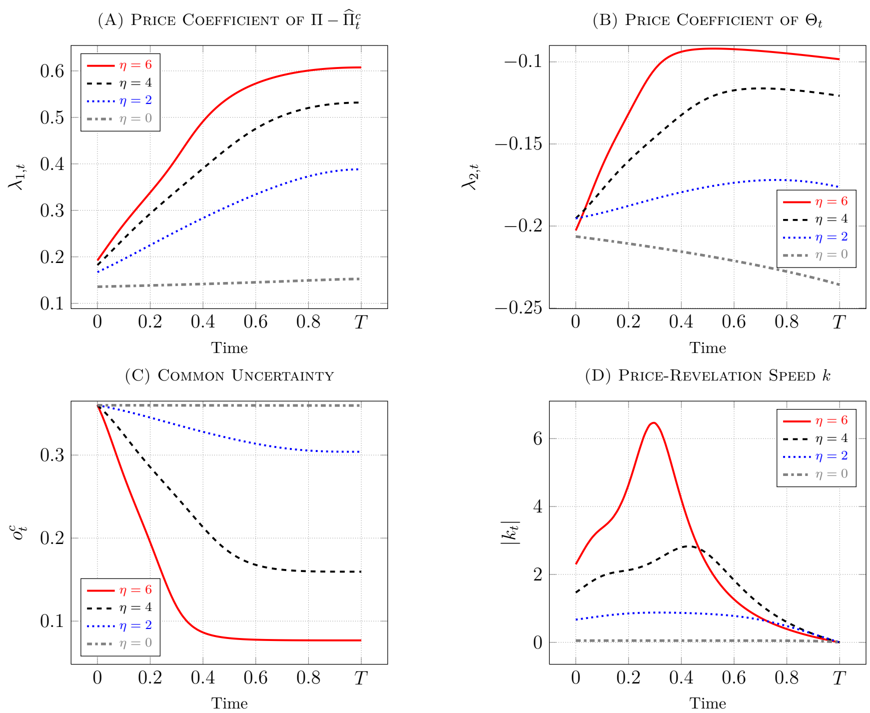
Proof. See Appendix E. Q.E.D.

3.3.1 Price Informativeness

Social interactions cause prices to become more informative. There are two ways to see this. First, increased price informativeness is directly implied by the behavior of the price coefficients λ_1 and λ_2 (plotted in Panels A and B of Figure 6, as functions of time and social interactions η). The weight λ_1 tilts average market expectations either towards common expectations $\widehat{\Pi}^c$ or towards the actual Π. In an economy without social interactions ($\eta = 0$), Panel A shows that average market expectations are strongly biased towards common expectations, i.e., prices reveal little information about Π. As social interactions intensify ($\eta > 0$), prices convey more information so that market learning significantly improves, and average market expectations shift closer to Π.

The compensation for risk, λ_2, plotted in Panel B, conveys a similar intuition. The evolution of λ_2 is the result of two opposite forces. On the one hand, the more fiercely managers speculate, the larger is the risk to be taken advantage of by better-informed managers: the compensation for risk $|\lambda_2|$ rises. On the other hand, the more fiercely managers speculate, the more information is revealed through prices: the compensation for risk falls. Social interactions cause the latter effect to initially dominate (see Panel B). Yet, the risk associated with residual uncertainty gives rise to an increase in $|\lambda_2|$ around T.

Figure 6: Price Coefficients and Price Informativeness



Price Coefficients, Price Informativeness and Common Uncertainty. Figure 6 plots the evolution of the price coefficients λ_1 and λ_2 in Panels A and B, respectively, common uncertainty o^c in Panel C, and the speed k of information propagation through prices in Panel D. Each is plotted for the cases of no social interaction (- - - -), a social interaction of two meetings per year (⋯⋯), one meeting per quarter (- - -) and one meeting every two months (——). The calibration is reported in Table 1.

Another way to apprehend increased informativeness is through the evolution of common uncertainty o^c. Common uncertainty measures the remaining uncertainty regarding Π given the history of prices. The evolution of common uncertainty in (12) is driven by a key variable, k. One can view k in (11) as controlling the speed at which information is revealed through prices. To see this, notice that (11) may be rewritten as

$$k_t = \frac{1}{\sigma_\Theta}\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right) + a_\Theta \frac{\lambda_{1,t}}{\lambda_{2,t}}\right).$$

The ratio $\frac{\lambda_1}{\lambda_2}$ captures the price's sensitivity to information and therefore its informativeness. As a result, $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\lambda_1}{\lambda_2}$ measures the speed at which price informativeness increases over time. Since, in my model, increased price informativeness is the result of increased speculative aggressiveness, the evolution of $|k|$ (Panel D) follows the same pattern as that of speculative intensities (Figure 5). Specifically, Panel D shows that social interactions vastly increase price informativeness, particularly around the initial date, precisely when managers speculate the most aggressively. By increasing price informativeness at great speed, social interactions cause market learning to rapidly improve and common uncertainty (Panel C) to be reduced substantially.

3.3.2 Equilibrium Implementation and Calibration

Propositions 3, 4 and 5 jointly define a system of equations whose boundary conditions depend on the terminal level $o^c_{T_-}$ of common uncertainty. Since $o^c_{T_-}$ is itself an endogenous coefficient, I need to guess a value for $o^c_{T_-}$ and then shoot towards the initial condition $o^c_0 = \sigma^2_\Pi$.[26] Depending on the meeting intensity η and the magnitude of residual uncertainty σ_δ, up to three equilibria are obtained for a certain range of the parameter space.[27] One of these equilibria may be immediately discarded, though. It is an equilibrium in which the volatility of prices is constantly null and the price thus fully reveals Π, i.e., managers do not trade.[28]

The other equilibria originate from the interaction between common uncertainty o^c and price informativeness k in (11). By increasing $|k|$, social interactions cause both o^c (in Panel C of Figure 6) to decrease substantially through (12) and speculation to become more aggressive. Increased speculative aggressiveness further increases $|k|$ and this feedback eventually gives rise to two equilibria. In one equilibrium, prices are highly informative and speculation is aggressive, while in the other equilibrium, the

[26]Since both boundary conditions in (74) and (75) depend on $o^c_{T_-}$ which is unknown, a shooting procedure is needed. The system of equations may be solved using a standard finite-difference scheme such as Runge-Kutta. For instance, the NDSolve function of Mathematica proves to work well with the StiffnessSwitching option. To alleviate the numerical procedure, it is convenient to make the system of equations in (47), (72) and (73) explicit. Appendix E shows how to proceed.

[27]Residual uncertainty is often a cause of equilibrium multiplicity: Grundy and McNichols (1989) show that multiplicity obtains in their setting if residual uncertainty lies within a certain range. In particular, Hirshleifer, Subrahmanyam, and Titman (1994) obtain four equilibria. He and Wang (1995) further argue that multiplicity may not arise if a_Θ is sufficiently large.

[28]I show this at the end of Appendix E.

opposite happens.[29] Following this intuition and Grundy and McNichols (1989), these equilibria may be ranked according to $\left|\frac{\lambda_1}{\lambda_2}\right|$.

I base the equilibrium selection on a well-established fact: the empirical literature widely documents the superior performance of *aggressive growth funds*. For instance, Barras, Scaillet, and Wermers (2010) find that aggressive growth funds have the highest proportion of skilled managers.[30] Hence, I pick the equilibrium in which speculation is the most aggressive, i.e., that in which the average speculative activity d_Δ is the highest.[31] For this equilibrium to obtain and managers to trade aggressively, residual uncertainty needs to be sufficiently low. For this reason, I set σ_δ below that used in He and Wang (1995) (see Table 1).

Description	Symbol	Value
Time Horizon	T	1
Volatility of Residual Uncertainty	σ_δ	$\sqrt{1/6}$
Initial Proportion of Informed Managers	$1-\omega_0$	0
Risk Aversion	γ	0.5
Supply Mean-Reversion	a_Θ	0.2
Supply Volatility	σ_Θ	2.5
Volatility of Dividend Π	σ_Π	0.6
Volatility of the Signal	σ_S	$\sqrt{2}$
Perfect Knowledge Threshold	N	12
Good Ideas Stay Local Threshold	K	5

TABLE 1: **Benchmark Calibration**
This calibration is adapted from Campbell and Kyle (1993), Wang (1993) and He and Wang (1995) and is chosen to be consistent with empirical estimates (see Appendix G).

I consider an horizon date of one year, a relatively short-term horizon. I let the initial fraction ω_0 of fund managers l be 1 so that every manager is initially located in Network A. Hence, managers start with one idea and *become heterogeneously informed as soon as they start to interact.*

Managers' absolute risk aversion is set below 1 to reflect the fairly high risk tolerance of institutional investors. The remainder of the calibration is adapted from Campbell and Kyle (1993), Wang (1993) and He and Wang (1995) and is chosen to be consistent with empirical estimates (see Appendix G).

[29] This may be illustrated by means of a phase diagram.

[30] Daniel, Grinblatt, Titman, and Wermers (1997) find that aggressive growth funds feature significant stock-picking skills. Hendricks, Patel, and Zeckhauser (1993) report that the performance of growth-oriented funds persists over a one-year evaluation horizon. Kosowski, Timmermann, Wermers, and White (2006) conclude that outperformance by aggressive growth funds is not an artifact of luck.

[31] See Banerjee (2010) who describes multiple equilibria often referring to *high and low volatility* equilibria. The author argues that these equilibria have both desirable theoretical and empirical properties. See also Spiegel (1998) and Watanabe (2008).

4 Market Timing and Performance

Can mutual fund managers time the market ? This question has been under intense empirical investigation at least since Treynor and Mazuy (1966) and Henriksson and Merton (1981). These early studies develop linear regression models of the CAPM-type including an additional convex term reflecting timing skills. Doing so, they conclude in favor of the Efficient Market Hypothesis (see also Henriksson (1984)). These studies have largely contributed to the view that managers do not exhibit timing skills. Subsequent works have attempted to refine these early models by addressing certain practical issues, causing timing abilities to look somewhat better.[32]

The empirical literature has been equally inconclusive as to the performance of managers. A host of empirical studies, starting with Jensen (1968) and later followed by Carhart (1997), find that most mutual funds fail to outperform, and often underperform, passive benchmarks, even before transactions costs.[33] In general, the empirical consensus is that performance does not persist, except for a small group of funds (e.g., aggressive-growth funds) whose performance is, however, relatively short-lived.[34]

This section shows that my model can rationalize why timing skills seem to be weak and performance not to be persistent. Social interactions cause most managers to eventually implement passive investment strategies. As a result, the traditional measure of performance—the alpha generated over a passive benchmark—rapidly becomes negative, even for managers with good ideas.

4.1 Market-Timing

This subsection demonstrates that social interactions weaken managers' timing ability. In particular, social interactions increase price informativeness and thereby cause timing to be short-lived. If a manager ever had good ideas, sooner or later the market catches

[32] Time-varying betas have been introduced (Ferson and Schadt (1996)), the dichotomy between the rebalancing and observation frequency of holdings has been accounted for (Goetzmann, Ingersoll, and Ivkovic (2000)), non-parametric approaches have been proposed (Kosowski, Timmermann, Wermers, and White (2006)), while another strand of the literature has focussed on portfolio holdings (Daniel, Grinblatt, Titman, and Wermers (1997)). See also Ferson and Khang (2002), Barras, Scaillet, and Wermers (2010), Wermers (2000) and Kacperczyk, Sialm, and Zheng (2005).

[33] See also Malkiel (1995) and Gruber (1996). In contrast, alternative investigations (e.g., Grinblatt and Titman (1992) and Hendricks, Patel, and Zeckhauser (1993)) conclude that outperformance may persist but do not account for trading costs.

[34] Timing ability and performance are hardly identifiable at a monthly frequency, except perhaps for a small subset of top performing funds. Recognizing that the observation frequency of returns affects inferences regarding timing and performance, Bollen and Busse (2005) show that performance becomes strikingly more apparent at a higher frequency. Chance and Hemler (2001) and Bollen (2001) further demonstrate that timing ability reveals itself in daily tests but vanishes monthly. Other empirical evidence includes Hendricks, Patel, and Zeckhauser (1993) who describe the "hot-hand" phenomenon in the mutual fund industry and Graham and Harvey (1996) who consider newsletters data.

up with her. Timing ability only persists for managers with highly accurate information.

To gauge managers' timing ability, empiricists run performance regressions on total returns. These regressions typically augment the CAPM with additional terms that purport to pin down the ability of managers. Admati and Ross (1985), among others, study the relevant performance-regression structure in a theoretical context and list various pitfalls related to such regressions.[35] Adopting their approach, my goal here is to determine which additional factor should be added to the CAPM to identify timing ability in my model.

To make the relevant regression structure apparent, it is convenient to focus on the *myopic* part of managers' portfolio in (13), which I denote by $\widehat{\theta}^l_t \equiv \frac{E[\mathrm{d}P_t|\mathcal{F}^l_t]}{\gamma \mathrm{var}[\mathrm{d}P_t|\mathcal{F}^l_t]}$. Adding the hedging part of individual demands does not change the structure, but makes computations less transparent. The return $\widehat{R}^l$ on manager l's myopic portfolio writes

$$\mathrm{d}\widehat{R}^l_t := \widehat{\theta}^l_t \mathrm{d}P_t = \underbrace{\beta_t \mathrm{d}P_t}_{\text{Market Returns}} + \underbrace{\frac{k_t}{\gamma\sqrt{\mathrm{var}[\mathrm{d}P_t]}}\left(1 - \frac{o^l_t}{o^c_t}\right)\frac{1}{\lambda_{1,t}}(P_t - \widehat{\Pi}^c_t - \lambda_{2,t}\Theta_t)\mathrm{d}P_t}_{\text{Timing Payoff}} \quad (16)$$
$$+ \underbrace{\frac{k_t}{\gamma\sqrt{\mathrm{var}[\mathrm{d}P_t]}}\left(1 - \frac{o^l_t}{o^c_t}\right)\epsilon^l_t \mathrm{d}P_t}_{\text{Noise Relative to Market Information}}$$

where $\beta_t \equiv \frac{A_{Q,t}\Psi^c_t}{\gamma \mathrm{var}[\mathrm{d}P_t]}$ represents the market price of risk under common perceptions.

A fund's returns have three components: market returns, a timing factor measuring the additional performance a manager contributes based on her ideas, and a third term that the econometrician treats as noise. Equation (16) contains two key elements. First, the timing factor is proportional to $1 - \frac{o^l}{o^c}$, a term that relates to the market learning process. Specifically, as social interactions intensify, market learning significantly improves and common uncertainty o^c is greatly reduced. In turn, $\frac{o^l}{o^c}$ gets closer to 1, which progressively erodes a manager's ability to time the market. Second, the timing factor is a function of $P\mathrm{d}P \equiv \frac{\mathrm{d}P^2 - \mathrm{d}\langle P\rangle}{2}$. The quadratic term $\mathrm{d}\langle P\rangle$ in the timing factor originates from managers' informational advantage. In particular, the excess returns generated by managers i over market returns $(\theta^i - \theta^c)\mathrm{d}P$ depend on the square of their informational advantage, $\Pi - \widehat{\Pi}^c$. The quadratic-returns component $\mathrm{d}\langle P\rangle$ precisely captures this informational advantage as the stock price is itself a function of $\Pi - \widehat{\Pi}^c$. Since timing is quadratic in stock returns, a regression similar to that of Treynor and Mazuy (1966) endogenously arises as a criterion to evaluate timing.

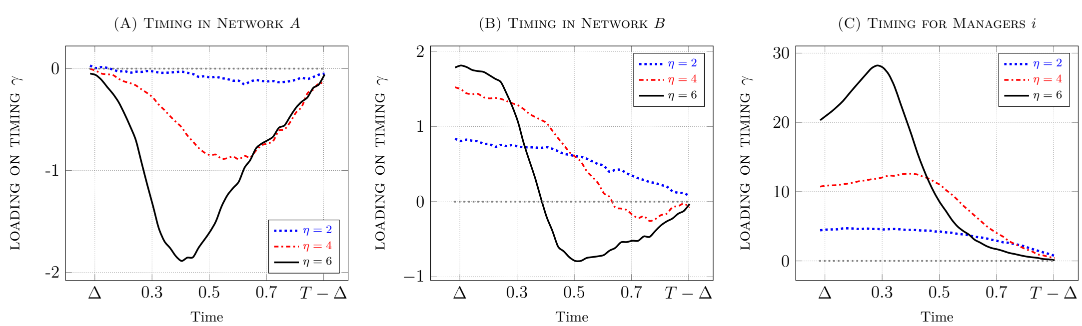
[35] See also Dybvig and Ross (1985) and Admati, Bhattacharya, Pfleiderer, and Ross (1986). Recently, Detemple and Rindisbacher (2012) show that a structural approach accounting for the nature of private information yields additional robustness.

Noise traders act randomly. Therefore, they are "sitting ducks" for managers and one should not give credit to a manager for making money on noise traders. For this reason, I do not run the Treynor and Mazuy (1966) regression over manager's total returns. Instead, I isolate returns $\widetilde{R}$ managers solely generate based on fundamental information. From now on, I consider a manager's *informational* holdings $\widetilde{\theta} \equiv \theta - d_\Theta \Theta$. Informational holdings contain managers' hedging demand (not only the myopic part), but do not include the fraction they contribute to the supply. Informational holdings are therefore exclusively related to managers' information. Finally, I run the Treynor and Mazuy (1966) regression over a rolling window, thus causing its coefficients to vary over time as in Ferson and Schadt (1996):

$$\widetilde{R}_t^l = \alpha_t^l + \beta_t^l \Delta P_t + \gamma_t^l (\Delta P_t)^2 + \epsilon_t^l, \quad \Delta \leq t \leq T - \Delta. \tag{17}$$

To evaluate managers' timing ability, I run the regression in (17) over managers' simulated returns.[36] To focus the analysis on a restricted number of managers, I plot the results for the *average* manager in each group (i.e., the average timing factor $\sum_n \frac{\mu^j(n)}{q^j} \gamma^j$, $j = A, B$) in Figure 7 below. The market coefficient β and the intercept α of (17) are statistically insignificant and are therefore omitted.

Figure 7: Timing Coefficient



Treynor and Mazuy (1966) Regression and Timing. Figure 7 plots the loading on market timing of the regression in (17) over a one-month Δ rolling window for an interaction intensity of one meeting twice per year (⋯⋯), one meeting every quarter (-·-·-) and one meeting every two months (——). Panels A, B and C respectively correspond to the average manager in Network A, in Network B, and managers i. The calibration is reported in Table 1.

When the intensity of interpersonal communications is low (the dotted lines), timing abilities across networks are as expected. Managers in Network A (Panel A) are unable

[36] I simulate portfolios for each agents type using the formulae of Appendix F.2. Then, I compute the cross-sectional average using the population dynamics.

to time the market, while managers in Network B and informed managers come out as successful market timers (Panels B and C). Importantly, for the latter managers, timing ability is long-lived.

As social interactions intensify, this result is partly reversed. While managers in Network A and informed managers respectively worsen and improve their timing ability, managers in Network B become progressively unable to time the market—timing vanishes (see Panel B). For an intensity of one meeting every two months (the solid lines), it becomes rapidly impossible to tell apart managers from Networks A and B. This result is surprising since social dynamics cluster good ideas in Network B and thereby allow managers to preserve their informational advantage over a longer time period. But the market learns at great speed, which progressively impairs their timing ability through the term $1 - \frac{o^l}{o^c}$ appearing in (16). That is, by improving the market learning process, social interactions prevent managers of Network B from keeping their informational advantage, despite good ideas staying localized. Only informed managers whose views are unaffected by the market remain successful market timers.

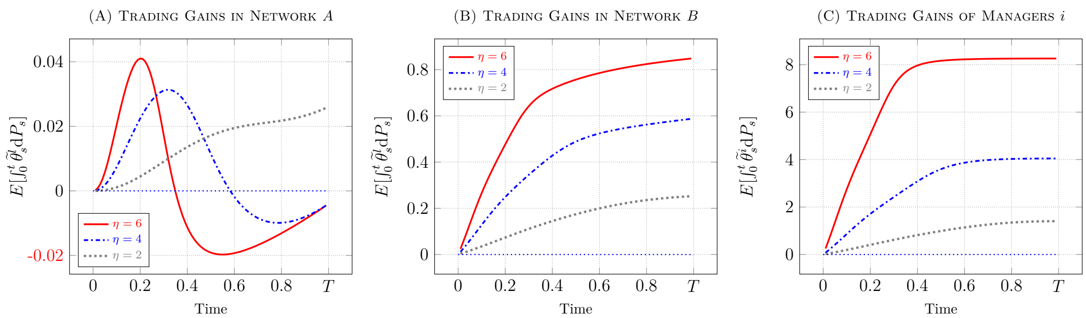
4.2 Performance

Social interactions cause market timing to be short-lived for most managers. I shall now show that social interactions carry similar implications regarding performance. Specifically, social interactions operate on price informativeness to produce non-persistent performance. In addition, only a small group of funds appear to be top performers, consistent with empirical findings. The underlying mechanism—word-of-mouth communication—contrasts with alternative explanations, which are either based on the structure of the mutual fund industry (Berk and Green (2004)), on career concerns (Chevalier and Ellison (1999b)) or possibly on market volatility. In particular, unlike Berk and Green (2004) whose mechanism relies on the competitive provision of funds by investors, the mechanism I highlight ignores flows into and out of the fund.

Importantly, even though managers with good ideas accumulate large trading gains, their performance, as measured by their alpha, does not persist. The reason is that social interactions cause most managers—even those with good ideas—to implement passive investment strategies for which they should not be rewarded. As a result, not only does performance become non-persistent, but it also becomes literally impossible to distinguish managers with good ideas from those with inferior ideas. Only exceptionally good ideas allow a manager to consistently generate abnormal returns.

It is insightful to first consider the *dollar value* managers create. In my model, funds neither pay out their income nor their capital gains and, therefore, the dollar value managers generate coincides with the fund's net asset value (NAV). To compute funds' expected NAV, $E[\int_0^t \widetilde{\theta}_s \mathrm{d}P_s]$, I assume that each fund starts with an initial wealth equal to zero and then evaluate how much (expected) value is added by the average fund in each group.[37] I plot the expected fund's NAV in Figure 8 below as a function of time and the intensity of meetings η.

Figure 8: Net Asset Value (NAV)



NAV and Social Interactions. Figure 8 plots a fund's NAV as a function of time for the average manager in Network A (Panel A), in Network B (Panel B) and manager i (Panel C). Each is plotted for an interaction of one meeting twice per year (⋯⋯), one meeting every quarter (-·-·-) and one meeting every two months (——). Table 1 reports the calibration.

Based on funds' NAV, managers' performance is as expected. Panel A shows that managers in Network A make some modest profits (on momentum; see below), but end up destroying value. Managers in Network B make significantly more money (see Panel B), while informed managers sharply come out as top performers (see Panel C). Moreover, a fund's NAV has three important properties. First, the average dollar value generated by informed managers and managers in Network B is increasing in the intensity of social interactions. The reason is that managers speculate more aggressively as social interactions intensify. This aggressive speculation creates *large spreads in dollar value* across networks. Second, *the ranking of managers is robust to the intensity of social interactions*. Finally, for managers with superior information, *performance measured in trading gains is persistent* (Panels B and C).

Although trading gains appear to provide an intuitive measure of performance, such a measure is misleading because it does not penalize managers for implementing passive strategies. I shall now show that, controlling for passive investment strategies, social interactions cause performance not to persist and to rapidly converge across networks.

To evaluate performance, empiricists usually choose a passive benchmark, over which abnormal returns added by a manager are computed. The benchmark collects returns

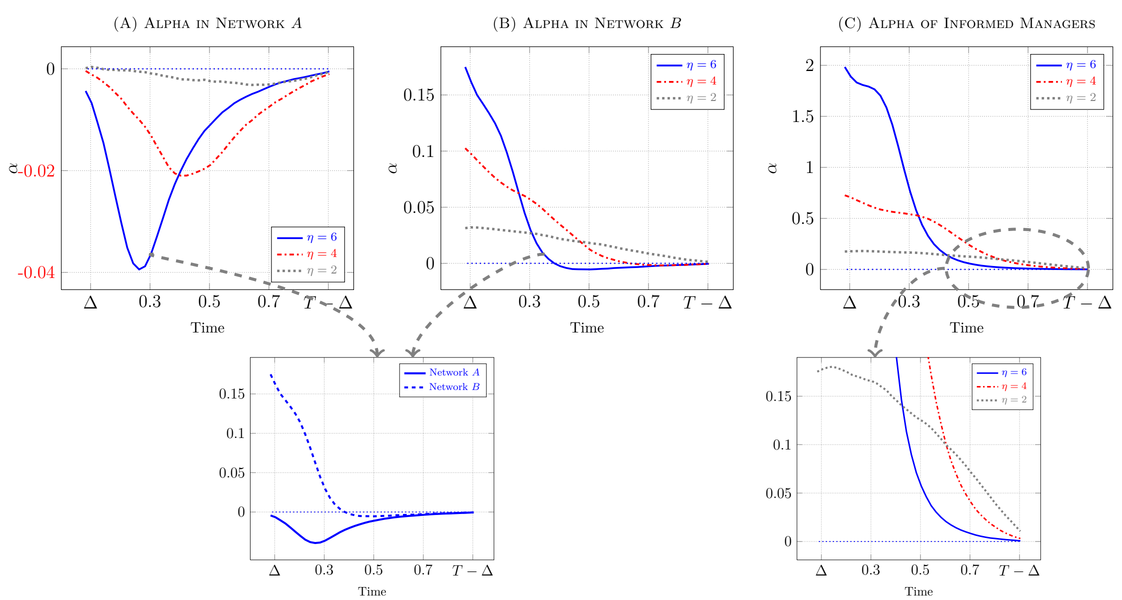
[37]In practice, one may recoup a fund's dollar value by adding back past dividends to the fund's NAV. A similar exercise is conducted in Bollen and Busse (2005), for instance.

on passive strategies for which empiricists deem managers should not be rewarded. In my model, two passive strategies are relevant to consider. First, as (16) indicates, one should not reward a manager for buying the market portfolio, since this strategy only requires the observation of the tape. Second, one should not reward a manager for pursuing a momentum strategy. Here, I anticipate the results of Sections 5 and 6, namely that social interactions produce momentum in stock returns and cause lesser-informed managers to implement momentum strategies; accordingly, I consider a benchmark regression that includes market returns and the (times-series) momentum factor of Moskowitz, Ooi, and Pedersen (2011):

$$\widetilde{R}_t^l = \alpha_t^l + \beta_t^l \Delta P_t + \delta_t^l \mathrm{sign}(\Delta P_{t-\Delta}) \Delta P_t + \epsilon_t^l, \quad \Delta \leq t \leq T - \Delta. \tag{18}$$

The regression in (18) is essentially the Carhart (1997) risk-factor model without Fama and French (1995) factors (there is only one stock in my model). Most empirical studies use this benchmark. I plot alpha in Figure 9 below, as a function of time and the meeting intensity η.

Figure 9: Alpha



Benchmark Regression, Alpha and Momentum. Figure 9 plots alpha and the momentum loading in the benchmark regression in (18) over a one-month Δ rolling window for an interaction of one meeting twice per year (⋯⋯), one meeting every quarter (-·-·-) and one meeting every two months (——). Each column corresponds to, respectively, the average manager in Network A, in Network B, and manager i. The calibration is reported in Table 1.

As long as social interactions are moderate (the dotted lines), managers can be accurately ranked based on their alpha. Managers in Network A rapidly destroy value, while managers in Network B and informed managers persistently create value. Their performance eventually becomes insignificant due to residual uncertainty.

As social interactions intensify, the picture becomes hazier. First, performance becomes dramatically non-persistent expect for a small fraction of the population, consistent with empirical findings. As seen from Panel B, managers of Network B do not maintain their performance. Their performance ceases to persist *a long time ahead of the announcement date*, which indicates that this phenomenon is exclusively driven by social interactions. Not only does their performance become non-persistent, but their alpha becomes negative. This implies that social interactions cause managers of Network B to eventually implement passive investment strategies (see Section 6). Overall, performance only persists for a small group of managers (informed managers), as apparent from the zoom on Panel C (see the picture located below Panel C).

Second, performance converges across networks so that it becomes difficult to tell managers apart. This is particularly striking when comparing the performance of managers in Networks A and B. For a large meeting intensity (the solid lines), the picture located below Panels A and B shows that it becomes literally impossible to distinguish managers in Network A from those in Network B, despite the large spread in trading gains across the two networks. While managers of Network B consistently accumulate trading gains (Panel B of Figure 8), they progressively implement passive investment strategies for which they should not be rewarded. As a result, managers of Network B eventually align with those of Network A. Only informed managers remain able to generate returns in excess of the passive benchmark.

I conclude this section with a comment on the implications of momentum for performance (momentum itself is the subject of the next section). Some empirical studies argue that managers' performance strongly relates to momentum. For instance, Carhart (1997) finds that the "*hot hands*" phenomenon of Hendricks, Patel, and Zeckhauser (1993) is driven by the one-year momentum effect.[38]

In light of my model, however, this conclusion may have to be reexamined. In the colorful words of Cohen, Coval, and Pástor (2005), "*it seems hard to argue that sitting on one's laurels and doing nothing is a managerial skill that should be given credit*". In particular, for managers of Network A, alpha is always negative, which indicates that their initial trading gains are made on momentum (and penalized by the benchmark); then, Panel A of Figure 8 clearly demonstrates that momentum profits are significantly

[38] Grinblatt, Titman, and Wermers (1995) and Wermers (1999) conclude that funds that invest on momentum are more likely perform. Brown and Goetzmann (1995) report that performance persistence is mostly accounted for by funds that repeatedly lag the S&P500 or a passive benchmark.

lower than the profits made on fundamental information (Panels B and C).

5 Momentum in Stock Returns

In the previous section, I consider momentum as a factor to be added to a benchmark regression, yet not eliciting the reason for its presence; this is the task of this section. Identifying momentum in a theoretical, rational context is fundamental in many respects. First, momentum is one of the most pervasive facts in Financial Economics.[39] Second, well-accepted explanations for momentum are mostly behavioral.[40] In contrast, momentum arises here as a rational phenomenon driven by social interactions, consistent with the empirical finding of Hong, Lim, and Stein (2000). Third, mutual fund managers are likely to play an important role in the momentum mechanism (Wermers (1999)). Moreover, momentum provides managers with an opportunity to make profits. For instance, Avramov and Wermers (2006) find that returns predictability is an essential determinant of managers' performance. Finally, the statistical properties of stock returns are central, as empiricists use returns to compute conditional expectations of managers' performance. Doing so, Ferson and Schadt (1996) show that usual measures of managers' performance are substantially improved.

It usually takes very specific assumptions for returns to feature momentum in a noisy rational-expectations equilibrium (REE): *i)* in the context of Wang (1993), momentum mechanically arises if the supply is extremely persistent. *ii)* In a REE *à la* Wang (1993) with a finite horizon, Holden and Subrahmanyam (2002) show that momentum obtains if the mass of informed agents—managers i in my model—is sufficiently large. *iii)* In a REE *à la* He and Wang (1995), Cespa and Vives (2012) demonstrate that returns exhibit momentum if the average precision across agents shows sufficient improvement over time. While assumptions *ii)* and *iii)* are admittedly somewhat *ad-hoc*, they happen to be endogenous implications of social dynamics in my model. Specifically, social interactions endogenously drive the mass of managers i, $1-\omega_t$, and the average precision $\int_{j_t \in I} (o_t^j)^{-1} \mathrm{d}\iota(j_t)$ over time, thus addressing *ii)* and *iii)*. Social interactions therefore generate momentum in stock returns and reaffirm the role of information as a main driver of serial correlation, as I shall now show.

To understand how social interactions produce momentum, it is first necessary to recall how prices are formed. The discussion of Subsection 3.1 implies that the price

[39] See Poterba and Summers (1988), Jegadeesh and Titman (1993), Rouwenhorst (1998), Menzly and Ozbas (2010) and Moskowitz, Ooi, and Pedersen (2011), among others.

[40] See Daniel, Hirshleifer, and Subrahmanyam (1998) or Barberis, Shleifer, and Vishny (1998).

has the form

$$P_t = \underbrace{\int_{j_t \in I} E[\Pi + \delta | \mathcal{F}_t^j] \mathrm{d}\iota(j_t)}_{\text{Average Market Expectations about the Fundamental}} + \underbrace{\lambda_{2,t}\Theta_t}_{\text{Compensation for Risk}} \tag{19}$$

where the first and the second term represent respectively the contribution of information and risk to prices.[41]

To illustrate how information and risk operate on prices, I consider the following two cases. First, let social interactions be intense, i.e., $\eta \to \infty$; then, every manager instantaneously becomes informed as soon as $t > 0$ and the price in (19) reduces to $P_t = \Pi + \lambda_{2,t}\Theta_t$. That is, the price has no informational content.[42] As a result, price variations are solely driven by supply shocks and the serial correlation over a time period Δ writes $\text{cov}(\Delta P_{t-\Delta}, \Delta P_t) = (\lambda_{2,t+\Delta} - \lambda_{2,t})/(\lambda_{2,t} - \lambda_{2,t-\Delta})$. Hence, unless the compensation for risk λ_2 evolves in a strong non-monotonic fashion, prices are strongly *positively* autocorrelated and the sign of the serial correlation is determined by the compensation for risk in an almost mechanical manner.

Second, let the economy be absent of social interactions, i.e., $\eta \to 0$. This case nests the setting of He and Wang (1995), except for allowing trading to be continuous. Serial correlation is computed over a one-month period Δ, as usually done in empirical studies, and is depicted in Panel A of Figure 10 below.[43] The contribution of risk (the dotted line) is positive, reminiscent of the perfect-information case just discussed, while the contribution of information (the dashed line) is essentially insignificant. In spite of this, returns are weakly *negatively* autocorrelated (the solid line). This negative correlation is due to the interaction between the respective contributions of information and risk to prices (the dashed-dotted line). The reason is that average market beliefs and the compensation for risk are inversely related: if the informational content of prices rises (λ_1 increases), the compensation for risk $|\lambda_2|$ has to decrease due to the weaker information asymmetry. Similar to the perfect-information case, the informational content of prices does not play a *direct* role. Consequently, as often happens in a REE (e.g., Wang (1993)), returns exhibit *reversal*.

Introducing a moderate meeting intensity of one meeting twice per year ($\eta = 2$ in Panel B) leads both risk *and information* to contribute an equal and positive fraction to returns autocorrelation. Yet, the interaction thereof still dominates and even magnifies stock returns reversal. The economic mechanism previously highlighted bites all the

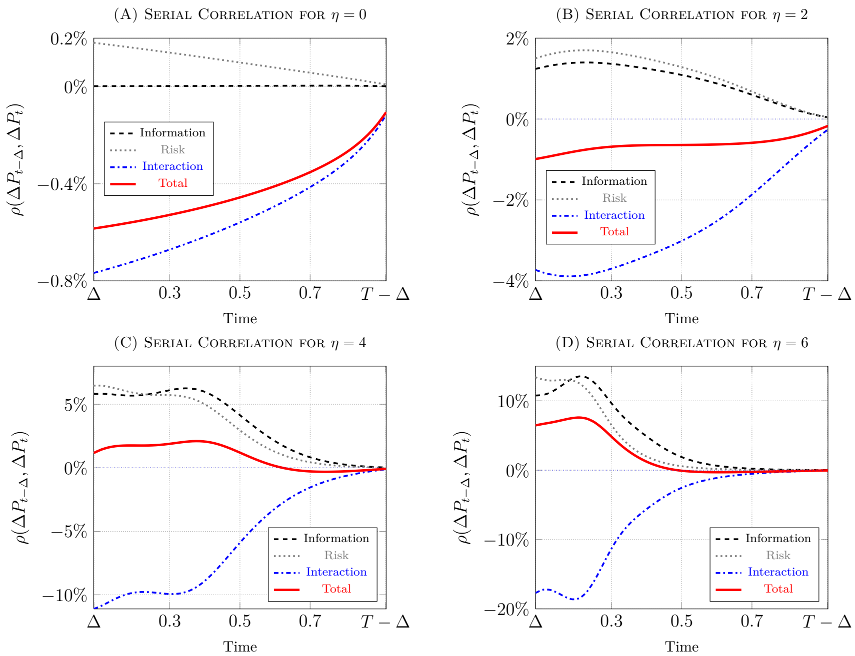
[41] See Appendix E.

[42] In this case, λ_2 solves the following system of coupled differential equations: $\lambda'_{2,t} = \gamma\sigma_\Theta^2\lambda_{2,t}^2 + \sigma_\Theta^2\lambda_{2,t}M_t^i + a_\Theta\lambda_{2,t}$, $\lambda_{2,T} = -\gamma\sigma_\delta^2$ and $(M_t^i)' = \sigma_\Theta^2(M_t^i)^2 - \sigma_\Theta^2\gamma^2\lambda_{2,t}^2 + 2a_\Theta M_t^i$, $M_T^i = \gamma^2\sigma_\delta^2$.

[43] See Appendix F.1.

same. As a positive shock in the supply occurs, prices experience a decrease (through the compensation for risk $\lambda_2 < 0$), which generates low current returns. Since social interactions are moderate and the proportion of well-informed managers is low, managers attribute the shock to noise trading and therefore consider it transitory. They hold on to their speculative positions, thereby inducing stock returns to reverse.

FIGURE 10: SERIAL CORRELATION IN RETURNS AND SOCIAL INTERACTIONS



Serial Correlation in Returns. Figure 10 plots serial correlation (—) in returns over a one-month period Δ for the cases of no social interaction (Panel A), of a meeting twice per year (Panel B), one meeting every quarter (Panel C) and one meeting every two months (Panel D). Each is decomposed into the contribution of average market expectations (- - -), of risk (⋯⋯) and of the interaction between both (-·-·-). Table 1 reports the calibration.

Further increasing the meeting intensity allows one to overturn this mechanism. The individual contribution of information and risk eventually overbalances that of their interaction and a first phase of *momentum* emerges (Panel C). Because social interactions produce an increasing fraction of well-informed agents (point *ii)* above), managers become increasingly concerned that price changes may be information-driven. Moreover, the way in which private information flows—through word-of-mouth communication—induces a steady increase in managers' average precision over time (point *iii)* above) and further causes prices to reveal more information. Now, suppose that prices increase. Since managers l start to attribute price changes to a misevaluation of Π, they tend to buy shares as they probably underestimated Π. By buying additional shares, managers push prices further up, which eventually leads to stock returns momentum. When social interactions are intense (Panel D), momentum can be as high as 8% monthly. Clearly, momentum follows from the diffusion of information through prices.

This initial phase of momentum does not carry all the way to T, though, as apparent from Panels C and D. The negative relation between information and risk is still at play and eventually gives rise to a late phase of reversal, although rather weak. Reversal is due to both the decrease in the informational content of prices (Panel D of Figure 6) and the surge in the compensation for risk resulting from the pending revelation of δ.

Both phenomena are consistent with well-known facts—the first phase of momentum corresponds to the early returns continuation documented by Jegadeesh and Titman (1993), and the second phase of reversal is in line with the late over-reaction evidence of De Bondt and Thaler (1985).

While the mechanism highlighted here builds on Andrei and Cujean (2011), the idea that social interactions drive momentum is already present in Hong and Stein (1999), yet it relies on a behavioral model. Furthermore, rational explanations for momentum are often based on risk, as in Berk, Green, and Naik (1999) or in Johnson (2002).[44] Importantly, I show that momentum is not the sole result of risk, but that it centrally depends on the diffusion of information through private conversations.

6 Momentum and Contrarian Strategies

Mounting evidence suggests that social interactions influence managers' portfolio decisions. In a survey conducted by Shiller and Pound (1989), 44% of institutional investors recognize that interpersonal communications impact their portfolio choice. In particular, fund managers' holdings are strongly related to those of managers operating in the same city (Hong, Kubik, and Stein (2005)).[45] Moreover, managers bias their portfolio towards firms to which they are connected through an educational network (Cohen, Frazzini, and Malloy (2008)). Similar evidence carries over to individual investors.[46]

In my model, interpersonal communications cause lesser-informed managers to follow the trend and better-informed managers to be contrarians. Yet, perhaps surprisingly,

[44] See Vayanos and Woolley (2010) for an alternative mechanism.

[45] In that respect, large cities appear to be particularly suited places for word-of-mouth communication, for it is where leading mutual fund families concentrate. Large cities are further reported by Christoffersen and Sarkissian (2009) to convey knowledge spillovers and to guide learning.

[46] Hong, Kubik, and Stein (2004) documents that investors find the market more attractive when more of their peers participate. Brown, Ivkovic, Smith, and Weisbenner (2008) uncover a strong causal relation between an individual's decision whether to own stocks and the average ownership of her community. See also Shive (2010). Massa and Simonov (2011) show that college-based interactions influence portfolio decisions. Other evidence includes Grinblatt and Keloharju (2001) or Ivkovic and Weisbenner (2005).

even good ideas may not keep managers from eventually aligning with the market consensus. I show that these trading patterns relate to the way in which managers bet on price convergence towards its fundamental value.

6.1 Aligning with the Market Consensus

Informational holdings—portfolios ignoring managers' market-making activity—are not observed by the econometrician. Therefore, they need to be conditioned on publicly available information, namely prices. I follow Brennan and Cao (1996) and regress informational portfolio changes $\Delta\widetilde{\theta}^l$ on price changes ΔP over a time period Δ, i.e., $E[\Delta\widetilde{\theta}^l|\Delta P] = \text{cov}(\Delta\widetilde{\theta}^l, \Delta P)/\text{var}(\Delta P)\Delta P$.[47] Although holdings are usually available quarterly, I consider a monthly window to remain consistent with the previous sections.

I plot the trading measure, $\text{cov}(\Delta\widetilde{\theta}^l|\Delta P)$, in Figure 11 below, as a function of time and social interactions intensity η. A first observation, and an important result, is that managers of Network A are strong momentum traders. As apparent in Panel A, $\text{cov}(\Delta\widetilde{\theta}^A, \Delta P)$ is positive, which implies that managers of Network A tend to buy shares after price increases (and vice-versa). Intuitively, managers of Network A hold few ideas and thus rely more on public information. Because price changes partially reveal better-informed managers' information, all the more so when social interactions are intense, managers of Network A tend to intensify their positive-feedback strategy.

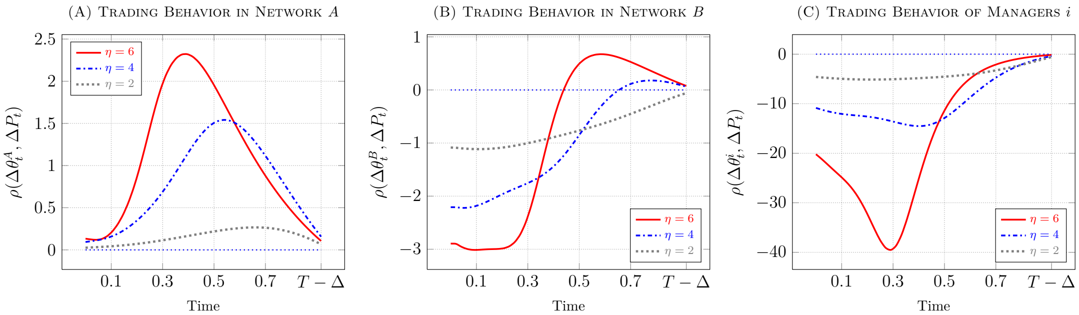
Unlike managers of Network A, informed managers can perfectly interpet price changes. If managers i observe a price change triggered by noise traders, they know it simply represents an inappropriate move. If managers i observe a price change driven by the market, they know it merely reflects an imperfect adjustment towards the stock's *actual* value, part of which, Π, managers i know. Hence, one should expect managers i to systematically trade against the market. Panel C shows that $\text{cov}(\Delta\widetilde{\theta}^i|\Delta P)$ is negative and managers i indeed come out as contrarian investors—they buy shares after price decreases (and vice-versa). If conversations often take place, they fiercely bet against the market to exploit their information before the market digests it.

Better-informed managers are contrarians while lesser-informed managers follow the trend. This result is similar to that of Brennan and Cao (1996) who show that it relates to agents' precision. An important difference is that, in my model, managers' precision is endogenized through their number of ideas (see Subsection 3.1).[48] Moreover, for the range of meeting intensities considered, momentum traders represent a vast majority of the population, a result supported by Grinblatt, Titman, and Wermers (1995) who find that 77% of managers in their sample implement momentum strategies.

[47] The derivation is provided in Appendix F.2.

[48] See also Colla and Mele (2010) and Watanabe (2008).

FIGURE 11: TRADING BEHAVIOR



Trading Behavior and Social Interactions. Figure 11 plots the contemporaneous correlation of portfolio and price changes over a one-month period Δ for the average manager in Network A (Panel A), in Network B (Panel B) and manager i (Panel C). Each is plotted as a function of time, for a social interaction of one meeting twice per year (······), one meeting every quarter (-·-·-·) and one meeting every two months (——). The calibration is reported in Table 1.

That informed managers pursue contrarian strategies is consistent with the empirical finding of Coval and Moskowitz (2001), who document a strong inverse relationship between herding activity and geographic proximity, in line with a local informational advantage. Fund managers break away from the herd in their local investments and "free-ride" on others' information by herding on distant investments. Feng and Seasholes (2004) reach similar conclusions regarding individual investors.[49]

The trading behavior of managers in Network B offers another intriguing result. One would expect managers of Network B, who have access to privileged information, to behave at least similarly to informed managers. But Panel B shows that this conjecture may not always be verified. As long as the market is at an early stage of learning, managers of Network B have sufficiently many ideas to behave like contrarians. Nevertheless, if managers entertain more than two meetings per year on average ($\eta > 2$), $\text{cov}(\Delta\widetilde{\theta}^B|\Delta P)$ switches sign and *they eventually align with the market consensus*. This result illustrates the strong impact of social interactions on trading. Specifically, word-of-mouth communication causes market learning to improve at great speed so that even good ideas cannot keep a manager from eventually following the trend. This tendency to rally to the market consensus is consistent with Wermers (1999).

[49] That informed managers i's strategy weakly relies on public information is consistent with the empirical conclusions of Kacperczyk and Seru (2007).

6.2 Betting on Price Convergence

I now provide the mechanism driving the momentum and contrarian strategies previously described. These strategies hinge upon the way in which managers bet on price convergence; this becomes apparent if one rewrites managers' portfolio in (15) as[50]

$$\theta_t^l - \underbrace{d_{\Theta,t}^l \Theta_t}_{\text{Market-Making}} \equiv \widetilde{\theta}_t^l = \underbrace{\frac{\varphi_t^l}{\lambda_{1,t}} E[P_t - E[\Pi + \delta | \mathcal{F}_t^c]|\Pi]}_{\text{a) Loading on Convergence}} + \underbrace{\left(\varphi_t^l - \frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^l\right) \frac{1}{n_t^l} \sum_{k=1}^{n_t^l} \epsilon_k^l}_{\text{b) Noise from Conversations}} - \underbrace{\varphi_t^l \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta}_{\text{c) Noise Traders}} \quad (20)$$

where $\varphi_t^l = \frac{\sigma_S^2}{\sigma_S^2 + o_t^c n_t^l} \frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^l + \frac{o_t^c n_t^l}{\sigma_S^2 + o_t^c n_t^l} d_{\Delta,t}^l$. Managers exploit the time series of prices according to their own views. First, managers can compute the price deviation from its fundamental $\Pi + \delta$ *as commonly perceived*, $P_t - E[\Pi + \delta | \mathcal{F}_t^c]$. Second, managers have access to at least one idea and can therefore refine the common estimate of price convergence. Ideally, managers would like to condition price convergence on Π, thus giving rise to a) in (20). Yet, due to b), this operation is overwhelmed by the errors contained in their set of ideas. Even for informed managers, who are not affected by b), this operation remains imperfect due to noise traders whose orders produce transitory price deviations through c).

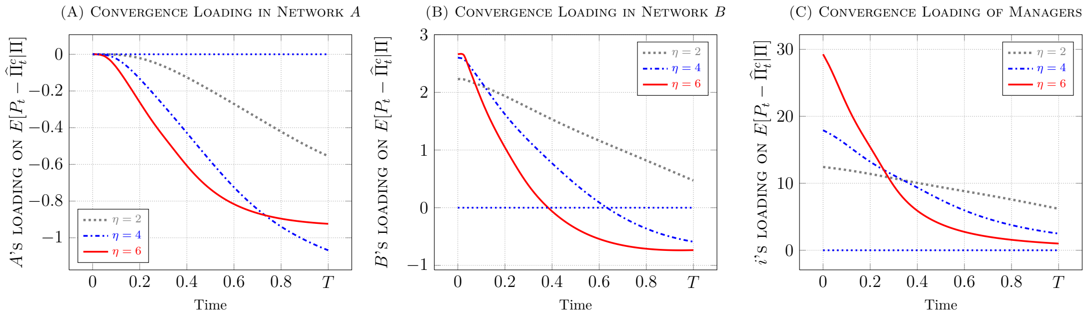
Ignoring noise perturbations in b) and c), the loading φ^l in a) determines how a manager bets on price convergence. φ^l is a weighted average of her speculative d_Δ^l and her scaled market-making $\frac{\lambda_1}{\lambda_2} d_\Theta^l$ activities with weights depending on both her number of ideas n^l and common uncertainty o^c. One therefore needs to investigate which of the two dominates. To do so, I plot a) in Figure 12, as a function of time and intensity η.

Consider first managers in Network A. Because managers of Network A have a moderate number of ideas, their loading φ^A on price convergence is tilted towards $\frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^A$. This term is negative due to the compensation for risk $\lambda_2 < 0$ and a) is therefore negative (Panel A). In turn, if the market thinks the stock is overpriced (that is, $P_t > E[(\Pi + \delta)|\mathcal{F}_t^c]$), managers of Network A follow the market and go short (and vice-versa). They are trend-chasers, indeed.

Informed managers do the opposite. Importantly, managers i can tell that market expectations, on average, will not converge to Π before the announcement date. Hence, in the meantime, they can make money by betting against the market. To see this, notice that their loading φ^i reduces to their speculative activity d_Δ^i. As a result, when

[50]See Appendices F.1 and F.2 for further details.

Figure 12: Betting on Price Convergence



Price Convergence and Social Interactions. Figure 12 plots the exposure to price convergence for the average manager in Network A (Panel A), in Network B (Panel B) and manager i (Panel C). Each is plotted as a function of time, for a social interaction of one meeting twice per year (⋯⋯), one meeting every quarter (-·-·-) and one meeting every two months (——). The calibration is reported in Table 1.

the market thinks the stock is overpriced, they go long by positively loading $\varphi^i > 0$ on a) (and vice-versa), as seen in Panel C. Needless to say, they are contrarians.

Managers in Network B speculate on price convergence in a non-trivial manner. Good ideas initially allow these managers to trade alongside informed managers (φ^B is tilted towards d^B_Δ). Yet, as interactions intensify, market learning shows a substantial improvement and common uncertainty o^c is greatly reduced. As a result, their loading φ^B shifts towards $\frac{\lambda_{1,t}}{\lambda_{2,t}} d^B_{\Theta,t}$ and managers of Network B eventually bet alongside the market (see the solid and dashed-dotted lines in Panel B). That is to say, *even good ideas cannot keep a manager from aligning with the market consensus.*

While this mechanism may have the flavor of informational cascades (Bikhchandani, Hirshleifer, and Welch (1992)), Bayesian managers do not disregard—but optimally give less and less weight to—their own ideas and increasingly rely on market information. This mechanism *is endogenously conveyed through the informational content of prices.*

7 Conclusion

I show that word-of-mouth communication can explain a number of stylized facts in the mutual fund literature. First, social interactions lead to greater efficiency and progressively erode managers' informational advantage, causing managers' performance not to persist. Second, social interactions generate momentum in stock returns. In my model, momentum arises as a rational phenomenon whose underlying mechanism depends on the diffusion of information among managers. Finally, social interactions give rise to trading patterns that are consistent with those identified in the mutual fund industry. Momentum trading becomes sooner or later an optimal strategy, except for a handful of top performers who consistently break away from the herd.

This paper suggests many potentially interesting avenues for future research. First, the social dynamics of my model may be adapted for the purpose of studying fads and fashions in the marketplace. Rumors may propagate and drive prices away from their fundamental value. If agents are allowed to act strategically, this may lead to a new theory of price manipulation. Second, the relation between word-of-mouth communication and prices is only one-way—social interactions impact prices, but prices do not impact the way in which agents interact. A challenging extension involves a full-fledged equilibrium in which social interactions and prices feed back both ways. Third, in the "limits to arbitrage" literature, social interactions may mitigate two kinds of risk: word-of-mouth communication may help arbitrageurs to, firstly, synchronize their trades to arbitrage bubbles away and, secondly, reduce convergence risk by accelerating the price adjustment towards its fundamental value. Finally, this paper focusses on managers' market-timing ability. An economy with multiple stocks is a natural extension of the present work and would allow one to study the effects of social interactions on selectivity. I leave investigations along these lines for future research.

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The Social Dynamics of Performance
Technical Appendix

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A Proof of Propositions 1 and 2

In this appendix, I derive the dynamics of Network A and Network B. The derivations of the density μ_t^A for Network A contained in equation (4) of Proposition 1 proceed along the same lines as those of Duffie and Manso (2007) and the references thereof, except for one aspect: agents meet in Network A with an intensity ηq_t^A that is network-specific. Since agents are restricted to meet with others currently positioned within the same network, types are drawn from a density that needs to be normalized by the size of the network such that it integrates up to one, i.e. $\frac{\mu_t^A}{q_t^A}$. Hence, for $n \in A$, the dynamics of μ_t^A evolve as

$$\frac{\mathrm{d}\mu_t^A(n)}{\mathrm{d}t} = \eta q_t^A \left(\sum_{k=1}^{n-1} \mu_t^A(n-k) \frac{\mu_t^A(k)}{q_t^A} - \mu_t^A \right)$$

which yields (4). The mass q_t^A is then obtained as follows: from (4), I can write

$$\mathrm{d}e^{\eta \int_0^t q_s^A \mathrm{d}s} \mu_t^A(n) = \eta e^{\eta \int_0^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_t^A(n-k) \mu_t^A(k) \mathrm{d}t.$$

Integrating, I obtain that

$$\mu_t^A(n) = e^{-\eta \int_0^t q_s^A \mathrm{d}s} \mu_0^A(n) + \eta \int_0^t e^{-\eta \int_u^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_u^A(n-k) \mu_u^A(k) \mathrm{d}u.$$

Hence, if $n = 1$, this implies that $\mu_t^A(1) = e^{-\eta \int_0^t q_s^A \mathrm{d}s}$ and

$$\mu_t^A(n) = \eta \int_0^t e^{-\eta \int_u^t q_s^A \mathrm{d}s} \sum_{k=1}^{n-1} \mu_u^A(n-k) \mu_u^A(k) \mathrm{d}u$$

for $n \in A \backslash \{1\}$. Accordingly, the mass q_t^A of Network A satisfies

$$\begin{aligned} q_t^A &= \sum_{n=1}^{K-1} \mu_t^A(n) \\ &= e^{-\eta \int_0^t q_s^A \mathrm{d}s} \left(1 + \eta \int_0^t e^{\eta \int_0^u q_s^A \mathrm{d}s} \sum_{n=2}^{K-1} \sum_{k=1}^{n-1} \mu_u^A(n-k) \mu_u^A(k) \mathrm{d}u \right). \end{aligned}$$

Differentiating this expression, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} q_t^A = -\eta (q_t^A)^2 + \eta \sum_{n=2}^{K-1} \sum_{m=1}^{n-1} \mu_t^A(n-m) \mu_t^A(m), \quad q_0^A = \omega_0.$$

Equation (5) in Proposition 2 is derived in two steps: *i)* I first take care of the set of agents who migrate to Network B. Taking discrete intervals of time Δ, the migration of agents to Network B satisfies

$$\mu^B_{t+\Delta} = (1 - \eta q^B_t \Delta)\mu^B_t + \mu^A_{t+\Delta} - \mu^A_t.$$

Rearranging and taking the limit on both sides, I get

$$\lim_{\Delta \to 0} \frac{\mu^B_{t+\Delta} - \mu^B_t}{\Delta} = -\eta q^B_t \mu^B_t + \lim_{\Delta \to 0} \frac{\mu^A_{t+\Delta} - \mu^A_t}{\Delta}.$$

Migrating agents have between K and $2(K-1)$ signals and their meetings further need to incorporate the restriction that both networks are disjoint: an agent who currently holds $K+1$ signals could not previously have K signals for, otherwise, she would already be in Network B. Likewise, an agent who holds $2(K-1)$ could only previously possess $K-1$ signals. Hence, the types k and $n-k$ whose meeting results into $n \in [K, 2(K-1)]$ have to be such that $1 \leq k \leq K-1$ and $1 \leq n-k \leq K-1$, which is equivalent to $1 \vee (n-(K-1)) \leq k \leq (K-1) \wedge (n-1)$ or $n-(K-1) \leq k \leq K-1$ given that $K \geq 2$. Therefore, migrating agents with $n \in [K, 2(K-1)]$ signals enter Network B at a rate $\frac{\mathrm{d}\mu^B_t(n)}{\mathrm{d}t} = -\eta q^B_t \mu^B_t(n) + \eta \sum_{k=n-(K-1)}^{K-1} \mu^A_t(n-k)\mu^A_t(k)$. First, there is no emigration term related to μ^A_t because agents achieving type $n \in [K, 2(K-1)]$ are not part of Network A. Their emigration takes now place with respect to Network B. Second, the convolution is truncated in such a way that the previous discussion holds.

Besides migrating agents, *ii)* I need to take care of the meetings among incumbents to Network B who necessarily achieve a type higher than $2(K-1)$. The derivation thereof proceeds along the same lines as that for Network A and may, therefore, be directly adapted: the types k and $n-k$ whose meeting results into $n \in [2K, N-1]$ have to be such that $K \leq k \leq N-1$ and $K \leq n-k \leq N-1$, which is equivalent to $K \vee (n-(N-1)) \leq k \leq (n-K) \wedge (N-1)$ or $K \leq k \leq n-K$ given that $N \geq 2K$. Hence, the convolution of incumbents is given by $\sum_{k=K}^{n-K} \mu^B_t(n-k)\mu^B_t(k)$ and the resulting density in (5) follows.

From (5), the mass q^B_t of agents located in Network B satisfies

$$q^B_t = \sum_{n=K}^{N-1} \mu^B_t(n) = \eta \int_0^t e^{-\eta \int_u^t q^B_s \mathrm{d}s} \sum_{n=K}^{N-1} \left(\begin{array}{c} \mathbf{1}_{\{n \in [K, 2(K-1)]\}} \sum_{k=n-(K-1)}^{K-1} \mu^A_u(n-k)\mu^A_u(k) \\ +\mathbf{1}_{\{[n \in [2K, N-1]]\}} \sum_{k=K}^{n-K} \mu^B_u(n-k)\mu^B_u(k) \end{array} \right) \mathrm{d}u.$$

Differentiating this expression, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} q^B_t = -\eta (q^B_t)^2 + \eta \sum_{n=K}^{N-1} \left(\begin{array}{c} \mathbf{1}_{\{n \in [K, 2(K-1)]\}} \sum_{m=n-(K-1)}^{K-1} \mu^A_t(n-m)\mu^A_t(m) \\ +\mathbf{1}_{\{n \in [2K, N-1]\}} \sum_{m=K}^{n-K} \mu^B_t(n-m)\mu^B_t(m) \end{array} \right), \quad q^B_0 = 0.$$

Finally, the mass of agents perfectly informed is determined by the sum of the agents holding $n \in C$ signals for the set of integers $C = \{N, ..., 2(N-1)\} \subset \mathbb{N}^*$. Denoting by $\mu_t^C(\mathbb{C}) = \iota(\{j : j_t \in \mathbb{C}\})$ the density for each classes of number $n \geq N$ of signals and where $\mathbb{C} \subseteq C$, I can write

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_t^C(n) = \eta \sum_{k=K\vee(n-(N-1))}^{(N-1)\wedge(n-K)} \mu_t^B(n-k)\mu_t^B(k), \;\; \mu_0^C(n) = \delta_0.$$

Migrating agents pile up in each class which have become irrelevant given that Network C indifferently grants access to perfect information. Hence, the mass q_t^C of agents located in Network C satisfies

$$\begin{aligned} q_t^C &= \sum_{n=N}^{2(N-1)} \mu_t^C(n) = \eta \int_0^t \sum_{n=N}^{2(N-1)} \sum_{k=K\vee(n-(N-1))}^{(N-1)\wedge(n-K)} \mu_s^B(n-k)\mu_s^B(k)\mathrm{d}s \\ &= 1 - q_t^A - q_t^B. \end{aligned}$$

■

B Proof of Proposition 3

In this appendix, I derive the Bayesian updating procedure in the presence of social interactions. In a first step, I derive the filter pertaining to the information accrued from the tape and, then, derive the filter related to the information processed through private discussions. When filtering from the price, agents apply a Kalman filter to the vector of unobservable variables $X_t \equiv (\Pi, \Theta_t)^\top$ using the information Y_t^c generated by the tape. The vector X_t has dynamics

$$\mathrm{d}X_t = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix} X_t \mathrm{d}t + \begin{bmatrix} 0 \\ \sigma_\Theta \end{bmatrix} \mathrm{d}B_t^\Theta, \; X_0 = \begin{bmatrix} \Pi \\ \Theta_0 \end{bmatrix}.$$

From (6), the price may be written as

$$P_t = \xi_t + (1 - \lambda_{1,t})\widehat{\Pi}_t^c$$

where $\xi_t \equiv \lambda_{1,t}\Pi + \lambda_{2,t}\Theta_t$. Furthermore, notice that observing the price is informationally equivalent to observing ξ_t, i.e. $\sigma(P_s : 0 \leq s \leq t) \Leftrightarrow \sigma(\xi_s : 0 \leq s \leq t)$ and thus $Y_t^c = \xi_t$. The dynamics of Y_t^c are obtained by applying Ito's lemma

$$\mathrm{d}Y_t^c = \left(\lambda_{1,t}'\Pi + (\lambda_{2,t}' - a_\Theta\lambda_{2,t})\Theta_t\right)\mathrm{d}t + \lambda_{2,t}\sigma_\Theta \mathrm{d}B_t^\Theta.$$

The Kalman filter under $\mathcal{F}^j$ for $j = c, l$ is obtained through the theorem below.

Theorem 1. *Denote the unobservable vector by X_t and the observable vector by Y_t with dynamics*

$$dX_t = (a_0 + a_1 X_t)\,dt + b\,dB_t$$
$$dY_t = (A_0 + A_1 X_t)\,dt + B\,dB_t$$

where $dB_t = dB_t^{\Theta}$. The conditional mean $\widehat{X}_t$ with respect to the information set $\mathcal{F}_t^Y = \sigma(Y_s : 0 \leq s \leq t)$ has dynamics

$$d\widehat{X}_t = (a_0 + a_1\widehat{X}_t)\,dt + (O_t A_1^\top + bB^\top)(BB^\top)^{-\frac{1}{2}}\,d\widehat{B}_t$$

where $O_t = E\left[(X - \widehat{X}_t)(X - \widehat{X}_t)^\top \middle| \mathcal{F}_t^Y\right]$ is the positive semi-definite conditional variance-covariance matrix of X_t given by the solution to the Ricatti equation

$$\dot{O}_t = a_1 O_t + O_t a_1^\top + bb^\top - (O_t A_1^\top + bB^\top)(BB^\top)^{-1}(A_1 O_t + B^\top b)$$

and where the filter innovation $\widehat{B}_t$ satisfying

$$d\widehat{B}_t = (BB^\top)^{-\frac{1}{2}}(dY_t - (A_0 + A_1\widehat{X}_t)\,dt)$$

is a Brownian motion with respect to the filtration $\mathcal{F}_t^Y$.

Proof. See Lipster and Shiryaev (2001), Theorem 12.7. Q.E.D.

To apply Theorem 1, I need an expression for the variance-covariance matrix O of the filter: notice that because $\xi_t \in \mathcal{F}_t^c \subseteq \mathcal{F}_t^l$, it follows that $\xi_t = \lambda_{1,t}\Pi + \lambda_{2,t}\Theta_t \equiv \lambda_{1,t}\widehat{\Pi}_t^c + \lambda_{2,t}\widehat{\Theta}_t^c \equiv \lambda_{1,t}\widehat{\Pi}_t^l + \lambda_{2,t}\widehat{\Theta}_t^l$. This means, in turn, that

$$E\left[(\Theta_t - \widehat{\Theta}_t^j)^2 \middle| \mathcal{F}_t^j\right] = E\left[\left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 (\Pi - \widehat{\Pi}_t^j)^2 \middle| \mathcal{F}_t^j\right] = \left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 o_t^j, \quad j = c, l$$

and

$$E\left[(\Theta_t - \widehat{\Theta}_t^j)(\Pi - \widehat{\Pi}_t^j) \middle| \mathcal{F}_t^j\right] = E\left[-\frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^j)^2 \middle| \mathcal{F}_t^j\right] = -\frac{\lambda_{1,t}}{\lambda_{2,t}} o_t^j, \quad j = c, l.$$

Accordingly, O^j for $j = c, l$ may be written as

$$O_t^j = o_t^j \begin{bmatrix} 1 & -\frac{\lambda_{1,t}}{\lambda_{2,t}} \\ -\frac{\lambda_{1,t}}{\lambda_{2,t}} & \left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^2 \end{bmatrix}.$$

Further observing that

$$a_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ a_1 = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix},\ b = \begin{bmatrix} 0 \\ \sigma_\Theta \end{bmatrix},\ A_0 = 0,\ A_1 = \begin{bmatrix} \lambda'_{1,t} \\ \lambda'_{2,t} - a_\Theta \lambda_{2,t} \end{bmatrix}^\top$$

and $B = \lambda_{2,t}\sigma_\Theta$ and working out the expression for the conditional mean $\widehat{X}_t$ in Theorem 1, I get the following dynamics under common information

$$\mathrm{d}\begin{bmatrix} \widehat{\Pi}^c_t \\ \widehat{\Theta}^c_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix}\begin{bmatrix} \widehat{\Pi}^c_t \\ \widehat{\Theta}^c_t \end{bmatrix}\mathrm{d}t + \frac{1}{\lambda^2_{2,t}\sigma_\Theta}\begin{bmatrix} o^c_t(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t})) \\ \lambda^2_{2,t}\sigma^2_\Theta + o^c_t\left(\frac{\lambda^2_{1,t}}{\lambda_{2,t}}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}) - \lambda_{1,t}\lambda'_{1,t}\right) \end{bmatrix}\mathrm{d}\widehat{B}^c_t$$

where $\widehat{B}^c_t$ is a one-dimensional Brownian motion with respect to $\mathcal{F}^c_t$ with

$$\mathrm{d}\widehat{B}^c_t = \frac{1}{\lambda_{2,t}\sigma_\Theta}\left(\mathrm{d}\xi_t - (\lambda'_{1,t}\widehat{\Pi}^c_t + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})\widehat{\Theta}^c_t)\mathrm{d}t\right). \tag{21}$$

Using the definition of k_t in (11), I get the equation (9) for the common filter. Furthermore, substituting the expression obtained above for O^c_t into the Ricatti equation of Theorem 1 and working out the equation, I obtain the following ordinary differential equation

$$\frac{\mathrm{d}o^c_t}{\mathrm{d}t} = -\frac{(o^c_t)^2}{\lambda^4_{2,t}\sigma^2_\Theta}(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}))^2 = -k^2_t(o^c_t)^2.$$

This yields the equation for the filtered variance under common information appearing in (12).

Similarly, when agents l do not meet anyone and, thus, collect information from only watching the tape, their forecasts evolve as

$$\mathrm{d}\begin{bmatrix} \widehat{\Pi}^l_t \\ \widehat{\Theta}^l_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -a_\Theta \end{bmatrix}\begin{bmatrix} \widehat{\Pi}^l_{t_-} \\ \widehat{\Theta}^l_{t_-} \end{bmatrix}\mathrm{d}t + \begin{bmatrix} o^l_t\left(n^l_{t_-}\right)k_t \\ \left(\sigma_\Theta - o^l_t\left(n^l_{t_-}\right)\frac{\lambda_{1,t}}{\lambda_{2,t}}k_t\right) \end{bmatrix}\mathrm{d}\widehat{B}^l_t \tag{22}$$

where

$$\mathrm{d}\widehat{B}^l_t = \frac{1}{\lambda_{2,t}\sigma_\Theta}(\mathrm{d}\xi_t - (\lambda'_{1,t}\widehat{\Pi}^l_{t_-} + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})\widehat{\Theta}^l_{t_-})\mathrm{d}t) \tag{23}$$

is a one-dimensional Brownian motion with respect to $\mathcal{F}^l_{t_-}$. The variance of their filter evolves as $\mathrm{d}o^l_t\left(n^l_t\right) = -k^2_t\left(o^l_t\left(n^l_{t_-}\right)\right)^2\mathrm{d}t$.

I now turn to show how both the beliefs about Π and $(\Theta_t)_{t\geq 0}$ are updated when an agent is met: whenever an agent l meets another agent holding m_t signals, she gets a sequence $\{S^l_k\}_{k=1}^{m_t}$ of incremental signals. Conveniently, by Gaussian theory, $\bar{S}^l_{m,t} \equiv \frac{1}{m_t}\sum_{k=1}^{m_t} S^l_k$ is a sufficient statistic for the latter. Notice that $\bar{S}^l_{m,t}$ is conditionally distributed as $\bar{S}^l_{m,t} \sim \mathcal{N}(\Pi, \frac{\sigma_S^2}{m_t})$. Then, by Bayes' rule, the conditional density $p^l_t(\Pi|\mathcal{F}^l_t)$ may be written in the following recursive way

$$p^l_t(\Pi|\mathcal{F}^l_t) = \frac{p^l_{t_-}(\Pi|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-\Pi)^2}{2\sigma_S^2/m_t}}}{\int_{\mathbb{R}} p^l_{t_-}(x|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-x)^2}{2\sigma_S^2/m_t}}\,\mathrm{d}x}.$$

Hence, at time 0, given the prior $\Pi \sim \mathcal{N}(0, \sigma_\Pi^2)$ and immediately after investor l receives her initial private signal (in which case, $n = 1$)

$$p^l_0(\Pi|\mathcal{F}^l_0) = \frac{e^{-\frac{1}{2}\left(\frac{\Pi}{\sigma_\Pi}\right)^2} e^{-\frac{1}{2}\left(\frac{S^l-\Pi}{\sigma_S}\right)^2}}{\int_{\mathbb{R}} e^{-\frac{1}{2}\left(\frac{x}{\sigma_\Pi}\right)^2} e^{-\frac{1}{2}\left(\frac{S^l-x}{\sigma_S}\right)^2}\,\mathrm{d}x} = \sqrt{\frac{\frac{1}{\sigma_S^2}+\frac{1}{\sigma_\Pi^2}}{2\pi}}\,e^{-\frac{((\Pi)\sigma_S^2+(\Pi-S^l)\sigma_\Pi^2)^2}{2\sigma_S^2\sigma_\Pi^2(\sigma_S^2+\sigma_\Pi^2)}}$$

which may be used as an initial condition for the above recursion. I can then compute the integral in the denominator of the expression above to obtain

$$\int_{\mathbb{R}} p^l_{t_-}(x|\mathcal{F}^l_{t_-})e^{-\frac{(\bar{S}^l_{m,t}-x)^2}{2\sigma_S^2/m_t}}\,\mathrm{d}x = \sqrt{\frac{\widehat{o}^l_t\left(n^l_t\right)}{o^l_t\left(n^l_{t_-}\right)}}\,e^{-\left(\frac{\left(\bar{S}^l_{m,t}\right)^2}{2\sigma_S^2/m_t}+\frac{\left(\widehat{\Pi}^l_{t_-}\right)^2}{2o^l_t\left(n^l_{t_-}\right)}\right)+\left(\frac{\widehat{\Pi}^l_{t_-}}{o^l_t\left(n^l_{t_-}\right)}+\frac{\bar{S}^l_{m,t}}{\sigma_S^2/m_t}\right)^2\frac{\widehat{o}^l_t\left(n^l_t\right)}{2}}$$

where $\widehat{o}^l_t\left(n^l_t\right) \equiv \left(\frac{1}{o^l_t\left(n^l_{t_-}\right)}+\frac{m_t}{\sigma_S^2}\right)^{-1}$. Substituting this expression back, I obtain

$$p^l_t(\Pi|\mathcal{F}^l_t) = \frac{1}{\sqrt{2\pi\widehat{o}^l_t\left(n^l_t\right)}}e^{-\frac{\left(\left(\frac{\widehat{\Pi}^l_{t_-}}{o^l_t\left(n^l_{t_-}\right)}+\frac{\bar{S}^l_{m,t}}{\sigma_S^2/m_t}\right)\widehat{o}^l_t\left(n^l_t\right)-\Pi\right)^2}{2\widehat{o}^l_t\left(n^l_t\right)}} = \frac{1}{\sqrt{2\pi o^l_t\left(n^l_t\right)}}e^{-\frac{1}{2}\frac{(\widehat{\Pi}^l_t-\Pi)^2}{o^l_t\left(n^l_t\right)}} \quad (24)$$

where the second equality follows from that the conditional distribution $p^l_t(\Pi|\mathcal{F}^l_t)$ is Gaussian for any t. Comparing the two expressions in (24) yields the updating rule for the variance

$$\frac{1}{o^l_t\left(n^l_t\right)} = \frac{1}{o^l_t\left(n^l_{t_-}\right)}+\frac{m_t}{\sigma_S^2} \quad (25)$$

and the mean

$$\frac{\widehat{\Pi}_t^l}{o_t^l\left(n_t^l\right)} = \frac{\widehat{\Pi}_{t_-}^l}{o_t^l\left(n_{t_-}^l\right)} + \frac{\bar{S}_{m,t}^l}{\sigma_S^2/m_t} = \frac{\widehat{\Pi}_{t_-}^l}{o_t^l\left(n_t^l\right)} + \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}. \tag{26}$$

I still need to take care of the updating rule for the noisy supply $(\Theta_t)_{t\geq 0}$: observing that information ξ_t accruing from the price is continuous, I can write $\widehat{\Theta}_t^l - \widehat{\Theta}_{t_-}^l = -\frac{\lambda_{1,t}}{\lambda_{2,t}}(\widehat{\Pi}_t^l - \widehat{\Pi}_{t_-}^l)$ and, in consideration of (26), the updating rule immediately follows

$$\frac{\widehat{\Theta}_t^l}{o_t^l\left(n_t^l\right)} = \frac{\widehat{\Theta}_{t_-}^l}{o_t^l\left(n_t^l\right)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}. \tag{27}$$

Alternatively, the conditional p.d.f. $p_t^l(\Theta_t|\mathcal{F}_t^l)$ for the noisy supply satisfies the recursion

$$p_t^l(\Theta_t|\mathcal{F}_t^l) = \frac{p_{t_-}^l(\Theta_t|\mathcal{F}_{t_-}^l)e^{-\frac{(\bar{S}_{m,t}^l-(\widehat{\Pi}_t^l+\frac{\lambda_{2,t}}{\lambda_{1,t}}(\widehat{\Theta}_t^j-\Theta_t)))^2}{2\sigma_S^2/m_t}}}{\int_{\mathbb{R}} p_{t_-}^l(x|\mathcal{F}_{t_-}^l)e^{-\frac{(\bar{S}_{m,t}^l-(\widehat{\Pi}_t^l+\frac{\lambda_{2,t}}{\lambda_{1,t}}(\widehat{\Theta}_t^j-x)))^2}{2\sigma_S^2/m_t}}\mathrm{d}x}$$

with initial condition

$$p_0^l(\Theta_0|\mathcal{F}_0^l) = \sqrt{\frac{\frac{\lambda_{2,0}^2}{\lambda_{1,0}^2\sigma_S^2} + \frac{2a_\Theta}{\sigma_\Theta^2}}{2\pi}}e^{-\frac{1}{2}\frac{(2a_\Theta\Theta_0\lambda_{1,0}^2\sigma_S^2+\lambda_{2,0}(S^l\lambda_{1,0}+\Theta_0\lambda_{2,0})\sigma_\Theta^2)^2}{2a_\Theta\lambda_{1,0}^4\sigma_S^4\sigma_\Theta^2+\lambda_{1,0}^2\lambda_{2,0}^2\sigma_S^2\sigma_\Theta^4}}.$$

In consideration of the updating rules in (25), (26) and (27), when an agent is met at time t, the filtered fundamental and the filtered noisy supply experience a jump whose size is respectively given by

$$\Delta\widehat{\Pi}_t^l \equiv \widehat{\Pi}_t^l - \widehat{\Pi}_{t_-}^l = \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}o_t^l\left(n_t^l\right) \text{ and } \Delta\widehat{\Theta}_t^l \equiv \widehat{\Theta}_t^l - \widehat{\Theta}_{t_-}^l = -\frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}o_t^l\left(n_t^l\right). \tag{28}$$

Likewise, the variance of posteriors experiences a jump of size

$$\Delta o_t^l \equiv o_t^l\left(n_t^l\right) - o_t^l\left(n_{t_-}^l\right) = -\frac{1}{\sigma_S^2}o_t^l\left(n_t^l\right)o_t^l\left(n_{t_-}^l\right)m_t.$$

Moreover, the respective filters of Π, a constant, and $(e^{a_\Theta t}\Theta_t)_{t\geq 0}$, a martingale, need to be martingales. Hence, I need to compensate their jump size adequately so as to enforce their martingality. To do so, I have to pin down the distribution of the jump size in posteriors or, equivalently, the distribution of $Z_{m,t}^l \equiv \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2/m_t}o_t^l\left(n_t^l\right)$. Let $\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)$ denote the required density. Moreover, notice that the probability of meeting someone in the time interval $[t, t+\mathrm{d}t)$ is $\mathbb{P}[N^l_{t+\mathrm{d}t} - N^l_t = 1] = \eta_t \mathrm{d}t$ and the probability of getting m incremental signals given $n^l_{t\_}$ currently held and conditional on meeting someone in $[t, t+\mathrm{d}t)$ is

$$\mathbb{P}[m_t = m | N^l_{t+\mathrm{d}t} - N^l_t = 1, n^l_{t\_}] = \frac{\mu^A_t(m)}{q^A_t}\mathbf{1}_{\{n^l_{t\_} \in A; m\in A\}} + \frac{\mu^B_t(m)}{q^B_t}\mathbf{1}_{\{n^l_{t\_} \in B; m\in B\}}.$$

Hence, the probability of getting m signals in $[t, t+\mathrm{d}t)$ given $n^l_{t\_}$ is

$$\mathbb{P}[\{m_t = m\} \cap \{N^l_{t+\mathrm{d}t} - N^l_t = 1\} | n^l_{t\_}] = \eta \left(\mu^A_t(m)\mathbf{1}_{\{n^l_{t\_} \in A; m\in A\}} + \mu^B_t(m)\mathbf{1}_{\{n^l_{t\_} \in B; m\in B\}} \right) \mathrm{d}t$$

and the distribution $\nu^l_t(\cdot)$ satisfies

$$\begin{aligned} \nu^l_t(m; \mathrm{d}t; \mathrm{d}Z) &= \mathbb{P}[\{m_t = m\} \cap \{N^l_{t+\mathrm{d}t} - N^l_t = 1\} | n^l_{t\_}] \times \mathbb{P}[Z^l_{m,t} \in \mathrm{d}Z | \mathcal{F}^l_{t\_}, m_t] \quad (29) \\ &= \eta \left(\mu^A_t(m)\mathbf{1}_{\{n^l_{t\_} \in A; m\in A\}} + \mu^B_t(m)\mathbf{1}_{\{n^l_{t\_} \in B; m\in B\}} \right) \mathrm{d}t \\ &\times \mathcal{N}\left(0, \frac{m_t \sigma^2_S (o^c_t)^2}{(\sigma^2_S + n^l_{t\_} o^c_t)(\sigma^2_S + n^l_t o^c_t)}\right) \mathrm{d}Z \end{aligned}$$

where the last expression follows from that

$$E\left[Z^l_{m,t} | \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t o^l_t(n^l_t)}{\sigma^2_S} E\left[\Pi + \frac{1}{m_t}\sum_{j=1}^{m_t} \epsilon^j - \widehat{\Pi}^l_{t\_} \middle| \mathcal{F}^l_{t\_}, m_t\right] = 0$$

and

$$V\left[Z^l_{m,t} | \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t^2 (o^l_t(n^l_t))^2}{\sigma^4_S} V\left[\Pi + \frac{1}{m_t}\sum_{j=1}^{m_t} \epsilon^j \middle| \mathcal{F}^l_{t\_}, m_t\right] = \frac{m_t^2 (o^l_t(n^l_t))^2}{\sigma^4_S}(o^l_t(n^l_{t\_}) + \frac{\sigma^2_S}{m_t}).$$

Since the expected jump size is null, the compensation is null and the filtered dynamics in (10) and the variance dynamics

$$\mathrm{d}o^l_t(n^l_t) = -k^2_t (o^l_t(n^l_{t\_}))^2 \mathrm{d}t - \frac{1}{\sigma^2_S} o^l_t(n^l_t) o^l_t(n^l_{t\_}) m_t \mathrm{d}N^l_t$$

immediately follow by putting together (22) and the jump sizes in (28).

Finally, letting $K^c_t \equiv \frac{1}{o^c_t}$, I can write $\mathrm{d}K^l_t\left(n^l_t\right) = \mathrm{d}K^c_t + \frac{m_t}{\sigma^2_S}\mathrm{d}N^l_t$ and obtain the explicit relation

$$K^l_t\left(n^l_t\right) = K^l_0 + K^c_t + \int_0^t \frac{m_s}{\sigma^2_S}\mathrm{d}N^l_s = K^c_t + \frac{n^l_t}{\sigma^2_S}$$

where K^c_t satisfies $\mathrm{d}K^c_t = k^2_t \mathrm{d}t$. This yields the expression for $o^l_t\left(n^l_t\right)$ in (12). ■

C Proof of Proposition 4

In this appendix, I turn to agents i and l's optimization problem. Following the notations of He and Wang (1995), I denote by Q the excess return on the price. The latter satisfies

$$\mathrm{d}Q_t = \mathrm{d}P_t - rP_t\mathrm{d}t + \mathbf{1}_{\{t=T\}}\Delta P_T. \tag{30}$$

Accordingly, agent l who currently holds $n^l_{t\_}$ signals chooses a portfolio strategy $\theta^l_{t\_} \equiv \theta^l(\Psi^l_{t\_}, n^l_{t\_}, t)$ with Ψ^l to be shortly described in order to maximize

$$\sup_{\theta^l} E\left[-e^{-\gamma W^l_T}\middle|\mathcal{F}^l_{t\_}\right] \text{ s.t. } \quad \mathrm{d}W^l_t = rW^l_t\mathrm{d}t + \theta^l_{t\_}\mathrm{d}Q_t \tag{31}$$

while agent i chooses a portfolio strategy $\theta^i_t \equiv \theta^i(\Psi_t, t)$ in order to maximize

$$\sup_{\theta^i} E\left[-e^{-\gamma W^i_T}\middle|\mathcal{F}^i_t\right] \text{ s.t. } \quad \mathrm{d}W^i_t = rW^i_t\mathrm{d}t + \theta^i_t\mathrm{d}Q_t. \tag{32}$$

I first solve the problems in (31) and (32) over $[0, T)$ and then solve the ones prevailing at the horizon date which, in turn, will provide boundary conditions. In doing so, I need to determine the state variables relevant to (31) and (32): due to the CARA form of utility, W will act as a trivial state variable. Besides t whose dependence is triggered by $T < \infty$, the other relevant state variables, Ψ, appear to be the ones driving Q.

I proceed first with (31) and, then, obtain the solution to (32) as the special case when $n^l \to \infty$: notice that, because $r = 0$, $Q_t = E[P_t|\mathcal{F}^l_t] = \lambda_{1,t}\widehat{\Pi}^l_t + \lambda_{2,t}\widehat{\Theta}^l_t + (1-\lambda_{1,t})\widehat{\Pi}^c_t$. The dynamics of $\widehat{\Pi}^l_t$ and $\widehat{\Theta}^l_t$ being obtained from (10), I just need to derive the dynamics of $\widehat{\Pi}^c_t$ with respect to $\mathcal{F}^l_{t\_}$: substitutions between the two Brownians in (21) and (23) lead to

$$\mathrm{d}\widehat{B}^c_t = \mathrm{d}\widehat{B}^l_t + \frac{1}{\lambda_{2,t}\sigma_\Theta}(\lambda'_{1,t}(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t) + (\lambda'_{2,t} - a_\Theta\lambda_{2,t})(\widehat{\Theta}^l_{t\_} - \widehat{\Theta}^c_t))\mathrm{d}t.$$

Using the equivalence relations between $\widehat{\Theta}^c_t$ and $\widehat{\Theta}^l_t$, this change of measure is written as

$$\begin{aligned}\mathrm{d}\widehat{B}^c_t &= \mathrm{d}\widehat{B}^l_t + \frac{1}{\lambda^2_{2,t}\sigma_\Theta}(\lambda'_{1,t}\lambda_{2,t} - \lambda_{1,t}(\lambda'_{2,t} - a_\Theta\lambda_{2,t}))(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t)\mathrm{d}t \\ &= \mathrm{d}\widehat{B}^l_t + k_t(\widehat{\Pi}^l_{t\_} - \widehat{\Pi}^c_t)\mathrm{d}t\end{aligned} \tag{33}$$

with the associated Radon-Nikodym derivative

$$\left.\frac{\mathrm{d}\widehat{\mathbb{P}}^c}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}^l_t} = e^{-\frac{1}{2}\int_0^t (k_s(\widehat{\Pi}^l_{s\_} - \widehat{\Pi}^c_s))^2\mathrm{d}s + \int_0^t k_s(\widehat{\Pi}^l_{s\_} - \widehat{\Pi}^c_s)\mathrm{d}\widehat{B}^c_s}. \tag{34}$$

I assume that (34) is a martingale (and not only a local martingale) and Girsanov's theorem applies. Then $\widehat{\Pi}_t^c$ satisfies the following dynamics

$$
\begin{aligned}
\mathrm{d}\widehat{\Pi}_t^c &= \frac{o_t^c}{\lambda_{2,t}^4 \sigma_\Theta^2}(\lambda_{1,t}'\lambda_{2,t} - \lambda_{1,t}(\lambda_{2,t}' - a_\Theta \lambda_{2,t}))^2(\widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c)\mathrm{d}t \\
&+ \frac{o_t^c}{\lambda_{2,t}^2 \sigma_\Theta}(\lambda_{1,t}'\lambda_{2,t} - \lambda_{1,t}(\lambda_{2,t}' - a_\Theta \lambda_{2,t}))\mathrm{d}\widehat{B}_t^l = o_t^c k_t^2(\widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c)\mathrm{d}t + o_t^c k_t \mathrm{d}\widehat{B}_t^l
\end{aligned}
$$

with respect to $\mathcal{F}_{t_-}^l$. An application of Ito's lemma then shows

$$
\begin{aligned}
\mathrm{d}Q_t &= ((\lambda_{1,t}' + (1-\lambda_{1,t})o_t^c k_t^2)(\widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c) + (\lambda_{2,t}' - a_\Theta \lambda_{2,t})\widehat{\Theta}_{t_-}^l)\mathrm{d}t \\
&+ (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t})o_t^c k_t)\mathrm{d}\widehat{B}_t^l.
\end{aligned} \tag{35}
$$

Inspection of (35) reveals that $\Psi_t^l := (1, \widehat{\Theta}_t^l, \Delta_t^l)^\top$ where the first element is introduced to capture linear dependencies along with some constant and where $\Delta_t^l \equiv \widehat{\Pi}_t^l - \widehat{\Pi}_t^c$ denotes the difference between investors l's estimate of the stock value and the estimate solely based on market information. By Ito's lemma, the latter satisfies

$$
\mathrm{d}\Delta_t^l = -o_t^c k_t^2 \Delta_{t_-}^l \mathrm{d}t + \left(o_t^l\left(n_{t_-}^l\right) - o_t^c\right) k_t \mathrm{d}\widehat{B}_t^l + \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t \mathrm{d}N_t^l.
$$

Moreover, using the expression for $\widehat{\Theta}_t^l$ in (10) in Proposition 3, I get

$$
\mathrm{d}\widehat{\Theta}_t^l = -a_\Theta \widehat{\Theta}_{t_-}^l \mathrm{d}t + \left(\sigma_\Theta - o_t^l\left(n_{t_-}^l\right)\frac{\lambda_{1,t}}{\lambda_{2,t}}k_t\right)\mathrm{d}\widehat{B}_t^l - \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t \mathrm{d}N_t^l.
$$

As a result, the excess return Q and the filtered state variables $\Psi_t^l \equiv E[\Psi_t | \mathcal{F}_t^l]$ follow the coupled process

$$
\begin{aligned}
\mathrm{d}Q_t &= A_{Q,t}\Psi_{t_-}^l \mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^l + C_{Q,t}\left(\Psi_{t_-}^l, n_t^l\right)\mathrm{d}N_t^l \\
\mathrm{d}\Psi_t^l &= A_{\Psi,t}\Psi_{t_-}^l \mathrm{d}t + B_{\Psi,t}^l\left(n_{t_-}^l\right)\mathrm{d}\widehat{B}_t^l + C_{\Psi,t}^l\left(\Psi_{t_-}^l, n_t^l\right)\mathrm{d}N_t^l
\end{aligned}
$$

over $[0, T)$ where

$$
A_{\Psi,t} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -a_\Theta & 0 \\ 0 & 0 & -k_t^2 o_t^c \end{pmatrix}, \quad B_{\Psi,t}^l\left(n_t^l\right) = \begin{pmatrix} 0 \\ \sigma_\Theta - \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{k_t \sigma_S^2}{K_t^c \sigma_S^2 + n_t^l} \\ \left(\frac{\sigma_S^2}{\sigma_S^2 K_t^c + n_t^l} - \frac{1}{K_t^c}\right) k_t \end{pmatrix},
$$

and

$$
C_{\Psi,t}^l\left(\Psi_t^l, n_t^l\right) = \begin{pmatrix} 0 & -\frac{\lambda_{1,t}}{\lambda_{2,t}} & 1 \end{pmatrix}^\top \frac{\bar{S}_{m,t}^l - \widehat{\Pi}_{t_-}^l}{\sigma_S^2} o_t^l\left(n_t^l\right) m_t
$$

along with

$$A_{Q,t} = [\ 0 \quad \lambda'_{2,t} - a_\Theta \lambda_{2,t} \quad \lambda'_{1,t} + (1-\lambda_{1,t}) o^c_t k^2_t\ ], \quad B_{Q,t} = \lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t}) o^c_t k_t \tag{36}$$

and $C_{Q,t} = 0$. As expected, the excess return does not jump, expect, perhaps, at the horizon date T when the stock pays out.

Finally, notice that Ψ^l_t does not constitute a sufficient statistic for (31) because, unlike He and Wang (1995), the variance o^l is not a sole function of time. Instead, o^l jumps at random times and one needs to include it as an additional state variable. In that respect, the relation between o^c_t and $o^l_t\left(n^l_t\right)$ described in Proposition 3 implies that one may choose to keep track either of n^l_t or o^l_t. Accordingly, I let the state variables for (31) be (W^l, Ψ^l, n^l, t) and let the associated value function J^l be of the form $J^l(W^l, \Psi^l, n^l, t)$. Using these, I can write the Hamilton-Jacobi-Bellman (HJB) equation associated with (31). By the standard martingale argument, it satisfies

$$\begin{aligned}
0 = \sup_{\theta^l_{t_-}} & \left\{ J^l_W A_{Q,t} \Psi^l_{t_-} \theta^l_{t_-} + \frac{1}{2} J^l_{WW} B^2_{Q,t} (\theta^l_{t_-})^2 + B_{Q,t} (B^l_{\Psi,t}(n^l_{t_-}))^\top J^l_{W\Psi} \theta^l_{t_-} \right\} \\
& + J^l_t + (J^l_\Psi)^\top A_{\Psi,t} \Psi^l_{t_-} + \frac{1}{2} \mathrm{tr}(J^l_{\Psi\Psi} B^l_{\Psi,t}(n^l_{t_-})(B^l_{\Psi,t}(n^l_{t_-}))^\top) \\
& + E^{\nu^l_t}[J^l(W^l_t, \Psi^l_{t_-} + C^l_{\Psi,t}\left(\Psi^l_{t_-}, n^l_t\right), n^l_{t_-} + m_t, t) - J^l(W^l_t, \Psi^l_{t_-}, n^l_{t_-}, t)]
\end{aligned}$$

with terminal boundary condition $J^l(W^l, \Psi^l, n^l, T) = -e^{-\gamma W^l_T}$ and where $\mathrm{tr}(\cdot)$ denotes the trace operator. The first-order condition reads

$$J^l_W A_{Q,t} \Psi^l_{t_-} + J^l_{WW} B^2_{Q,t} \theta^l_{t_-} + B_{Q,t} (B^l_{\Psi,t}(n^l_{t_-}))^\top J^l_{W\Psi} = 0 \tag{37}$$

and the second-order condition for optimality is $J^l_{WW} < 0$. Substituting the first-order condition into the HJB equation, I obtain the following PDE

$$\begin{aligned}
& J^l_t + (J^l_\Psi)^\top A_{\Psi,t} \Psi^l_{t_-} + \frac{1}{2} \mathrm{tr}(J^l_{\Psi\Psi} B^l_{\Psi,t}(n^l_{t_-})(B^l_{\Psi,t}(n^l_{t_-}))^\top) \\
& + E^{\nu^l_t}[J^l(W^l_t, \Psi^l_{t_-} + C^l_{\Psi,t}\left(\Psi^l_{t_-}, n^l_t\right), n^l_{t_-} + m_t, t) - J^l(W^l_t, \Psi^l_{t_-}, n^l_{t_-}, t)] \\
& - \frac{1}{2} \frac{(J^l_W A_{Q,t} \Psi^l_{t_-} + B_{Q,t}(B^l_{\Psi,t}(n^l_{t_-}))^\top J^l_{W\Psi})^2}{J^l_{WW} B^2_{Q,t}} = 0.
\end{aligned} \tag{38}$$

To solve (38), I make the following observation: if my setting were absent of social relations, Ψ^l would follow a bi-dimensional OU process and the setting would be affine quadratic. As shown in Cheng and Scaillet (2007), I could then conjecture that

$$J^l(W^l, \Psi^l, n^l, t) = -\exp\left(-\gamma W^l - \frac{1}{2} (\Psi^l)^\top M^l_t(n^l) \Psi^l\right) \tag{39}$$

where $M_t^l(n^l)$ is a $(3\times 3)-$symmetric matrix of coefficients to be determined and that satisfies boundary conditions pinned down by the problem to be solved at the horizon date. Yet, due to social interactions, learning is not purely Brownian and the combination of the jump in posteriors along with the quadratic form of (39) make the setup not LQJD (Linear-Quadratic-Jump-Diffusion) but QJD. Hence, unlike the setting of Cheng and Scaillet (2007) or Piazzesi (2005), for instance, the quadratic state variables jump along with the affine ones and the jump size in (38) is of the form

$$\begin{aligned} &E^{\nu_t^l}[J^l(W^l,\Psi_{t_-}^l + C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right),n_{t_-}^l+m_t,t) - J^l(W_t^l,\Psi_{t_-}^l,n_{t_-}^l,t)] = \\ &\qquad J^l(W^l,\Psi_{t_-}^l,n_{t_-}^l,t)\times \\ &E^{\nu_t^l}\left[\exp\left(-\frac{1}{2}\left(\begin{array}{c} \left(C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right)\right)^\top M_t^l(n_t^l)C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right) \\ +(\Psi_{t_-}^l)^\top(M_t^l(n_t^l)-M_t^l(n_{t_-}^l))\Psi_{t_-}^l \\ +(\Psi_{t_-}^l)^\top M_t^l(n_t^l)C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right) \\ +\left(C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right)\right)^\top M_t^l(n_t^l)\Psi_{t_-}^l \end{array}\right)\right)-1\right]. \end{aligned} \tag{40}$$

In this context, Chen, Filipovic, and Poor (2004) have shown that the exponential quadratic ansatz in (39) for the value function fails to hold. Instead, for (39) to hold and in the spirit of the approximation considered in Duffie, Gârleanu, and Pedersen (2007) and Vayanos and Weill (2008), I linearize the jump size in (40) by means of a first-order Taylor approximation of the latter around zero:

$$\begin{aligned} &E^{\nu_t^l}[J^l(W^l,\Psi_{t_-}^l + C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_t^l\right),n_{t_-}^l+m_t,t) - J^l(W_t^l,\Psi_{t_-}^l,n_{t_-}^l,t)] \\ \approx & -\frac{1}{2}\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_{t_-}^l+m\right)\right)^\top M_t^l(n_{t_-}^l+m)C_{\Psi,t}^l\left(\Psi_{t_-}^l,n_{t_-}^l+m\right)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right) \\ &-\frac{1}{2}\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(C_{\Psi,t}^l(\Psi_{t_-}^l,n_{t_-}^l+m)\right)^\top M_t^l(n_{t_-}^l+m)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\Psi_{t_-}^l \\ &-\frac{1}{2}(\Psi_{t_-}^l)^\top\left(\int_{\mathbb{N}^*\times\mathbb{R}}M_t^l(n_{t_-}^l+m)C_{\Psi,t}^l(\Psi_{t_-}^l,n_{t_-}^l+m)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right) \\ &-\frac{1}{2}(\Psi_{t_-}^l)^\top\left(\int_{\mathbb{N}^*\times\mathbb{R}}\left(M_t^l(n_{t_-}^l+m)-M_t^l(n_{t_-}^l)\right)\nu_t^l(m;\mathrm{d}t;\mathrm{d}Z)\right)\Psi_{t_-}^l \\ \equiv & -\frac{1}{2}\left(\phi_{0,t_-}^l(M^l)+\left(\phi_{1,t_-}^l(M^l)\right)^\top\Psi_{t_-}^l+(\Psi_{t_-}^l)^\top\phi_{1,t_-}^l(M^l)+(\Psi_{t_-}^l)^\top\phi_{2,t_-}^l(M^l)\Psi_{t_-}^l\right). \end{aligned}$$

This allows to make the setting LQJD and to preserve its tractablity. An upper bound on the error induced by the linearization is provided in Appendix D. Further substituting the ansatz in (38), I obtain the following matrix differential equation

$$\dot{M}_t^l(n_{t_-}^l) = -\eta\phi_{2,t_-}^l(M^l) - \mathrm{diag}(\phi_{1,t_-}^l(M^l))I_{(1:3,1)}^{(3)} - I_{(1,1:3)}^{(3)}\mathrm{diag}(\phi_{1,t_-}^l(M^l)) \quad (41)$$
$$M_t^l(n_{t_-}^l)\left(\frac{B_{\Psi,t}^l(n_{t_-}^l)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right) + \left(\frac{B_{\Psi,t}^l(n_{t_-}^l)A_{Q,t}}{B_{Q,t}} - A_{\Psi,t}\right)^\top M_t^l(n_{t_-}^l)$$
$$-\frac{A_{Q,t}^\top A_{Q,t}}{(B_{Q,t})^2} - \left(\mathrm{tr}\left(M_t^l(n_{t_-}^l)B_{\Psi,t}^l(n_{t_-}^l)(B_{\Psi,t}^l(n_{t_-}^l))^\top\right) + \eta\phi_{0,t_-}^l(M^l)\right)I_{11}^{(3)}$$

where $I_{i,j}^{(N)}$ is a $N \times N-$index matrix with its elements being zero except elements (i, j) being 1.

Since, as per (29), the expected jump size is null, it is immediate that

$$\phi_{1,t}^l(M^l) = [\begin{array}{ccc} 0 & 0 & 0 \end{array}]^\top \;\; \forall l \in \mathcal{I}_2. \quad (42)$$

To determine the coefficients $\phi_{0,t}^l$ and $\phi_{2,t}^l$, I denote by $c_{\Psi,t} \equiv \left[\begin{array}{ccc} 0 & -\frac{\lambda_{1,t}}{\lambda_{2,t}} & 1 \end{array}\right]^\top$ the jump scale in such a way that $C_\Psi^l = c_\Psi Z^l$. I then integrate over the different realizations of the incremental number m_t of signals: clearly, since agents only meet others within their own network, ϕ's need to be network-specific. This yields

$$\phi_{0,t_-}^l(M^l) = \begin{cases} \sum_{m\in A}\mu_t^A(m)c_{\Psi,t}^\top M_t^l(n_{t_-}^l+m)c_{\Psi,t}\frac{m\sigma_S^2}{(\sigma_S^2K_t^c+n_{t_-}^l)(\sigma_S^2K_t^c+n_{t_-}^l+m)} & \text{if } n_{t_-}^l \in A \\ \sum_{m\in B}\mu_t^B(m)c_{\Psi,t}^\top \times\left(\begin{array}{c} \mathbf{1}_{\{n_{t_-}^l+m\in A\cup B\}}\left(M_t^l(n_{t_-}^l+m)c_{\Psi,t}\frac{m\sigma_S^2}{(\sigma_S^2K_t^c+n_{t_-}^l)(\sigma_S^2K_t^c+n_{t_-}^l+m)}\right) \\ +\mathbf{1}_{\{n_{t_-}^l+m\notin A\cup B\}}\left(M_t^i c_{\Psi,t}\frac{\sigma_S^2}{\sigma_S^2K_t^c+n_{t_-}^l}\right)\end{array}\right) & \text{if } n_{t_-}^l \in B\end{cases}$$

and

$$\phi_{2,t_-}^l(M^l) = \begin{cases} \sum_{m\in A}\mu_t^A(m)\left(M_t^l(n_{t_-}^l+m) - M_t^l(n_{t_-}^l)\right) & \text{if } n_{t_-}^l \in A \\ \sum_{m\in B}\mu_t^B(m)\left(\begin{array}{c} \mathbf{1}_{\{n_{t_-}^l+m\in A\cup B\}}\left(M_t^l(n_{t_-}^l+m) - M_t^l(n_{t_-}^l)\right) \\ +\mathbf{1}_{\{n_{t_-}^l+m\notin A\cup B\}}\left(M_t^i - M_t^l(n_{t_-}^l)\right)\end{array}\right) & \text{if } n_{t_-}^l \in B\end{cases}$$

The solution to agents i's problem is directly obtained as a particular case when $n^l \to \infty$: the HJB equation for (32) is

$$0 = \sup_{\theta_t^i}\left\{J_W^i A_{Q,t}\Psi_t\theta_t^i + \frac{1}{2}J_{WW}^i B_{Q,t}^2(\theta_t^i)^2 + B_{Q,t}(B_{\Psi,t}^i)^\top J_{W\Psi}^i\theta_t^i\right\} \quad (43)$$
$$+ J_t^i + (J_\Psi^i)^\top A_{\Psi,t}\Psi_t + \frac{1}{2}\mathrm{tr}(J_{\Psi\Psi}^i B_{\Psi,t}^i(B_{\Psi,t}^i)^\top)$$

with $J^i(W^i,\Psi,T)=-e^{-\gamma W^i_T}$ and $\Psi_t=(1,\Pi-\widehat{\Pi}^c_t,\Theta_t)^\top$ and $B^i_{\Psi,t}=\lim_{n^l\to\infty}B^l_{\Psi,t}(n^l)$. After substitution of the first-order condition and using the conjecture $J^i(W^i,\Psi,t)=-e^{-\gamma W^i-\frac{1}{2}(\Psi)^\top M^i_t\Psi}$, which, in this case, is exact, I obtain

$$\begin{aligned}\dot{M}^i_t &= M^i_t\left(\frac{B^i_{\Psi,t}A_{Q,t}}{B_{Q,t}}-A_{\Psi,t}\right)+\left(\frac{B^i_{\Psi,t}A_{Q,t}}{B_{Q,t}}-A_{\Psi,t}\right)^\top M^i_t \\ &\quad -\frac{A^\top_{Q,t}A_{Q,t}}{(B_{Q,t})^2}-\operatorname{tr}\left(M^i_tB^i_{\Psi,t}(B^i_{\Psi,t})^\top\right)I^{(3)}_{11}.\end{aligned}\tag{44}$$

I can go one step further and simplify the matrix differential equations above: first, considering (41) and (42) and (44) along with the terminal condition $M^{j,(1,2)}(T)=M^{j,(1,3)}(T)=0$ for $j=l,i$ which follows from (52) below and since the differential equations for $M^{j,(1,2)}$ and $M^{j,(1,3)}$ have no generator term, $M^l_t(n^l_t)$ is of the form

$$M^l_t(n^l_t)=\begin{bmatrix} M^{l,(1,1)}_t(n^l_t) & 0 & 0 \\ 0 & M^{l,(2,2)}_t(n^l_t) & M^{l,(2,3)}_t(n^l_t) \\ 0 & M^{l,(2,3)}_t(n^l_t) & M^{l,(3,3)}_t(n^l_t)\end{bmatrix}\tag{45}$$

where M^i_t follows from $n^l\to\infty$. (45) implies that (41) and (44) may be decoupled from $M^{l,(1,1)}_t(n^l_t)$ and $M^{i,(1,1)}_t$: I shall redefine $c^\star_{\Psi,t}\equiv\begin{bmatrix}-\frac{\lambda_{1,t}}{\lambda_{2,t}} & 1\end{bmatrix}^\top$ and

$$\begin{aligned}A^\star_{\Psi,t}&=\begin{bmatrix}-a_\Theta & 0\\ 0 & -o^c_tk^2_t\end{bmatrix},\quad B^{\star,l}_{\Psi,t}(n^l_t)=\begin{bmatrix}\sigma_\Theta-\frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{k_t\sigma^2_S}{\sigma^2_SK^c_t+n^l_t}\\ (\frac{\sigma^2_S}{\sigma^2_SK^c_t+n^l_t}-\frac{1}{K^c_t})k_t\end{bmatrix},\\ A^\star_{Q,t}&=\begin{bmatrix}\lambda'_{2,t}-a_\Theta\lambda_{2,t}\\ \lambda'_{1,t}+(1-\lambda_{1,t})\frac{k^2_t}{K^c_t}\end{bmatrix}^\top,\quad B^{\star,i}_{\Psi,t}=\begin{bmatrix}\sigma_\Theta\\ -o^c_tk_t\end{bmatrix}\end{aligned}\tag{46}$$

along with matrices $M^{\star,l}$ and $M^{\star,i}$ whose dependence on $\phi^{\star,l}_2\equiv\phi^l_2(M^{\star,l})$ is emphasized

$$M^{\star,l}_t(n^l_t,\phi^{\star,l}_{2,t})=\begin{bmatrix}M^{l,(2,2)}_t(n^l_t,\phi^{\star,l}_{2,t}) & M^{l,(2,3)}_t(n^l_t,\phi^{\star,l}_{2,t})\\ M^{l,(2,3)}_t(n^l_t,\phi^{\star,l}_{2,t}) & M^{l,(3,3)}_t(n^l_t,\phi^{\star,l}_{2,t})\end{bmatrix}\text{ and }M^{\star,i}_t=\begin{bmatrix}M^{i,(2,2)}_t & M^{i,(2,3)}_t\\ M^{i,(2,3)}_t & M^{i,(3,3)}_t\end{bmatrix}.$$

Then, reorganizing (41) and (44) respectively yields

$$\begin{aligned}\dot{M}^{\star,l}_t(n^l_{t_-},\phi^{\star,l}_{2,t_-}) &= -\eta\phi^{\star,l}_{2,t_-}+M^{\star,l}_t(n^l_{t_-},\phi^{\star,l}_{2,t_-})\left(\frac{B^{\star,l}_{\Psi,t}(n^l_{t_-})A^\star_{Q,t}}{B_{Q,t}}-A^\star_{\Psi,t}\right)\\ &\quad+\left(\frac{B^{\star,l}_{\Psi,t}(n^l_{t_-})A^\star_{Q,t}}{B_{Q,t}}-A^\star_{\Psi,t}\right)^\top M^{\star,l}_t(n^l_{t_-},\phi^{\star,l}_{2,t_-})-\frac{(A^\star_{Q,t})^\top A^\star_{Q,t}}{(B_{Q,t})^2},\end{aligned}\tag{47}$$

$$\dot{M}_t^{\star,i} = M_t^{\star,i}\left(\frac{B_{\Psi,t}^{\star,i}A_{Q,t}^{\star}}{B_{Q,t}} - A_{\Psi,t}^{\star}\right) + \left(\frac{B_{\Psi,t}^{\star,i}A_{Q,t}^{\star}}{B_{Q,t}} - A_{\Psi,t}^{\star}\right)^{\top} M_t^{\star,i} - \frac{(A_{Q,t}^{\star})^{\top}A_{Q,t}^{\star}}{(B_{Q,t})^2}. \quad (48)$$

Substituting (39) into (37) and the ansatz for agents i into the first-order condition in (43) yields

$$\theta_t^l \equiv \theta^l(\widehat{\Theta}_{t_-}^l, \widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c, n_{t_-}^l, t) = \frac{A_{Q,t}^{\star} - B_{Q,t}(B_{\Psi,t}^{\star,l}(n_{t_-}^l))^{\top} M_t^{\star,l}(n_{t_-}^l, \phi_{2,t_-}^{\star,l})}{\gamma B_{Q,t}^2}\begin{bmatrix}\widehat{\Theta}_{t_-}^l \\ \widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c\end{bmatrix}$$

and

$$\theta_t^i \equiv \theta^i(\Theta_t, \Pi - \widehat{\Pi}_t^c, t) = \frac{A_{Q,t}^{\star} - B_{Q,t}(B_{\Psi,t}^{\star,i})^{\top} M_t^{\star,i}}{\gamma B_{Q,t}^2}\begin{bmatrix}\Theta_t \\ \Pi - \widehat{\Pi}_t^c\end{bmatrix},$$

delivering the respective optimal portfolio choices in (13) and (14) in Proposition 4.

Boundary conditions to (48) and (47) are provided by agents' optimization problem at the very last round of trading. Here, I need to take care of the jump in (30):

$$\begin{aligned} J^l(W_{T_-}^l, \Psi_{T_-}^l, n_T^l, T) &= \sup_{\theta_{T_-}^l} E\left[-e^{-\gamma W_{T_-}^l - \gamma\theta_{T_-}^l \Delta P_T}\middle| \mathcal{F}_{T_-}^l\right] \quad (49)\\ &= \sup_{\theta_{T_-}^l} -e^{-\gamma W_{T_-}^l - \gamma\theta_{T_-}^l E\left[\Delta P_T|\mathcal{F}_{T_-}^l\right] + \frac{1}{2}\gamma^2\left(\theta_{T_-}^l\right)^2 V\left[\Delta P_T|\mathcal{F}_{T_-}^l\right]} \end{aligned}$$

where I used the Laplace transform of a normal random variable. Solving for the first-order condition yields the optimal portfolio

$$\theta_{T_-}^l = \frac{E\left[\Delta P_T|\, \mathcal{F}_{T_-}^l\right]}{\gamma V\left[\Delta P_T|\, \mathcal{F}_{T_-}^l\right]}. \quad (50)$$

Using that $\Delta P_T = \Pi + \delta - P_{T_-} = (\delta + (1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T)$, I obtain, for agents l and i,

$$\theta_{T_-}^l = \frac{(1-\lambda_{1,T_-})(\widehat{\Pi}_{T_-}^l - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\widehat{\Theta}_{T_-}^l}{\gamma(o_{T_-}^l(n_T^l) + \sigma_\delta^2)}, \quad \theta_{T_-}^i = \frac{(1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T}{\gamma\sigma_\delta^2}. \quad (51)$$

Substituting (51) into the terminal value function in (49), I get

$$J^l(W_{T_-}^l, \Psi_{T_-}^l, n_T^l, T) = \exp\left(-\gamma W_{T_-}^l - \frac{1}{2}\frac{\left((1-\lambda_{1,T_-})(\widehat{\Pi}_{T_-}^l - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\widehat{\Theta}_{T_-}^l\right)^2}{o_{T_-}^l(n_T^l) + \sigma_\delta^2}\right) \quad (52)$$

for agents l. Similarly, I obtain $J^i(W_{T_-}^i, \Psi, T) = e^{-\gamma W_{T_-}^i - \frac{1}{2}\frac{\left((1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T\right)^2}{\sigma_\delta^2}}$ for agents i. But given the shape of the value function conjecture in (39), the boundary conditions are

$$M^{\star,l}_{T_-}(n^l_T) = \frac{1}{o^l_{T_-}(n^l_T) + \sigma^2_\delta} \begin{bmatrix} \lambda^2_{2,T_-} & -\lambda_{2,T_-}(1-\lambda_{1,T_-}) \\ -\lambda_{2,T_-}(1-\lambda_{1,T_-}) & (1-\lambda_{1,T_-})^2 \end{bmatrix} \tag{53}$$

and

$$M^{\star,i}_{T_-} = \frac{1}{\sigma^2_\delta} \begin{bmatrix} \lambda^2_{2,T_-} & -\lambda_{2,T_-}(1-\lambda_{1,T_-}) \\ -\lambda_{2,T_-}(1-\lambda_{1,T_-}) & (1-\lambda_{1,T_-})^2 \end{bmatrix}. \tag{54}$$

Notice that these boundary conditions are given in terms of the terminal price coefficients λ_{1,T_-} and λ_{2,T_-}. The boundary conditions for these will be derived in Appendix E. ■

D On the Approximation Error

In this appendix, I derive the dual counterpart to the primal problem in (31): I use the dual formulation to obtain a probabilistic characterization to the first-order Taylor approximation used in Proposition 4. Then, I use the dual approach along the lines of Haugh, Kogan, and Wang (2006) to provide an upper bound on the approximation error taking prices as given.

The primal problem in (31) may be alternatively written as

$$\sup_{\theta^l} E\left[-e^{-\gamma W^l_{T_-} + A} \middle| \mathcal{F}^l_{t_-} \right] \text{s.t. } \mathrm{d}W_t = \theta_{t_-}\mathrm{d}P_t = \theta_{t_-}(A_{Q,t}\Psi^l_{t_-}\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}^l_t) \tag{55}$$

where A is a $\mathcal{F}_{T_-}$−measurable random variable given in (52). I denote by $(\theta^\star_t)_{t\geq 0}$ the optimal portfolio strategy associated with (55) and by $V^\star_t$ the value function associated with (55) and evaluated at the optimal strategy $(\theta^\star_t)_{t\geq 0}$. Also, I denote by $(\widetilde{\theta}_t)_{t\geq 0}$ the portfolio strategy in (13) which is associated with the value function $\widetilde{V}_t$ whose jump has been linearized. Clearly, because the policy $(\widetilde{\theta}_t)_{t\geq 0}$ is suboptimal, it is immediate that

$$\widetilde{V}_t \leq V^\star_t, \ \ \forall t \in [0, T). \tag{56}$$

That is, the approximated value function $\widetilde{V}_t$ provides a lower bound to the optimal value function $V^\star_t$. To get a sense of what $V^\star_t$ and, thereby, the magnitude of the approximation error may be, it is necessary to include an upper bound on $V^\star_t$ as well. The latter is provided by duality theory: consider a fictitious market which is comprised of the original risky stock with equilibrium price P and dynamics

$$\mathrm{d}P_t = \sigma_t(\kappa_t\mathrm{d}t + \mathrm{d}\widehat{B}^l_t)$$

where $\kappa_t \equiv \frac{\mu_t}{\sigma_t}$ denotes its market price of risk. Furthermore, I introduce a market completion in the form of a fictitious asset with price S and dynamics

$$\mathrm{d}S_t = \mathrm{d}N_t^l - g_t \mathrm{d}t.$$

An agent facing such a market builds a portfolio strategy (θ, ψ) where θ and ψ denote the fractions invested in P and S, respectively. Her wealth therefore evolves according to

$$\mathrm{d}W_t = \theta_{t\_}\mathrm{d}P_t + \psi_{t\_}\mathrm{d}S_t = \theta_{t\_}\sigma_t(\kappa_t \mathrm{d}t + \mathrm{d}\widehat{B}_t^l) + \psi_{t\_}(\mathrm{d}N_t^l - g_t \mathrm{d}t). \tag{57}$$

The dual state variable $H \in \mathcal{K}$ to (57), or equivalently its Lagrange multiplier, is assumed to satisfy

$$\mathrm{d}H_t = \beta_t H_{t\_}\mathrm{d}\widehat{B}_t^l + \delta_t H_{t\_}\mathrm{d}M_t^l \tag{58}$$

where $M_t^l = N_t^l - \int_0^t \eta_s \mathrm{d}s$ is a $\widehat{\mathbb{P}}^l$−martingale and $\mathcal{K}$ is assumed to be as follows.

Assumption 1. *The space $\mathcal{K}$ is such that $(\beta, \delta) \in \mathbb{R} \times (-1, +\infty)$ are square integrable and predictable processes satisfying the conditions*

$$\int_0^t \beta_s^2 ds < \infty, \ \int_0^t \delta_s \eta_s ds < \infty \ \ \forall t \in [0, T) \text{ and such that } H_t > 0, \ E[H_t] = 1,$$
$$H_t W_t \text{ a local martingale } \ \forall t \in [0, T) \text{ and } E[H_T(\log(H_T) - A)] < \infty.$$

The dual state variable in (58) is assumed to take an exponential martingale form to enforce its positivity. Furthermore, an application of Ito's lemma yields

$$\begin{aligned}\mathrm{d}H_t W_t = &H_{t\_}\theta_{t\_}\sigma_t(\kappa_t + \beta_t)\mathrm{d}t + H_{t\_}(\theta_t\sigma_t + W_{t\_}\beta_t)\mathrm{d}\widehat{B}_t^l \\ &+ (\psi_{t\_}(1+\delta_t) + \delta_t W_{t\_})H_{t\_}\mathrm{d}N_t^l - (\psi_{t\_}g_t + W_{t\_}\delta_t\eta_t)H_{t\_}\mathrm{d}t.\end{aligned}$$

Since, from Assumption 1, $(H_t W_t)_{t\geq 0}$ is restricted to be a local martingale, it must be that $\beta_t = -\kappa_t$ and $(N_t^l)_{t\geq 0}$ should be appropriately compensated, i.e.

$$\psi_t g_t + W_t \delta_t \eta_t = \eta_t(\psi_t(1+\delta_t) + \delta_t W_t)$$

and thus $\delta_t = \frac{g_t}{\eta_t} - 1$. Therefore, any candidate dual state variable H for the above fictitious market takes the following form

$$\begin{aligned}H_T &= e^{-\frac{1}{2}\int_0^T \kappa_s^2 \mathrm{d}s - \int_0^T \kappa_s \mathrm{d}\widehat{B}_t^l} e^{\int_0^T (\eta_s - g_s)\mathrm{d}s} \prod_{0\leq s\leq t}\left(1 + \left(\frac{g_s}{\eta_s} - 1\right)\Delta N_s^l\right) \\ &= \mathcal{E}\left(-\kappa \widehat{B}^l + \left(\frac{g}{\eta} - 1\right) M^l\right)_T\end{aligned}$$

where $\mathcal{E}(\cdot)$ denotes the Doléans-Dade exponential. The remaining conditions in Assumption 1 are imposed so that H defines a proper change of measure and such that the dual problem is well-defined. The optimization problem faced by an agent in the fictitious market is then given by

$$\begin{aligned} V_t^{(g)} &\equiv \sup_{(\theta,\psi)} E\left[-e^{-\gamma W_{T_-}^{\theta,\psi}(g)+A}\right] \\ &\text{s.t. } \mathrm{d}W_t^{\theta,\psi}(g) = \theta_{t_-}\mathrm{d}P_t + \psi_{t_-}\mathrm{d}S_t^{(g)} \\ &\text{s.t. } \psi_t^\star(g) = 0 \ \ \forall t \in [0,T). \end{aligned} \tag{59}$$

That is, at the optimum, $g^\star$ is such that an agent finds it optimal not to invest in the fictitious asset. The portfolio strategies associated with the fictitious problem in (59) may be obtained as follows: by the martingale representation theorem, the discounted wealth may be written as

$$\begin{aligned} H_t W_t^{\theta,\psi} &= W_0 + \int_0^t \varphi_s \mathrm{d}\widehat{B}_s^l + \int_0^t \phi_s \mathrm{d}M_s^l \\ &= W_0 + \int_0^t H_s(\theta_s\sigma_s - W_s^{\theta,\psi}\kappa_s)\mathrm{d}\widehat{B}_s^l + \int_0^t H_s\left(\psi_s\frac{g_s}{\eta_s} + W_s^{\theta,\psi}\left(\frac{g_s}{\eta_s}-1\right)\right)\mathrm{d}M_s^l \end{aligned}$$

where (φ,ϕ) are predictable processes. Accordingly, for any arbitrary dual candidate $H^{(g)}$, the portfolio strategy is

$$\theta_t^{(g)} = (\sigma_t)^{-1}(\varphi_t^{(g)}\left(H_t^{(g)}\right)^{-1} + \kappa_t W_t^{(g)}) \text{ and } \psi_t^{(g)} = \left(\frac{g_t}{\eta_t}\right)^{-1}\left(\phi_t^{(g)}\left(H_t^{(g)}\right)^{-1} + \left(1-\frac{g_t}{\eta_t}\right)W_t^{(g)}\right). \tag{60}$$

Also, for any arbitrary candidate $H^{(g)}$, I can write the optimization problem in a static form à la Cox-Huang as

$$\sup_{W_T^{(g)}} E_0\left[-e^{-\gamma W_{T_-}^{(g)}+A}\right] \text{ s.t. } E_0\left[H_T^{(g)}W_T^{(g)}\right] \le W_0. \tag{61}$$

Solving the associated Lagrangian, the value function $V^{(g)}$ for any $H^{(g)}$ may be written as

$$V_0^{(g)} = -e^{-\gamma W_0 - E_0[H_{T_-}^{(g)}(\log(H_{T_-}^{(g)})-A)]}. \tag{62}$$

Since $W_t^{(g)} \ge W_t^\star$, (62) constitutes an upper bound to $V_t^\star$, i.e. $V_t^\star \le V_t^{(g)} \ \ \forall t \in [0,T)$, and is expected to be strictly equal to it at the optimum $V_t^\star = \inf_H V_t^{(g)}$. Hence, taking into account (56), $V_t^\star$ is bounded by

$$\widetilde{V}_t \ \ \forall(\widetilde{\theta}_t)_{t\ge 0} \text{ given} \le V_t^\star \le V_t^{(g)} \ \ \forall g > 0 \text{ given}$$

and, thus, the approximation error is bounded by

$$0 \le \left| V_t^{\star} - \widetilde{V}_t \right| \le \left| V_t^{(g)} - \widetilde{V}_t \right|$$

for given prices. The upper bound in (62) requires the computation of $E_0[H_T^{(g)}(\log(H_T^{(g)}) - A)]$: since $H \in \mathcal{K}$, H represents a change of measure and I can accordingly define the following Radon-Nikodym derivative

$$\left.\frac{\mathrm{d}\widetilde{\mathbb{P}}^l}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}_t^l} = H_t. \tag{63}$$

By Girsanov's theorem, this change of measure implies that $\widetilde{B}_t^l = \widehat{B}_t^l + \int_0^t \kappa_s \mathrm{d}s$ is a $\widetilde{\mathbb{P}}^l-$Brownian motion and

$$\widetilde{M}_t^l = M_t^l - \int_0^t (g_s - \eta_s)\mathrm{d}s$$

is a $\widetilde{\mathbb{P}}^l-$martingale and $(\widetilde{N}_t^l)_{t\ge 0}$ is a $\widetilde{\mathbb{P}}^l-$Poisson process with intensity g_t. Hence,

$$E_0[H_T^{(g)}(\log(H_T^{(g)}) - A)] = \widetilde{E}_0[\log(H_T^{(g)})] - \widetilde{E}_0[A].$$

Moreover, applying the change of measure in (63) and Ito's lemma shows that

$$\mathrm{d}\log(H_t^{(g)}) = \frac{1}{2}\kappa_t^2 \mathrm{d}t - \kappa_t \mathrm{d}\widetilde{B}_t^l - \eta_t \left(\frac{g_t}{\eta_t} - 1\right)\mathrm{d}t + \log\left(\frac{g_t}{\eta_t}\right)\mathrm{d}\widetilde{N}_t^l$$

such that

$$\widetilde{E}_0[\log(H_T^{(g)})] = \frac{1}{2}\widetilde{E}_0\left[\int_0^T \kappa_s^2 \mathrm{d}t\right] + \widetilde{E}_0\left[\int_0^T \left(\log\left(\frac{g_t}{\eta_t}\right) g_t - g_t + \eta_t\right)\mathrm{d}t\right].$$

To actually compute an upper bound, I further need to pick a particular g: one such choice includes $g = \eta$ as a natural candidate for reasons that will become clear shortly. With this candidate at hand, the dual state variable $H_t^{(\eta)} = \mathcal{E}(-\kappa\widehat{B}^l)_t$ does not jump and the upper bound reduces to

$$V_0^{(\eta)} = -e^{-\gamma W_0 - \widetilde{E}_0[\frac{1}{2}\int_0^T \kappa_s^2 \mathrm{d}s - A]} \equiv -e^{-\gamma W_0 - \frac{1}{2}f(\Psi^l, n, 0)}.$$

Recalling that the market price of risk is given by $\kappa_t = \frac{A_{Q,t}\Psi_t^l}{B_{Q,t}}$ and that the change of measure in (63) implies that $(\Psi^l)_{t\ge 0}$ still evolves as an OU process under $\widetilde{\mathbb{P}}^l$

$$\mathrm{d}\Psi_t^l = \left(A_{\Psi,t} - B_{\Psi,t}\frac{A_{Q,t}}{B_{Q,t}}\right)\Psi_{t\_}^l \mathrm{d}t + B_{\Psi,t}\mathrm{d}\widetilde{B}_t^l + C_{\Psi,t}\mathrm{d}N_t^l$$

where the particular choice of $g = \eta$ implies that $(N_t^l)_{t\geq 0}$ is the same Poisson process under both measures, I can write the following PDE for the function f

$$0 = \left(\frac{A_{Q,t}\Psi_t^l}{B_{Q,t}}\right)^2 + f_t + (f_\Psi)^\top \left(A_{\Psi,t} - B_{\Psi,t}\frac{A_{Q,t}}{B_{Q,t}}\right)\Psi_t^l + \frac{1}{2}\mathrm{tr}\left(J_{\Psi\Psi}B_{\Psi,t}B_{\Psi,t}^\top\right)$$
$$+ E^\nu\left[f(\Psi_{t_-}^l + C_{\Psi,t}, n+m, t) - f(\Psi_{t_-}^l, n, t)\right]$$

with boudary condition $f(\Psi^l, n, T) = A$. Given that $(\Psi^l)_{t\geq 0}$ follows a Gaussian process under $\widetilde{\mathbb{P}}^l$ and that f is quadratic, one may conjecture that $f(\Psi^l, n, t) \equiv (\Psi^l)^\top R_t(n)\Psi^l$ and substitution of this conjecture into the above PDE after separation of variables yields (41) and inspection of A, implies that $R_t(n) = M_t(n)$, or

$$V_t^{(\eta)} = -e^{-\gamma W_t - \frac{1}{2}(\Psi_t^l)^\top M_t(n)\Psi_t^l}$$

which is nothing but the conjecture in (39). One may then obtain the unconditional upper bound at time 0, by integrating out the vector Ψ_0^l. To that purpose, one should notice that the latter is distributed as $\Psi_0^l \sim \mathcal{N}(\mathbf{0}, \Sigma_0)$ with

$$\Sigma_0 = \begin{bmatrix} \frac{\sigma_\Theta^2}{2a_\Theta} - \left(\frac{\lambda_{1,0}}{\lambda_{2,0}}\right)^2 \frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} & \frac{\lambda_{1,0}}{\lambda_{2,0}}\frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} \\ \frac{\lambda_{1,0}}{\lambda_{2,0}}\frac{\sigma_\Pi^4}{\sigma_\Pi^2+\sigma_S^2} & \frac{\sigma_\Pi^2\sigma_S^2}{\sigma_\Pi^2+\sigma_S^2} \end{bmatrix}.$$

Therefore,

$$E^{\Psi_0^l}\left[V_0^{(\eta)}\right] = -e^{-\gamma W_0 - \frac{1}{2}M_0^{l,(1,1)}(1)} \int_{\mathbb{R}^2} e^{-\frac{1}{2}\left(\Psi_0^{(\star,l)}\right)^\top M_0^{\star,l}(1)\Psi_0^{(\star,l)}} \mathrm{d}\Phi\left(\Psi_0^{(\star,l)}\right)$$
$$= -e^{-\gamma W_0 - \frac{1}{2}M_0^{l,(1,1)}(1)}\left|I + \Sigma_0 M_0^{\star,l}(1)\right|^{-\frac{1}{2}}$$

where the second equality follows from completing the square in the bivariate normal distribution. Given this explicit form for the upper bound, a probabilistic interpretation to the linearization is obtained by inspection of the portfolio strategy implied by $g = \eta$: the static problem in (61) induces the first-order condition $U'(W_T) = \lambda_t \frac{H_T^{(\eta)}}{H_t^{(\eta)}}$ where λ_t denotes the Lagrange multiplier of the static constraint at time t. Furthermore, the envelope theorem implies that $\frac{\partial V_t^{(\eta)}}{\partial W_t} = \lambda_t$. Thus, $\frac{H_T^{(\eta)}}{H_t^{(\eta)}} = U'(W_T)\left(\frac{\partial V_t^{(\eta)}}{\partial W_t}\right)^{-1}$ and, as a result, $\frac{H_s^{(\eta)}}{H_t^{(\eta)}} = \frac{\partial V_s^{(\eta)}}{\partial W_s} / \frac{\partial V_t^{(\eta)}}{\partial W_t}$ $\forall s \geq t$, which finally implies that $\mathrm{d}\log(H_t^{(\eta)}) = \mathrm{d}\log\left(\frac{\partial V_t^{(\eta)}}{\partial W_t}\right)$. Matching the diffusion terms yields

$$\theta_t^{(\eta)} = -(\sigma_t)^{-1}\left(\frac{\partial^2 V_t}{\partial W_t^2}\right)^{-1}\left(\frac{\partial V_t}{\partial W_t}\kappa_t + \left(\frac{\partial^2 V_t}{\partial W_t \partial \Psi_t^l}\right)^\top B_{\Psi,t}\right),$$

which, after substitution of the explicit upper bound $V_t^{(\eta)}$, needless to say, yields the portfolio policy in (13). Further matching the jump terms yields

$$0 = \log\left(V_t^{(\eta)}(W_{t_-} + \psi_t, \Psi_{t_-}^l + C_{\Psi,t})\right) - \log\left(V_t^{(\eta)}(W_{t_-}, \Psi_{t_-}^l)\right).$$

Substituting the explicit form and using the portfolio policy in (60) shows that

$$\psi_t^{(\eta)} \equiv \phi_t^{(\eta)}\left(H_t^{(\eta)}\right)^{-1} = -\frac{1}{2\gamma}\left(\left(\Psi_{t_-}^l + C_{\Psi,t}\right)^\top M_t(n+m)\left(\Psi_{t_-}^l + C_{\Psi,t}\right) - \left(\Psi_{t_-}^l\right)^\top M_t(n)\Psi_{t_-}^l\right).$$

Hence, the approximate policy $(\widetilde{\theta}_t)_{t\geq 0}$ boils down to pick the strategy $(\theta_t^{(\eta)})_{t\geq 0}$ and make it an optimal one by setting $\psi_t^{(\eta)} \approx 0$ since, as per the fictitious problem in (59), an optimal strategy must be so that $\psi_t = 0$. Alternatively, the Taylor approximation of the jump size tantamount to set the loading ϕ_t on the Poissonian risk in the martingale representation of the discounted wealth to zero.

I finally compute the value function $\widetilde{V}_t$ associated with the approximate strategy $(\theta_t^{(\eta)})_{t\geq 0}$ and $\psi_t = 0$. Since the approximate wealth satisfies

$$\widetilde{W}_T = W_0 + \int_0^T \theta_s^{(\eta)} \mathrm{d}P_s = W_T^{(\eta)} - \int_0^T \psi_s \mathrm{d}S_s^{(\eta)},$$

it remains to compute

$$\widetilde{V}_0 = E_0\left[-e^{-\gamma\left(W_0 + \int_0^T \theta_s^{(\eta)}\sigma_s(\kappa_s \mathrm{d}s + \mathrm{d}\widehat{B}_s^l)\right) + A}\right].$$

I introduce the following change of measure

$$\left.\frac{\mathrm{d}\bar{\mathbb{P}}^l}{\mathrm{d}\widehat{\mathbb{P}}^l}\right|_{\mathcal{F}_t^l} = e^{-\gamma\int_0^T \theta_s^{(\eta)}\sigma_s \mathrm{d}\widehat{B}_s^l - \frac{1}{2}\gamma^2\int_0^T\left(\theta_s^{(\eta)}\sigma_s\right)^2 \mathrm{d}s} \tag{64}$$

such that

$$\widetilde{V}_0 = -e^{-\gamma W_0}\bar{E}_0\left[e^{-\gamma\int_0^T \theta_s^{(\eta)}\sigma_s\left(\kappa_s - \frac{1}{2}\gamma\theta_s^{(\eta)}\sigma_s\right)\mathrm{d}s + A}\right] \equiv -e^{-\gamma W_0}h(\Psi^l, n, 0).$$

Recalling that $\theta_t^{(\eta)} = \frac{A_{Q,t} - B_{Q,t}B_{\Psi,t}^\top M_t^l(n)}{\gamma B_{Q,t}}\Psi_t^l$, I can write, after simplifications,

$$h(\Psi^l, n, 0) = \bar{E}_0\left[e^{-\frac{1}{2}\int_0^T\left(\Psi_t^l\right)^\top \frac{A_{Q,t}^\top A_{Q,t} - B_{Q,t}^2 M_t^l(n) B_{\Psi,t} B_{\Psi,t}^\top M_t^l(n)}{B_{Q,t}^2}\Psi_t^l \mathrm{d}t + A}\right]. \tag{65}$$

The computation of the function $h(\cdot)$ is generally complicated precisely because it involves a quadratic jump. Therefore, for the purpose of computing the lower bound, I resort to Monte Carlo Simulations. I simulate the dynamics of Ψ^l under $\bar{\mathbb{P}}^l$ using a standard Euler scheme. Given the change of measure in (64), $\bar{B}^l_t = \widehat{B}^l + \gamma \int_0^t \theta_s^{(\lambda)} \sigma_s \mathrm{d}s$ is a $\bar{\mathbb{P}}^l-$Brownian motion by Girsanov's theorem and Ψ^l satisfies

$$\mathrm{d}\Psi^l_t = \left(A_{\Psi,t} - B_{\Psi,t} \frac{A_{Q,t} - B_{Q,t} B^\top_{\Psi,t} M^l_t(n)}{B_{Q,t}} \right) \Psi^l_{t\_} \mathrm{d}t + B_{\Psi,t} \mathrm{d}\bar{B}^l_t + C_{\Psi,t} \mathrm{d}N^l_t.$$

I further draw m out of the cross-sectional distribution of types by drawing a uniform random variable according to $\mathcal{U}_{[0,1]}$ and cumulate μ_t. I then compute the integral in (65) by means of the trapezoid rule.

E Proof of Proposition 5

In this appendix, I derive the equilibrium equations for the price coefficients λ_1 and λ_2. To that purpose, I first observe that, from (13), individual portfolios take the form

$$\theta^l(\Psi^l_t, n^l_t, t) = d^l_{\Theta,t}(n^l_t) \widehat{\Theta}^l_t + d^l_{\Delta,t}(n^l_t) \Delta^l_t. \tag{66}$$

Aggregating the latter over the population of agents first calls for the average beliefs $\int_{j \in I} \widehat{\Pi}^j_t \mathrm{d}\iota(j)$. I recall that, given that $\xi_t \in \mathcal{F}^c_t \subseteq \mathcal{F}^l_t$, I can write

$$\xi_t = \lambda_{1,t} \Pi + \lambda_{2,t} \Theta_t \equiv \lambda_{1,t} \widehat{\Pi}^c_t + \lambda_{2,t} \widehat{\Theta}^c_t \equiv \lambda_{1,t} \widehat{\Pi}^l_t + \lambda_{2,t} \widehat{\Theta}^l_t.$$

It then follows that $\widehat{\Theta}^c_t = \frac{\xi_t - \lambda_{1,t} \widehat{\Pi}^c_t}{\lambda_{2,t}}$ and $\widehat{\Theta}^l_t = \frac{\xi_t - \lambda_{1,t} \widehat{\Pi}^l_t}{\lambda_{2,t}}$: using these expressions along with equation (9) for the filter $\widehat{\Pi}^c_t$ of Proposition 3, an application of Ito's lemma delivers

$$\mathrm{d}\frac{\widehat{\Pi}^c_t}{o^c_t} = k_t \mathrm{d}\widehat{B}^c_t + k^2_t \widehat{\Pi}^c_t \mathrm{d}t = k_t \mathrm{d}\widehat{B}^l_t + k^2_t \widehat{\Pi}^l_{t\_} \mathrm{d}t \tag{67}$$

where the second equality follows from the change of measure in (33). Similarly, using (10), one may write

$$\mathrm{d}\frac{\widehat{\Pi}^l_t}{o^l_t} = k_t \mathrm{d}\widehat{B}^l_t + k^2_t \widehat{\Pi}^l_{t\_} \mathrm{d}t + \bar{S}^l_{m,t} \frac{m_t}{\sigma^2_S} \mathrm{d}N^l_t \tag{68}$$

where the jump size directly follows from (26). Bunching (67) and (68) together, I can write

$$\mathrm{d}\left(\frac{\widehat{\Pi}^l_t}{o^l_t\left(n^l_t\right)} - \frac{\widehat{\Pi}^c_t}{o^c_t} \right) = \frac{\bar{S}^l_{m,t}}{\sigma^2_S} m_t \mathrm{d}N^l_t.$$

This equation may then be solved using the initial conditions $\widehat{\Pi}^c_0 = 0$ and $o^c_0 = \sigma^2_\Pi$ and applying the updating rule in (26) to $\widehat{\Pi}^l_0$. Doing so, I obtain the following lemma.

Lemma 1. *A manager l's expectations $\widehat{\Pi}^l$ satisfies*

$$\widehat{\Pi}_t^l = \frac{o_t^l\left(n_t^l\right)}{o_t^c}\widehat{\Pi}_t^c + \widehat{\Pi}_0^l \frac{o_t^l\left(n_t^l\right)}{o_0^l(1)} + \frac{o_t^l\left(n_t^l\right)}{\sigma_S^2}\int_0^t \bar{S}_{m,t}^l m_s dN_s^l = \frac{o_t^l\left(n_t^l\right)}{o_t^c}\widehat{\Pi}_t^c + \frac{o_t^l\left(n_t^l\right)}{\sigma_S^2}\bar{S}_{n,t}^l n_t^l \tag{69}$$

taking into account that $\mathrm{d}n_t^l = (m_t \mathbf{1}_{\{m_t+n_{t\_}^l \in A\cup B\}} + \infty \mathbf{1}_{\{m_t+n_{t\_}^l \notin A\cup B\}})\mathrm{d}N_t^l, n_0^l = 1$ when integrating. Moreover, recalling that $o_t^l = \frac{o_t^c \sigma_S^2}{\sigma_S^2 + n_t^l o_t^c}$ and that the law of large numbers implies $\int_{j_t\in I_2} \bar{S}_{n,t}^j \mathrm{d}\iota(j_t) = \omega_t \Pi$ for all n, I finally write the average beliefs as

$$\begin{aligned}\int_{j_t\in I} \widehat{\Pi}_t^j \mathrm{d}\iota(j_t) &= \int_{j_t\in I_2}\left(\frac{o_t^j\left(n_t^l\right)}{o_t^c}\widehat{\Pi}_t^c + \frac{o_t^j\left(n_t^l\right)}{\sigma_S^2}\bar{S}_{n,t}^j n_t^l\right)\mathrm{d}\iota(j_t) + (1-\omega_t)\Pi \\ &= \sum_{n\in A\cup B}\mu_t(n)\left(\frac{\sigma_S^2}{\sigma_S^2 + n o_t^c}\widehat{\Pi}_t^c + \frac{o_t^c n}{\sigma_S^2 + n o_t^c}\Pi\right) + (1-\omega_t)\Pi \\ &\equiv \alpha_t \widehat{\Pi}_t^c + (1-\alpha_t)\Pi\end{aligned} \tag{70}$$

where, similar to He and Wang (1995), $\alpha_t \equiv \sum_{n\in A\cup B}\mu_t(n)\frac{\sigma_S^2}{\sigma_S^2 + n o_t^c}$ and where

$$\mu_t(n) = \begin{cases} \mu_t^A(n) & \text{if } n\in A \\ \mu_t^B(n) & \text{if } n\in B \end{cases}.$$

Equation (70) allows to characterize the impact of percolation on higher-order beliefs (HOB). Since beliefs of higher order collapse to a linear combination of the first-order expectations and denoting agent l's expectation of $\int_{j_t\in I}\widehat{\Pi}_t^j \mathrm{d}\iota(j_t)$ by $\widehat{\Pi}_t^{(l,2)} \equiv E[\int_{j_t\in I}\widehat{\Pi}_t^j \mathrm{d}\iota(j_t)|\mathcal{F}_t^l]$, one may readily observe that $\widehat{\Pi}_t^{(l,2)} = \alpha_t\widehat{\Pi}_t^c + (1-\alpha_t)\widehat{\Pi}_t^l$ in such a way that the second-order expectation writes

$$\widehat{\Pi}_t^{(2)} \equiv \int_{j_t\in I}\widehat{\Pi}_t^{(j,2)}\mathrm{d}\iota(j_t) = (1-(1-\alpha_t)^2)\widehat{\Pi}_t^c + (1-\alpha_t)^2\Pi.$$

Iterating, the $k-$th order expectation $\widehat{\Pi}_t^{(k)}$ writes

$$\widehat{\Pi}_t^{(k)} = \int_{j_t\in I}\widehat{\Pi}_t^{(j,k)}\mathrm{d}\iota(j_t) = (1-(1-\alpha_t)^k)\widehat{\Pi}_t^c + (1-\alpha_t)^k\Pi.$$

Observing that the weight α_t is increasing in the cross-sectional average number of signals, and that the latter is roughly increasing like $e^{\eta t}$, the following pattern occurs. Because information percolation produces some inertia in updating initially, HOB play a stronger role at the beginning of the economy as compared with a setup in which signals continuously accrue. Yet, after some time, percolation takes off and signals accrue at a rate beyond that of a continuous flow of signals. This ultimately causes the effect of HOB to decrease after some time has elapsed.

This also allows to show how the conjecture for linear prices in Definition 1 obtains. Following Hong and Wang (2000), the price may be written as

$$P_t = \int_{j_t \in I} E[F_t | \mathcal{F}_t^j] \mathrm{d}\iota(j_t) + \lambda_{2,t}\Theta_t$$

with $F_t = E\left[e^{-r(T-t)}(\Pi + \delta) \middle| \mathcal{F}_t\right] = \Pi + \delta$ denoting the expected value of the terminal dividend under full information $\mathcal{F}_t$ discounted at the riskfree rate and where the second equality follows from $r = 0$. The term $\lambda_{2,t}\Theta_t$ represents a discount for inventory risk. Notice that $\widehat{\Theta}_t^c$ does not appear in the price as ξ_t implies that

$$\lambda_{1,t}(\Pi - \widehat{\Pi}_t^c) = \lambda_{2,t}(\Theta_t - \widehat{\Theta}_t^c)$$

since $E[\xi_t | \mathcal{F}_t^c] = \lambda_{1,t}\widehat{\Pi}_t^c + \lambda_{2,t}\widehat{\Theta}_t^c$ and, given $\xi_t \subseteq \mathcal{F}_t^c$, $E[\xi_t | \mathcal{F}_t^c] = \xi_t$. Putting everything together and taking into account that all agents have null prior regarding δ, I can write

$$P_t = (1 - \alpha_t)\Pi + \alpha_t \widehat{\Pi}_t^c + \lambda_{2,t}\Theta_t$$

and the conjecture in (6) follows. This expression should be considered to hold on $[0, T)$ so as to allow a discontinuity in the stock price at the horizon when it attains the final payoff $\Pi + \delta$.

Using (70), I can further compute the average beliefs regarding the supply

$$\int_{j_t \in I} \widehat{\Theta}_t^j \mathrm{d}\iota(j_t) = \frac{\lambda_{1,t}}{\lambda_{2,t}}\Pi + \Theta_t - \frac{\lambda_{1,t}}{\lambda_{2,t}} \int_{j_t \in I} \widehat{\Pi}_t^j \mathrm{d}\iota(j_t) = \Theta_t + \alpha_t \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^c).$$

Applying the law of large numbers, I then obtain the aggregate demand

$$\int_{j_t \in I} \theta_t^j \mathrm{d}\iota(j_t) = \sum_{n \in A \cup B} \mu_t(n) \begin{pmatrix} \frac{A_Q^{(2)} - B_Q D_l^{(2)}(n)}{\gamma B_Q^2}\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}\frac{\sigma_S^2}{\sigma_S^2 + o_t^c n}(\Pi - \widehat{\Pi}_t^c)\right) \\ + \frac{A_Q^{(3)} - B_Q D_l^{(3)}(n)}{\gamma B_Q^2}\frac{o_t^c n}{\sigma_S^2 + o_t^c n}(\Pi - \widehat{\Pi}_t^c)) \end{pmatrix} \\ + (1 - \omega_t)\left(\frac{A_Q^{(2)} - B_Q D_i^{(2)}}{\gamma B_Q^2}\Theta_t + \frac{A_Q^{(3)} - B_Q D_i^{(3)}}{\gamma B_Q^2}(\Pi - \widehat{\Pi}_t^c)\right) \quad (71)$$

where $D_j = B_\Psi^j M^j(t)$ for $j = l, i$. Clearing the markets yields

$$A_Q^{\star,(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}A_Q^{\star,(1)} = \begin{pmatrix} -\frac{\lambda_{1,t}}{\lambda_{2,t}}\gamma B_Q^2 + \sum_{n \in A \cup B}\mu_t(n) B_Q\left(D_l^{\star,(2)}(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}}D_l^{\star,(1)}(n)\right)\frac{o_t^c n}{\sigma_S^2 + o_t^c n} \\ +(1 - \omega_t)B_Q\left(D_i^{\star,(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}D_i^{\star,(1)}\right) \end{pmatrix}$$

$$\times \frac{1}{\sum_{n \in A \cup B}\mu_t(n)\frac{o_t^c n}{\sigma_S^2 + o_t^c n} + 1 - \omega_t} \quad (72)$$

and,

$$\sum_{n\in A\cup B}\mu_t(n)\frac{A_Q^{\star,(1)}-B_QD_l^{\star,(1)}(n)}{\gamma B_Q^2}+(1-\omega_t)\frac{A_Q^{\star,(1)}-B_QD_i^{\star,(1)}}{\gamma B_Q^2}=1. \tag{73}$$

These equilibrium equations are obtained from the market-clearing condition $\int_{j_t\in I}\theta_t^j \mathrm{d}\iota(j_t)=\Theta_t$ and $A_Q^\star$, $A_\Psi^\star$ and $B_\Psi^{\star,j}$ in (46) and (72) follows from (73). Both equations (72) and (73) determine the equilibrium behavior of $\lambda_{1,t}$ and $\lambda_{2,t}$ over $[0,T)$. Yet, they lack boundary conditions: at the very date before the economy ends, agents hold myopic portfolios of the kind in (50). Aggregating these, I obtain

$$\begin{aligned}\int_{j_T\in I}\theta_{T_-}^j \mathrm{d}\iota(j_T) &= \int_{j_T\in I_2}\frac{\widehat{\Pi}_{T_-}^j-P_{T_-}}{\gamma(o_{T_-}^j+\sigma_\delta^2)}\mathrm{d}\iota(j_T)+(1-\omega_T)\frac{\Pi-P_{T_-}}{\gamma\sigma_\delta^2}\\ &=\sum_{n\in A\cup B}\mu_T(n)\frac{\frac{\sigma_S^2}{\sigma_S^2+o_{T_-}^c n}\widehat{\Pi}_{T_-}^c+\frac{o_{T_-}^c n}{\sigma_S^2+o_{T_-}^c n}\Pi-P_{T_-}}{\gamma(o_{T_-}^l(n)+\sigma_\delta^2)}+(1-\omega_T)\frac{\Pi-P_{T_-}}{\gamma\sigma_\delta^2}=\Theta_T.\end{aligned}$$

Using that

$$P_{T_-}=\lambda_{1,T_-}\Pi+(1-\lambda_{1,T_-})\widehat{\Pi}_{T_-}^c+\lambda_{2,T_-}\Theta_T$$

and simplifying yields

$$\begin{aligned}\lambda_{1,T_-}=&\left(\sum_{n\in A\cup B}\mu_T(n)\frac{(o_{T_-}^c n+\sigma_S^2)\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)^{-1}\\ &\times\left(\sum_{n\in A\cup B}\mu_T(n)\frac{o_{T_-}^c n\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)\end{aligned} \tag{74}$$

and

$$\lambda_{2,T_-}=-\gamma\sigma_\delta^2\left(\sum_{n\in A\cup B}\mu_T(n)\frac{(o_{T_-}^c n+\sigma_S^2)\sigma_\delta^2}{\sigma_S^2 o_{T_-}^c+\sigma_\delta^2(o_{T_-}^c n+\sigma_S^2)}+1-\omega_T\right)^{-1}. \tag{75}$$

This provides the two required boundary conditions associated with (72) and (73) and takes care of the final jump in prices. Indeed, knowing λ_{1,T_-} and λ_{2,T_-} and, thus, P_{T_-}, the jump size is then simply obtained as $\Delta P_T=\Pi+\delta-P_{T_-}$. Substituting (74) and (75) into (53) and (54) makes all the boundary conditions depending on $o_{T_-}^c$ which represents the only unknown terminal condition, hence the shooting method suggested in the main text in Subsection 3.2.

When solving the equilibrium, one needs to jointly solve the HJB equations in (47) and (48) in Proposition 4 along with the price equations (72) and (73) and the equation (12) for the common variance in Proposition 3. The resulting system of differential equations is not explicit, for the derivatives $\lambda_{1,t}'$ and $\lambda_{2,t}'$ appear through the coefficient k_t. The solving procedure is alleviated if one is able to transform the system of equations to be solved into an explicit one. As it turns out, a trick which works as a continuous-time dynamic equivalent of Admati (1985)'s lemma is available: considering the second equilibrium equation in (73) and observing that the derivatives of $\lambda_{1,t}$ and $\lambda_{2,t}$ are contained in $A_{Q,t}$, one may spell out the left-hand side of (72) and write

$$\begin{aligned} A^{\star,(2)}_{Q,t} - \frac{\lambda_{1,t}}{\lambda_{2,t}} A^{\star,(1)}_{Q,t} &= \lambda'_{1,t} + (1-\lambda_{1,t}) k_t^2 o_t^c - \frac{\lambda_{1,t}}{\lambda_{2,t}} (\lambda'_{2,t} - a_\Theta \lambda_{2,t}) \\ &= \lambda_{2,t} \sigma_\Theta k_t + (1-\lambda_{1,t}) k_t^2 o_t^c. \end{aligned} \tag{76}$$

Substituting this expression in place of the left-hand side of (72) produces a quadratic equation in k_t. The latter has two real roots, one of which is of the form $k_t = \frac{\sigma_\Theta \lambda_{2,t}}{o_t^c(\lambda_{1,t}-1)}$. This root may be verified, by substitution into the equilibrium equations, to correspond to the fully-revealing equilibrium. That is, one in which the diffusion of the price P_t is constantly null over $[0,T)$ and the price thus reveals Π. As this equilibrium is trivial, I shall discard it and for obvious reasons only consider the second root. Substituting the second root allows to make the system of equilibrium equations explicit and to considerably facilitate the numerical implementation of the equilibrium. ■

F Serial Correlation, Trading Strategies, and Measures of Performance

In this appendix, I compute the different quantities plotted in the results sections.

F.1 Serial Correlation

Integrating the supply in (1) along with common expectations in (9), I can write

$$\Theta_t = \Theta_0 e^{-a_\Theta t} + \sigma_\Theta \int_0^t e^{a_\Theta (s-t)} \mathrm{d}B_s^\Theta, \tag{77}$$

$$\widehat{\Pi}_t^c = \Pi \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \equiv E[\widehat{\Pi}_t^c | \Pi] + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \tag{78}$$

where I used that $\widehat{\Pi}_0^c = 0$. Accordingly, P_t may be written as

$$\begin{aligned} P_t = {} & \lambda_{1,t}\Pi + \lambda_{2,t} \left(\Theta_0 e^{-a_\Theta t} + \sigma_\Theta \int_0^t e^{a_\Theta (s-t)} \mathrm{d}B_s^\Theta \right) \\ & + (1-\lambda_{1,t}) \left(\Pi \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}s + \int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \mathrm{d}B_s^\Theta \right). \end{aligned}$$

As a result, the price difference $\Delta P_t := P_{t+\Delta} - P_t$ over $[t, t+\Delta]$ is given by

$$\begin{aligned}
\Delta P_t = &\left[\begin{array}{c} \lambda_{1,t+\Delta} - \lambda_{1,t} + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right)\int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t+\Delta})\int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]\Pi \\
&+ \int_0^t \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right)\mathrm{d}B_s^\Theta \\
&+ \int_t^{t+\Delta}\left(\sigma_\Theta \lambda_{2,t+\Delta}e^{a_\Theta(s-(t+\Delta))} + (1-\lambda_{1,t+\Delta})o_s^c k_s e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\right)\mathrm{d}B_s^\Theta \\
&+ (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{-a_\Theta t}\Theta_0.
\end{aligned}$$

Similarly, I can compute the price difference $\Delta P_{t-\Delta}$ over $[t-\Delta, t]$ and get

$$\begin{aligned}
&\mathrm{var}(\Delta P_{t-\Delta}) = (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})^2 e^{-2a_\Theta(t-\Delta)}\frac{\sigma_\Theta^2}{2a_\Theta} + \\
&\left[\begin{array}{c} \lambda_{1,t} - \lambda_{1,t-\Delta} + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right)\int_0^{t-\Delta} o_s^c k_s^2 e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t})\int_{t-\Delta}^{t} o_s^c k_s^2 e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]^2 \sigma_\Pi^2 \\
&+ \int_0^{t-\Delta} \left(\begin{array}{c} (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{a_\Theta(s-(t-\Delta))}\sigma_\Theta \\ + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right) o_s^c k_s e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u} \end{array}\right)^2 \mathrm{d}s \\
&+ \int_{t-\Delta}^{t}\left(\sigma_\Theta \lambda_{2,t}e^{a_\Theta(s-t)} + (1-\lambda_{1,t})o_s^c k_s e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\right)^2\mathrm{d}s, \text{ along with} \\
&\mathrm{cov}(\Delta P_{t+\Delta}, \Delta P_{t-\Delta}) = (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{-a_\Theta t}(\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{-a_\Theta(t-\Delta)}\frac{\sigma_\Theta^2}{2a_\Theta} + \\
&\left[\begin{array}{c} \lambda_{1,t+\Delta} - \lambda_{1,t} + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right)\int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t+\Delta})\int_t^{t+\Delta} o_s^c k_s^2 e^{-\int_s^{t+\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right] \\
&\times \left[\begin{array}{c} \lambda_{1,t} - \lambda_{1,t-\Delta} + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right)\int_0^{t-\Delta} o_s^c k_s^2 e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \\ +(1-\lambda_{1,t})\int_{t-\Delta}^{t} o_s^c k_s^2 e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right]\sigma_\Pi^2 \\
&+ \int_0^{t-\Delta} \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right) \\
&\times \left(\begin{array}{c} (\lambda_{2,t}e^{-a_\Theta \Delta} - \lambda_{2,t-\Delta})e^{a_\Theta(s-(t-\Delta))}\sigma_\Theta \\ + \left((1-\lambda_{1,t})e^{-\int_{t-\Delta}^{t} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t-\Delta})\right) o_s^c k_s e^{-\int_s^{t-\Delta} o_u^c k_u^2 \mathrm{d}u} \end{array}\right)\mathrm{d}s \\
&+ \int_{t-\Delta}^{t} \left(\begin{array}{c} (\lambda_{2,t+\Delta}e^{-a_\Theta \Delta} - \lambda_{2,t})e^{a_\Theta(s-t)}\sigma_\Theta \\ + \left((1-\lambda_{1,t+\Delta})e^{-\int_t^{t+\Delta} o_u^c k_u^2 \mathrm{d}u} - (1-\lambda_{1,t})\right) o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u} \end{array}\right) \\
&\times \left(\sigma_\Theta \lambda_{2,t}e^{a_\Theta(s-t)} + (1-\lambda_{1,t})o_s^c k_s e^{-\int_s^{t} o_u^c k_u^2 \mathrm{d}u}\right)\mathrm{d}s
\end{aligned}$$

where the variance of $\Delta P_{t-\Delta}$ and the covariance between $\Delta P_{t-\Delta}$ and ΔP_t follow from Ito isometry. The econometrician then computes the serial correlation of stock returns by projecting ΔP_t onto $\Delta P_{t-\Delta}$. Applying the projection theorem, it follows that

$$E[\Delta P_t|\Delta P_{t-\Delta}] = \beta_t(\Delta)\Delta P_{t-\Delta}$$

where, as in Wang (1993), $\beta_t(\Delta) = \frac{\text{cov}(\Delta P_t, \Delta P_{t-\Delta})}{\text{var}(\Delta P_{t-\Delta})}$. Hence, as in Banerjee, Kaniel, and Kremer (2009), returns exhibit momentum whenever $\beta_t(\Delta) > 0$ and reversal whenever $\beta_t(\Delta) < 0$. This is the measure often used in the empirical literature as in Jegadeesh and Titman (1993), for instance.

F.2 Trading Strategies

From (66) and using the relation in (69) along with $\widehat{\Theta}_t^l = \Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^l)$, the portfolio strategy of an agent l holding n signals may be re-expressed as

$$\begin{aligned}\theta_t^l(n) &= d_{\Theta,t}^l(n)\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \alpha_t^l(n)\widehat{\Pi}_t^c - (1-\alpha_t^l(n))\bar{S}_{n,t}^l)\right) \\ &\quad + d_{\Delta,t}^l(n)(1-\alpha_t^l(n))(\bar{S}_{n,t}^l - \widehat{\Pi}_t^c) \\ &= d_{\Theta,t}^l(n)\Theta_t + \varphi_t^l(n)(\Pi - \widehat{\Pi}_t^c) + (\varphi_t^l(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^l(n))\frac{1}{n}\sum_{k=1}^n \epsilon^k\end{aligned} \tag{79}$$

where

$$\varphi_t^l(n) \equiv \alpha_t^l(n)\frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^l(n) + (1-\alpha_t^l(n))d_{\Delta,t}^l(n)$$

and where $\alpha_t^l(n) \equiv \frac{\sigma_S^2}{\sigma_S^2 + o_t^c n}$.

As in Brennan and Cao (1997) or He and Wang (1995), I further isolate the part of θ^l that is solely associated with private information. Since, from (71), agent l contributes a fraction $\mu_t(n)d_{\Theta,t}^l(n)$ to the per capita supply shock, I denote by $\widetilde{\theta}^l := \theta_t^l(n) - d_{\Theta,t}^l(n)\Theta_t$, the part of agent l's portfolio that is absent of market-making concerns. From (78), it then follows that

$$\begin{aligned}\widetilde{\theta}_t^l(n) &= \varphi_t^l(n)\left(1 - \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)\Pi - \varphi_t^l(n)\int_0^t o_s^c k_s e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}B_s^\Theta \\ &\quad + (\varphi_t^l(n) - \frac{\lambda_{1,t}}{\lambda_{2,t}} d_{\Theta,t}^l(n))\frac{1}{n}\sum_{k=1}^n \epsilon^k.\end{aligned}$$

Further assuming, similar to Watanabe (2008), that agent l remains of type n over Δ and dropping the index n for convenience, the informational portfolio variation over Δ

is given by

$$\Delta\widetilde{\theta}^l_t = \begin{pmatrix} \varphi^l_{t+\Delta} - \varphi^l_t + \left(\varphi^l_{t+\Delta} e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - \varphi^l_t\right) \int_0^t o^c_s k^2_s e^{-\int_s^t o^c_u k^2_u \mathrm{d}u} \mathrm{d}s \\ -\varphi^l_{t+\Delta} \int_t^{t+\Delta} o^c_s k^2_s e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} \mathrm{d}s \end{pmatrix} \Pi$$
$$- \left(\varphi^l_{t+\Delta} e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - \varphi^l_t\right) \int_0^t e^{-\int_s^t o^c_u k^2_u \mathrm{d}u} o^c_s k_s \mathrm{d}B^\Theta_s - \varphi^l_{t+\Delta} \int_t^{t+\Delta} e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} o^c_s k_s \mathrm{d}B^\Theta_s$$
$$+ \left(\varphi^l_{t+\Delta} - \varphi^l_t - \left(d^l_{\Theta,t+\Delta} \frac{\lambda_{1,t+\Delta}}{\lambda_{2,t+\Delta}} - d^l_{\Theta,t} \frac{\lambda_{1,t}}{\lambda_{2,t}}\right)\right) \frac{1}{n} \sum_{k=1}^n \epsilon^k$$

The econometrician, who can only make sense of $\Delta\widetilde{\theta}^l$ with respect to what she observes, considers $E[\Delta\widetilde{\theta}^l_t|\Delta P_t]$. Applying the projection theorem, one writes

$$E[\Delta\widetilde{\theta}^l_t|\Delta P_t] = \rho_t(\Delta)\Delta P_t \tag{80}$$

with $\rho_t(\Delta) = \frac{\mathrm{cov}(\Delta\widetilde{\theta}^l_t, \Delta P_t)}{\mathrm{var}(\Delta P_t)}$ and where, by Ito isometry and $\{\epsilon^k\}_{k=1}^n \perp \Pi \perp (B^\Theta_t)_{t\geq 0}$,

$$\mathrm{cov}(\Delta\widetilde{\theta}^l_t, \Delta P_t) = \begin{pmatrix} \varphi^l_{t+\Delta} - \varphi^l_t + \left(\varphi^l_{t+\Delta} e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - \varphi^l_t\right) \int_0^t o^c_s k^2_s e^{-\int_0^t o^c_u k^2_u \mathrm{d}u} \mathrm{d}s \\ -\varphi^l_{t+\Delta} \int_t^{t+\Delta} o^c_s k^2_s e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} \mathrm{d}s \end{pmatrix}$$
$$\times \begin{pmatrix} \left((1-\lambda_{1,t+\Delta}) e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - (1-\lambda_{1,t})\right) \int_0^t e^{-\int_s^t o^c_u k^2_u \mathrm{d}u} o^c_s k^2_s \mathrm{d}s \\ +\lambda_{1,t+\Delta} - \lambda_{1,t} + (1-\lambda_{1,t+\Delta}) \int_t^{t+\Delta} e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} o^c_s k^2_s \mathrm{d}s \end{pmatrix} \sigma^2_\Pi$$
$$- \int_0^t \begin{matrix} \left(\varphi^l_{t+\Delta} e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - \varphi^l_t\right) e^{-\int_s^t o^c_u k^2_u \mathrm{d}u} o^c_s k_s \\ \times \begin{pmatrix} \sigma_\Theta (\lambda_{2,t+\Delta} e^{-a_\Theta \Delta} - \lambda_{2,t}) e^{a_\Theta (s-t)} \\ + \left((1-\lambda_{1,t+\Delta}) e^{-\int_t^{t+\Delta} o^c_u k^2_u \mathrm{d}u} - (1-\lambda_{1,t})\right) o^c_s k_s e^{-\int_s^t o^c_u k^2_u \mathrm{d}u} \end{pmatrix} \end{matrix} \mathrm{d}s$$
$$- \varphi^l_{t+\Delta} \int_t^{t+\Delta} \begin{matrix} e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} o^c_s k_s \\ \times \left(\sigma_\Theta \lambda_{2,t+\Delta} e^{-a_\Theta (s-(t+\Delta))} + (1-\lambda_{1,t+\Delta}) e^{-\int_s^{t+\Delta} o^c_u k^2_u \mathrm{d}u} o^c_s k_s\right) \end{matrix} \mathrm{d}s.$$

Hence, as in Brennan and Cao (1997), agent l follows the trend whenever $\rho_t(\Delta) > 0$ and pursues a contrarian strategy whenever $\rho_t(\Delta) < 0$. This is the measure of momentum strategies used in Grinblatt, Titman, and Wermers (1995), for instance. Notice that the measure of trading behavior in (80) is not equivalent to that of Wang (1993), for instantaneous covariances ignore the contribution of private discussions. To see this, observe that prices have continuous sample paths

$$\mathrm{d}P_t = E[\mathrm{d}P_t|\mathcal{F}^l_t] + (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t}) o^c_t k_t) \mathrm{d}\widehat{B}^l_t.$$

Moreover, defining $f^l_t(n) = \frac{A_{Q,t} - B_{Q,t}(B^l_{\Psi,t}(n))^\top M^l_t(n)}{\gamma B^2_{Q,t}}$ along with the jump size of this function

$$\Delta f_t^l(n) = -\frac{(B_{\Psi,t}^l(n+m_t))^\top M_t^l(n+m_t) - (B_{\Psi,t}^l(n))^\top M_t^l(n)}{\gamma B_{Q,t}},$$

an application of Ito-Tanaka's formula yields

$$\mathrm{d}\theta_t^l = E[\mathrm{d}\theta_t^l|\mathcal{F}_t^l] + f_t^l(n_{t\_}^l)B_{\Psi,t}^l(n_{t\_}^l)\mathrm{d}\widehat{B}_t^l + (\Delta f_t^l(n_{t\_}^l)\Psi_{t\_}^l + f_t^l(n_{t\_}^l + m_t)C_{\Psi,t}^l)\mathrm{d}N_t^l.$$

Hence, I get

$$E[\mathrm{d}\theta_t^l \mathrm{d}P_t|\mathcal{F}_t^l] = (\lambda_{2,t}\sigma_\Theta + (1-\lambda_{1,t})o_t^c k_t)\frac{A_{Q,t} - B_{Q,t}(B_{\Psi,t}^l(n_{t\_}^l))^\top M_t^l(n_{t\_}^l)}{\gamma B_{Q,t}^2}B_{\Psi,t}^l(n_{t\_}^l)\mathrm{d}t$$

where the contribution of private meetings gets flushed out.

F.3 Measures of Performance

In the sequel, I derive a fund's NAV and the structure of the regression in (17). In so doing, I consider the performance solely induced by the informational portfolio $\widetilde{\theta}^l$, for one should not give credit to a manager for making money on noise traders.

***i)* Net Asset Value.** One may want to make the following preliminary observation: were agents risk-neutral, their expected trading gains would be null, for prices P would be martingales and, thus, $E[\int_0^T \theta_t^l \mathrm{d}P_t] = 0$ as in Hirshleifer, Subrahmanyam, and Titman (1994). Due to the serial correlation in returns, this result is expected to be changed: using (79), expected gains up to time τ may be expressed as

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = E\left[\int_0^\tau \varphi_t^l(\Pi - \widehat{\Pi}_t^c)A_{Q,t}\Psi_t\mathrm{d}t\right] + E\left[\int_0^\tau \left(\varphi_t^l - \frac{\lambda_{1,t}}{\lambda_{2,t}}d_{\Theta,t}^l\right)\frac{1}{n}\sum_{k=1}^n \epsilon^k \mathrm{d}P_t\right]$$
$$+ E\left[\int_0^\tau \widetilde{\theta}_t^l B_{Q,t}\mathrm{d}B_t^\Theta\right] + \mathbf{1}_{\{\tau=T\}}E[\widetilde{\theta}_{T\_}^l \Delta P_T].$$

where I set $W_0^l = 0$: since agents have CARA utility, this assumption is immaterial. Since $\{\epsilon^k\}_{k=1}^n \perp (\Psi_t)_{t\geq 0}$ and that ϵ^k has zero mean for all k and that the third term inside the expectation is an Ito integral, it follows, under regularity conditions, that

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = E\left[\int_0^\tau \varphi_t^l(\Pi - \widehat{\Pi}_t^c)\left(A_{Q,t}^{(2)}\Theta_t + A_{Q,t}^{(3)}(\Pi - \widehat{\Pi}_t^c)\right)\mathrm{d}t\right] + \mathbf{1}_{\{\tau=T\}}E[\widetilde{\theta}_{T\_}^l \Delta P_T].$$

Moreover, using (51), one may write

$$\widetilde{\theta}_{T\_}^l = \frac{(1-\alpha_{T\_}^l)\frac{1}{n}\sum_{k=1}^n \epsilon^k + (1-\alpha_{T\_}^l - \lambda_{1,T\_})(\Pi - \widehat{\Pi}_{T\_}^c)}{\gamma(o_{T\_}^l + \sigma_\delta^2)}$$

and it follows, applying Fubini's theorem, that

$$E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] = \int_0^\tau \varphi_t^l E\left[(\Pi - \widehat{\Pi}_t^c)\left(A_{Q,t}^{(2)}\Theta_t + A_{Q,t}^{(3)}(\Pi - \widehat{\Pi}_t^c)\right)\right]\mathrm{d}t + \mathbf{1}_{\{\tau=T\}}\frac{1-\alpha_{T_-}^l}{\gamma(o_{T_-}^l + \sigma_\delta^2)}E\left[(\Pi - \widehat{\Pi}_{T_-}^c)\left((1-\lambda_{1,T_-})(\Pi - \widehat{\Pi}_{T_-}^c) - \lambda_{2,T_-}\Theta_T\right)\right].$$

Substituting (77) and (78), and using Ito isometry along with $\Pi \perp \Theta_0 \perp (B_t^\Theta)_{t\geq 0}$, I get

$$\begin{aligned} E\left[\int_0^\tau \widetilde{\theta}_t^l \mathrm{d}P_t\right] &= -\sigma_\Theta \int_0^\tau \varphi_t^l A_{Q,t}^{(2)} \int_0^t o_s^c k_s e^{-a_\Theta(t-s) - \int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\mathrm{d}t \\ &+ \int_0^\tau \varphi_t^l A_{Q,t}^{(3)}\left(\left(1 - \int_0^t o_s^c k_s^2 e^{-\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)^2 \sigma_\Pi^2 + \int_0^t (o_s^c k_s)^2 e^{-2\int_s^t o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)\mathrm{d}t \\ &+ \mathbf{1}_{\{\tau=T\}}\frac{1-\alpha_{T_-}^l - \lambda_{1,T_-}}{\gamma(o_{T_-}^l + \sigma_\delta^2)} \times \\ &\left(\begin{array}{c} (1-\lambda_{1,T_-})\left(\begin{array}{c}\left(1 - \int_{[0,T)} o_s^c k_s^2 e^{-\int_{[s,T)} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\right)^2 \sigma_\Pi^2 \\ + \int_{[0,T)} (o_s^c k_s)^2 e^{-2\int_{[s,T)} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s\end{array}\right) \\ + \lambda_{2,T_-}\sigma_\Theta \int_{[0,T)} o_s^c k_s e^{-a_\Theta(T-s) - \int_{[s,T)} o_u^c k_u^2 \mathrm{d}u}\mathrm{d}s \end{array}\right). \end{aligned}$$

***ii)* Performance Regression.** Denoting by $\widehat{\theta}^c = \frac{A_Q^{(1)}\widehat{\Theta}^c}{\gamma B_Q^2}$ the myopic market portfolio and using that $\xi \in \mathcal{F}^c \subset \mathcal{F}^l$, I can write a manager l's myopic portfolio $\widehat{\theta}^l$ as

$$\widehat{\theta}_{t_-}^l = \widehat{\theta}_t^c + \frac{A_{Q,t}^{(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}A_{Q,t}^{(1)}}{\gamma B_{Q,t}^2}(\widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c).$$

Using (36) and (76), it follows that $A_{Q,t}^{(2)} - \frac{\lambda_{1,t}}{\lambda_{2,t}}A_{Q,t}^{(1)} = B_{Q,t}k_t$ and I get

$$\widehat{\theta}_{t_-}^l = \widehat{\theta}_t^c + \frac{k_t}{\gamma B_{Q,t}}(\widehat{\Pi}_{t_-}^l - \widehat{\Pi}_t^c) = \widehat{\theta}_t^c + \frac{k_t}{\gamma B_{Q,t}}(1-\alpha_t^l)(\bar{S}_t^l - \widehat{\Pi}_t^c)$$

where the last equality follows from (69). Further observing that $\Pi - \widehat{\Pi}_t^c = \frac{1}{\lambda_{1,t}}(P_t - \widehat{\Pi}_t^c - \lambda_{2,t}\Theta_t)$, one obtains (16).

The difference between the returns generated by managers i and market returns follows from observing that

$$\mathrm{d}R_t^i := \widetilde{\theta}_t^i \mathrm{d}P_t = d_{\Delta,t}^i(\Pi - \widehat{\Pi}_t^c)(A_{Q,t}^{(1)}\Theta_t\mathrm{d}t + (A_{Q,t}^{(2)} - B_{Q,t}k_t)(\Pi - \widehat{\Pi}_t^c)\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^c)$$

and

$$\mathrm{d}R_t^c := \widetilde{\theta}_t^c \mathrm{d}P_t = d_{\Theta,t}^c \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^c)\left(A_{Q,t}^{(1)}\left(\Theta_t + \frac{\lambda_{1,t}}{\lambda_{2,t}}(\Pi - \widehat{\Pi}_t^c)\right)\mathrm{d}t + B_{Q,t}\mathrm{d}\widehat{B}_t^c\right).$$

G Description of the Calibration

This appendix provides further comments on the calibration of Subsection 3.3.2, which is used in the results sections.

I set a_Θ above 0.05, as estimated by Campbell and Kyle (1993), for this parameter value would make the supply counterfactually persistent. The chosen calibration matches that used in Huang and Wang (1997) and Hong and Wang (2000). The volatility of the supply σ_Θ is consistent with that estimated by Campbell and Kyle (1993) who show that this parameter would be related to risk aversion in equilibrium. Inspection of Section 3 in Campbell and Kyle (1993) shows that Θ is equivalent to X in their setting. This further implies that σ_Θ is of the form $\frac{1-\Phi_2\eta\sigma_M\sigma_N}{\psi r\sigma_M^2}\text{diff}(\mathrm{d}N_t)$. After correcting a mistake in the first and third equations of B.12 and B.7, respectively, and substituting the estimates of Table 8 indicates that my calibration is obtained for a risk aversion of 8. This parameter lies within the range used by Wang (1993) and is below that used by Hong and Wang (2000). I choose to set the risk aversion parameter well below 8 to be consistent with Koijen (2012) who structurally estimates fund managers' relative risk aversion and reports a risk aversion of 5.51 and stock holdings of $ 93 millions on average.

The volatility of ideas σ_S is chosen to be higher than σ_Π and both are set to the calibration of He and Wang (1995). A similar estimate for σ_Π is obtained in Banerjee (2010). The volatility of ideas is chosen so as to reflect that working ideas are diffuse but not too imprecise. Notice that respectively increasing γ, σ_Θ, σ_S and σ_Π or decreasing a_Θ would only make my results stronger.

To fix ideas regarding the chosen network thresholds, suppose one shuts down the price-learning channel so that $o_t^c \equiv \sigma_\Pi^2$. Then, at the chosen value for N, the marginal effect $\frac{1}{\sigma_\Pi^2}\frac{\partial}{\partial n}o_t^l(n)\Big|_{n=N} = -\frac{\sigma_\Pi^2\sigma_S^2}{(\sigma_S^2+\sigma_\Pi^2 n)^2}$ of an additional idea is less than -2%. In other words, the contribution of an additional idea becomes negligible: social interactions have mostly exhausted their informational role. Since learning from prices will make this contribution even weaker, N is a natural level for perfect knowledge to step in.

To further make sense of the chosen K, suppose an enforcement technology exists so that agents optimally pre-commit to switch after getting K ideas or more at a cost β. Assuming away time-inconsistency issues, they would compare the value $V_T^A = \sum_{n\in A}\mu_T^A(n)J^l(n,T)$ of Network A with that $e^{\gamma\beta}V_T^B = e^{\gamma\beta}\sum_{n\in B\cup\{\infty\}}\mu_T^B(n)J^l(n,T)$ of Network B at time T. The value K would then be optimal if $\beta = \frac{1}{\gamma}\log\left(\frac{E[V_T^A|\mathcal{F}_0^l]}{E[V_T^B|\mathcal{F}_0^l]}\right)$. In turn, an interaction intensity of one meeting twice per year ($\eta = 2$), every quarter ($\eta = 4$) and every two months ($\eta = 6$) imply a cost β of 0.34, 0.44 and 0.438, respectively.

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