

# Asset Pricing on FOMC Announcements

Julien Cujean\*      Samuel Jaeger<sup>†</sup>

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## Abstract

Asset-pricing facts on FOMC announcements have changed strikingly in the last decade. The pre-announcement drift has disappeared, and other known facts—the announcement premium and a stronger CAPM—now concentrate on a subset of announcements. We propose these distinct patterns correspond to two equilibrium outcomes. The drift can be switched on and off across equilibria, which has implications for return informativeness, beta dispersion and premia, and how they are responsible for a stronger CAPM. The model reveals the key in understanding the facts is to condition on good and bad news, and market noise. High-frequency data shows the drift is mostly present and highly informative upon good news, and the equilibrium shift is likely unrelated to the Fed’s improved guidance but to the rise of market noise.

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\*University of Berne, Institute for Financial Management, Engehaldenstrasse 4, 3012 Bern, Switzerland, [julien cujean.com](http://julien cujean.com), [julien.cujean@unibe.ch](mailto:julien.cujean@unibe.ch).

<sup>†</sup>University of Berne, Institute for Financial Management, Engehaldenstrasse 4, 3012 Bern, Switzerland, [samuel.jaeger@unibe.ch](mailto:samuel.jaeger@unibe.ch).

# 1 Introduction

It is now common in the literature to list two facts regarding asset returns on FOMC announcements: on these occasions most of the risk premium is realized and the CAPM works. That there is an announcement premium is not surprising to the extent that the announcement resolves uncertainty (Epstein and Turnbull, 1980); what is surprising is that it builds ahead of the announcement (Lucca and Moench, 2015). This “pre-announcement drift” could reflect prior information, but the absence of a positive relation between pre- and post-announcement returns seems to discard this possibility (Laarits, 2022). Adding to the puzzle, the drift has in fact disappeared over the last decade (Boguth, Gregoire, and Martineau, 2019; Kurov, Wolfe, and Gilbert, 2021a), coinciding with the introduction of a press conference (PC) following some of the announcements. High returns and a stronger CAPM now concentrate on such “PC days” (Bodilsen, Eriksen, and Grønberg, 2021).

In this paper we ask, what does information ahead of announcements imply for pre-announcement returns and their beta formulation and can it be definitely ruled out as a driver of the facts? Why did the drift disappear, may it reappear and why do facts now concentrate on a subset of days? How are the facts related? For instance, the announcement premium comes with a compression in betas (Bodilsen et al., 2021; Andersen, Thyrgaard, and Todorov, 2021), and these two facts could together or separately explain a better-performing CAPM. We first develop a theory based on private information and then revisit the facts by testing our theoretical predictions.

In a dynamic rational-expectations framework with residual uncertainty in payoffs (e.g., He and Wang (1995)) a public announcement may cause (at least) two equilibria to arise. The particularity of an announcement is clear at high frequency (in continuous time)—it conveys a *discrete* amount of news (Wachter and Zhu, 2021). Suppose first no announcement is made and consider an economy with multiple stocks whose payoffs have a common-factor structure. A continuum of investors get an initial private signal about the factor and noise trading is continuous. Under the condition that noise is unpredictable, meaning that noise traders are not high-frequency traders, there always exists a no-trade equilibrium and there may exist another equilibrium in which informed trading takes place. Suppose now an announcement is made. In this context with multiple stocks no equilibrium exists without the announcement coming with a similarly discrete amount of noise. This lump in noise allows the announcement to reveal additional information through prices as they react to it. Although the announcement eliminates the no-trade equilibrium pre-announcement, there may still exist two equilibria. In one equilibrium the price reaction to the announcement is weak (hereafter, *E1*) whereas in the other it is strong (hereafter, *E2*). There are two ways of telling these two equilibria apart according to how their asset-pricing implications differ.

A first way to tell the two equilibria apart, and our first main argument, is that the pre-announcement drift is information-driven. In both equilibria investors believe that others speculate on the announcement outcome ahead of it and trade exploiting the resulting price informativeness. The difference across equilibria is that investors anticipate a weak or a strong reaction of prices at the announcement, which has consequences for the way the premium builds ahead of the announcement.

Note that the presence of a drift is a dynamic phenomenon. Because investors trade on short-term price swings there is a wedge between the weight Bayes' rule assigns to fundamentals and that assigned by prices (Cespa and Vives, 2012). Investors exploit price informativeness by chasing the trend, which causes prices to reveal more fundamental information than what Bayes' rule can justify. This mechanism feeds upon itself and since resolution of uncertainty commands a premium (Epstein and Turnbull, 1980) a drift builds up prior to the announcement. When reaction to the announcement is anticipated to be weak (strong) the pre-announcement drift is equally weak (strong). Hence, a strong drift is associated with high price informativeness.

However, there is a complication in linking the drift to price informativeness pre-announcement. The issue is that there is a pronounced asymmetry in the drift and market reaction across good and bad news, a second key prediction. That uncertainty resolution commands a premium is nondirectional—there is a premium on announcements whether the announcement outcome is good or bad. Suppose now that we condition pre-announcement returns based on whether news is good or bad. Intuitively, we should see an upward (downward) drift ahead of positive (negative) news. Yet, because prices are informative they partially resolve uncertainty and thus come with a premium. This premium adds to the existing positive return on good news, but partly offsets the negative drift on bad news. Therefore, whereas the drift is strong on good news it is substantially weaker on bad news. We conclude that a strong drift and thus high pre-announcement price informativeness concentrate upon good news, and thus conditioning on the news outcome is key.

A second way in which the two equilibria differ is in terms of cross-sectional dispersion in betas, which exhibits markedly different patterns around the announcement. Although betas become systematically more compressed post-announcement, they may move in opposite directions pre-announcement depending on the extent of market noise around the announcement. Specifically, if there is little noise beta compression also occurs prior to the announcement in  $E2$ , whereas betas become suddenly more dispersed in  $E1$  close to the announcement. Furthermore, the extent of noise also determines how the CAPM performs. There are two scenarios: either market noise is high and the Securities Market Line (SML) is downward sloping or instead noise is low and the CAPM works “too well,” in the sense that the SML slope exceeds the market premium. In both equilibria this “excess slope” is explained by dispersion in beta and value. On the announcement, however, the origins of excess slope differ across equilibria. In  $E1$  it is entirely explained by dispersion in fundamental value across stocks, and in  $E2$  it is additionally explained by compression in betas.

The point is that the drift can be “switched on and off” across the two equilibria, and that each equilibrium is associated with different asset-pricing patterns. In the model the announcement is characterized by two parameters, its informativeness and how much market noise comes with it and without which an equilibrium does not exist. Interestingly, for a given level of market noise both equilibria  $E1$  and  $E2$  exist for a large range of values of announcement informativeness, but the converse is not true. Namely, for a given level of informativeness,  $E2$  disappears as market noise rises. Hence, the key aspect in the announcement triggering equilibrium multiplicity is market

noise, as opposed to its informativeness, our second main point.

It is interesting to contrast this theoretical conclusion with the Fed adopting new communication policies over the last decade and concurrent changes in asset-pricing facts (e.g., disappearance of the drift). As of April 2011 the Fed introduced press conferences (PC) held after the publication of FOMC statements. PCs are “*intended to further enhance the clarity and timeliness of the Federal Reserve’s monetary policy communication*” (Federal Reserve, 2011). Improved forward guidance could result in improved informativeness of the announcement, and perhaps intuitively may have acted as a trigger for a switch from one equilibrium to the other.<sup>1</sup> Yet, our theory indicates that this intuition is likely incorrect, as an equilibrium shift is driven by a corresponding shift in market noise. Incidentally, adapting the approach of Boguth, Gregoire, and Martineau (2022) in measuring market noise, we find that it has increased notably in the last decade. That equilibrium  $E2$  disappears as noise rises suggests that the apparent shift from  $E2$  to  $E1$  is in fact unrelated to the introduction of PCs but rather a consequence of a rise in market noise in the period following their introduction. But this possibility requires first establishing empirically that recent changes in the facts indeed correspond to an equilibrium shift towards equilibrium  $E1$ , which we do by exploiting the distinct asset-pricing implications across the two equilibria.

We collect intraday stock prices for S&P 500 firms from the NYSE Trade and Quote (TAQ) database. We focus on the period between January 2001 and December 2022 and on days during which FOMC announcements took place, a sample of 167 announcements. We then split the sample into two subsamples: sample non-PC excludes PCs and thus runs from January 2001 and June 2018 (with most (82 out of 112) announcements taking place prior to 2011) and sample PC only contains PCs and spans the period from May 2011 to December 2022. We then test whether asset-pricing implications regarding pre-announcement returns, beta dispersion and SML slope differ across the two subsamples as the theory predicts they do across equilibria.

Our theory is that the drift reflects information but that informativeness of pre-announcement returns concentrates on good news. That is, we must first condition on good and bad news to examine how informative pre-announcement returns are. We define “good news” (“bad news”) as those announcements that come with a daily return falling in the upper (lower) quantile of all announcement days. We find a strong asymmetry within the non-PC sample along the lines of  $E2$ : the drift is almost entirely cashed in when the news is good (it is on average 54.5 bps on good news against 4.5 bps on bad news). In contrast, within the PC sample pre-announcement returns exhibit virtually no drift, yet there is a marked asymmetry in market reaction across good and bad news, similar to the  $E1$  outcome.

The defining difference between equilibrium  $E2$  and  $E1$  is that returns are highly informative

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<sup>1</sup>For instance, the average CBOE Volatility Index (VIX)—commonly used as a proxy for uncertainty—was significantly higher before A days in the pre-PC era. More specifically, between January 2001 and March 2011, the average VIX at the open of the announcement day was 22.6, while the average VIX in period April 2011-December 2022 was 17.92. The difference is statistically significant at the 1% level with a t-statistic of 3.54

in the former but they are not in the latter. To examine the link between the asymmetry in the drift we uncover in the data and informativeness we adapt the measure of informed trading of [Weller \(2017\)](#) and calculate the frequency at which pre-announcement returns correctly predict the announcement outcome. Remarkably, we find that the strong drift ahead of good news in the non-PC sample is informative—82% of the time the market correctly anticipates the outcome. In contrast, in the PC sample pre-announcement returns do just as well as a simple coin flip on good and bad news (successful predictions occur exactly 50% of the time in both cases). That is, the same way informativeness differs across  $E1$  and  $E2$  pre-announcement returns are informative in the non-PC sample and uninformative in the PC sample.

Depending on the extent of noise around the announcement, beta dispersion may move in opposite directions across equilibria at this time. We follow [Andersen et al. \(2021\)](#) to compute a measure of beta dispersion at high frequency. We find that betas become progressively more compressed over the day, consistent with [Bodilsen et al. \(2021\)](#) and [Andersen et al. \(2021\)](#), and that this pattern holds whether or not an announcement takes place. This pattern also extends to the non-PC sample. In the PC sample, although betas also become more compressed over the day they become markedly more dispersed at the announcement (beta dispersion spikes). These findings concur with the respective patterns in beta dispersion across equilibria  $E1$  and  $E2$  under the low noise scenario.

The CAPM works “too well” in the model under the low-noise scenario, and dispersion in beta and value together explain this result. Running intraday [Fama and MacBeth \(1973\)](#) regressions we first confirm that the SML slope exceeds the market premium on announcement days during which the two have the same sign. In other words, the CAPM indeed works too well in both samples on these days. Next, the theory implies that excess slope is summarised by the ratio of dispersion in value to dispersion in beta. We find that this ratio explains around 20% of variation in excess slope and that controlling for it excess slope disappears in both samples. Overall we conclude that the PC (non-PC) sample shares the asset-pricing properties of equilibrium  $E1$  ( $E2$ ).

We conduct several robustness checks regarding the asymmetry in the drift, which could be driven by factors other than those our theory envisages. Intuitively, since speculating on bad news requires shorting the market, a natural explanation for the asymmetry is that short-selling is costly; we find that such costs are unlikely drivers of the asymmetry. Another possibility is that there is an asymmetry in the signal itself. In particular, the Fed tends to announce easing policies following a period of low market returns, the so-called “Fed put” (e.g., [Cieslak and Vissing-Jorgensen \(2020\)](#)). We find that the Fed put could in part drive the asymmetry but in an unexpected way—the market seems to speculate on it but does not correctly anticipate it. Other factors are known to explain a strong drift. For instance, the comovement between bond and stock returns (e.g., [Cieslak and Schrimpf \(2019\)](#) or [Laarits \(2022\)](#)) and measures of risk (the “risk shift” measure of [Kroencke, Schmeling, and Schrimpf \(2021\)](#) and the VIX ([Lucca and Moench, 2015](#))). We find that both could play a role, which is consistent with our theory, e.g., when pre-announcement returns are

(un)informative there is a (no) drift and market reaction is weaker (stronger) on the announcement and thus “risk shift” is weak (strong). Finally, we control for surprises in the Fed Fund target rate following [Kuttner \(2001\)](#) and find that they appear unrelated.

We contribute to the current debate regarding asset-pricing facts on announcements and the mechanism underlying them. Adding to the literature we argue that examining the facts critically requires conditioning on good and bad announcement outcomes, and market noise. Specifically, conditioning on “good news” reveals that pre-announcement returns are highly informative, and that most facts concentrate upon this outcome; a rise in market noise eliminates the drift equilibrium. We argue these facts are driven by informed speculation. In contrast, given the formerly documented absence of relation between pre- and post-announcement returns, the literature has focused on alternative mechanisms to explain the facts, among which are non-additive preferences ([Ai and Bansal, 2018](#)), disaster risk ([Wachter and Zhu, 2021](#)), type of news release ([Laarits, 2022](#)), and endogenous timing of information acquisition ([Ai, Bansal, and Han, 2022](#); [Cocoma, 2022](#)).

Methodologically, we present a new solution method for continuous-time, rational-expectations equilibria with discrete public signals, a common-factor structure in payoffs, and multiple stocks. In particular, we extend the noisy rational-expectations framework of [He and Wang \(1995\)](#) to multiple stocks and continuous trading, and build on their solution method. However, we introduce a new, central step that consists in “guessing and verifying” the form of equilibrium risk premia, which allows us to boil down complicated matrix Riccati equations to just a few ODEs. Thus, an equilibrium with multiple stocks is not more difficult to solve than one with a single stock. We also exploit the important observation of [Cespa and Vives \(2012\)](#) that under residual uncertainty there is a distortion in the weight prices assign to fundamentals relative to Bayes’ rule; we demonstrate that there exist great simplifications in taking linear combinations of equilibrium risk premia and of the coefficients of investors’ value function based on this relative distortion.

## 2 Background

We want to understand how announcements affect risk premia and betas. The purpose of this section is to provide this intuition in a simple context in which the CAPM holds.

Consider an economy with 3 dates indexed by  $t = 1, 2, 3$ . The market consists of one riskless asset and  $N$  risky stocks indexed by  $n = 1, \dots, N$ . Trading takes place at dates 1 and 2 and consumption at date 3. There is no intermediate consumption, which means that the riskfree rate is exogenous, and normalized to 0. Stocks pay a liquidating dividend,  $\tilde{\mathbf{D}} \equiv [\tilde{D}_1 \dots \tilde{D}_N]'$ , at date

3, which is unobservable at all trading dates and has a common factor structure:

$$\tilde{\mathbf{D}} = \begin{bmatrix} D \\ D \\ \vdots \\ D \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} \tilde{F} + \begin{bmatrix} \tilde{\epsilon}_1 \\ \tilde{\epsilon}_2 \\ \vdots \\ \tilde{\epsilon}_N \end{bmatrix} = \mathbf{1}D + \Phi\tilde{F} + \tilde{\epsilon}, \quad (1)$$

where  $D$  is a positive scalar and  $\mathbf{1}$  is a  $N \times 1$  vector of ones. The common factor  $\tilde{F}$  and each stock-specific component  $\tilde{\epsilon}_n$  are independently normally distributed with means zero and precisions  $\tau_F$  and  $\tau_\epsilon$ . Without loss of generality, we assume that  $\Phi'\Phi = 1$  (this assumption is equivalent to scaling  $\tau_F$ ). We fix the aggregate number of shares for all stocks to  $\mathbf{M}$  (hereafter the *market portfolio*), a  $N \times 1$  vector; we denote the average loading by  $\bar{\Phi} \equiv \mathbf{M}'\Phi$ .

The economy is populated with a continuum of identical investors indexed by  $i \in [0, 1]$ , who choose their portfolios at each trading date and derive utility from their terminal wealth,  $u(\tilde{W}_3)$ . Investors know the structure of realized payoffs in Eq. (1), but do not observe the common factor  $\tilde{F}$ . Each investor  $i$  forms expectations about it based on available information. The only source of information is a public announcement about  $\tilde{F}$  made at date 2:

$$\tilde{A} = \tilde{F} + \tilde{v}, \quad (2)$$

with  $\tilde{v} \sim \mathcal{N}(0, \tau_A^{-1})$  independent of all other variables. The announcement provides market information, as opposed to firm-specific information (on  $\tilde{\epsilon}$ ); this specification is motivated by our later focus on announcements made by the Fed and that concern the whole economy. We assume that markets are complete, which allows us to show in the extended model how market noise affects conclusions. We refer to the continuum as “the agent.” This model corresponds to the framework of [Epstein and Turnbull \(1980\)](#) with an arbitrary but finite number of stocks and in which we rule out intermediate consumption.

For a given stochastic discount factor (SDF),  $\tilde{\xi}_t$ , at date  $t$ , the agent solves the problem:

$$\max_{\tilde{W}_3} \mathbb{E}_0[u(\tilde{W}_3)] \quad \text{s.t.} \quad \mathbb{E}_0[\tilde{\xi}_3 \tilde{W}_3] \leq W_0. \quad (3)$$

From optimality conditions we obtain the dynamics of the SDF, with  $\tilde{\xi}_3 \propto u'(\tilde{W}_3)$  and:

$$\tilde{\xi}_2 \propto \mathbb{E}_2[\tilde{\xi}_3 | \tilde{A}] \quad \text{and} \quad \xi_1 \propto \mathbb{E}_1[\tilde{\xi}_3]. \quad (4)$$

Let  $\tilde{\mathbf{P}}_t$  denote the  $N \times 1$  vector of equilibrium stock prices at date  $t$ , as given by the expected present value of future cashflows:

$$\mathbf{P}_1 = \mathbb{E}_1 \left[ \tilde{\xi}_3 / \xi_1 \tilde{\mathbf{D}} \right] \quad \text{and} \quad \tilde{\mathbf{P}}_2 = \mathbb{E}_2 \left[ \tilde{\xi}_3 / \xi_2 \tilde{\mathbf{D}} \mid \tilde{A} \right]. \quad (5)$$

Thus the expected dollar return,  $\tilde{\mathbf{R}}_2 \equiv \tilde{\mathbf{P}}_2 - \mathbf{P}_1$ , on each asset from date 1 to date 2 is:

$$\mathbb{E}_1[\tilde{\mathbf{R}}_2] = \mathbb{E} \left[ \left( \frac{1}{\tilde{\xi}_2} - \frac{1}{\xi_1} \right) \tilde{\xi}_3 \tilde{\mathbf{D}} \right]. \quad (6)$$

In words, there is an announcement premium to the extent that the SDF at dates 1 and 2 differ.<sup>2</sup> To determine how they differ we follow [Epstein and Turnbull \(1980\)](#) and further assume the agent has CARA utility,  $u(W) \equiv \exp(-\gamma W)$  with  $\gamma$  the coefficient of absolute risk aversion. The SDF at both dates becomes:

$$\tilde{\xi}_2 \propto \exp(-\gamma \mathbb{E}_2[\mathbf{M}'\tilde{\mathbf{D}}|\tilde{A}] + \gamma^2/2 \mathbb{V}_2[\mathbf{M}'\tilde{\mathbf{D}}|\tilde{A}]) \quad (7)$$

$$\neq \exp(-\gamma \mathbb{E}_1[\mathbf{M}'\tilde{\mathbf{D}}] + \gamma^2/2 \mathbb{V}_1[\mathbf{M}'\tilde{\mathbf{D}}]) \propto \xi_1 \quad (8)$$

and the associated stock prices satisfy:

$$\tilde{\mathbf{P}}_2 = \mathbb{E}_2[\tilde{\mathbf{D}}|\tilde{A}] - \gamma \mathbb{V}_2[\tilde{D}|\tilde{A}]\mathbf{M} \quad \text{and} \quad \mathbf{P}_1 = \mathbb{E}_1[\tilde{\mathbf{D}}] - \gamma \mathbb{V}_1[\tilde{D}]\mathbf{M}. \quad (9)$$

As is customary in this setup prices correspond to expected cashflows net of a discount for risk conditional on available information. By the law of iterated expectations we find that:

$$\mathbb{E}_1[\tilde{\mathbf{R}}_2] = \gamma \bar{\Phi} \underbrace{(\tau_1^{-1} - \tau_2^{-1})}_{\text{resolution of uncertainty}} \Phi, \quad (10)$$

where  $\tau_1 \equiv 1/\mathbb{V}_1[\tilde{F}]$  and  $\tau_2 \equiv 1/\mathbb{V}_2[\tilde{F}|\tilde{A}]$  denote the precision regarding the common factor conditional on available information at dates 1 and 2, respectively.

**Corollary 1.** (*Epstein and Turnbull, 1980*) *There is a positive announcement premium to the extent that the announcement resolves uncertainty (it has informational content).*

In this context systematic risk entirely explains the announcement premium. In particular, defining  $\tilde{\mathbf{R}}_3 \equiv \tilde{\mathbf{D}} - \tilde{\mathbf{P}}_2$  the CAPM holds at both trading dates:

$$\mathbb{E}[\tilde{\mathbf{R}}_2] = \underbrace{\frac{\text{Var}_1[\tilde{\mathbf{P}}_2]\mathbf{M}}{\text{Var}_1[\mathbf{M}'\tilde{\mathbf{P}}_2]}}_{\equiv \beta_1} \mathbb{E}_0[\mathbf{M}'\tilde{\mathbf{R}}_2] \quad \text{and} \quad \mathbb{E}[\tilde{\mathbf{R}}_3] = \underbrace{\frac{\text{Var}_2[\tilde{\mathbf{D}}|\tilde{A}]}{\text{Var}_2[\mathbf{M}'\tilde{\mathbf{D}}|\tilde{A}]}}_{\equiv \beta_2} \mathbb{E}_0[\mathbf{M}'\tilde{\mathbf{R}}_3], \quad (11)$$

where the betas satisfy:

$$\beta_1 = \Phi/\bar{\Phi} \quad \text{and} \quad \beta_2 = \frac{\tau_1^{-1}\bar{\Phi}\Phi + \tau_\epsilon^{-1}\mathbf{M}}{\tau_1^{-1}\bar{\Phi}^2 + \tau_\epsilon^{-1}\|\mathbf{M}\|^2}. \quad (12)$$

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<sup>2</sup>There are other ways of generating an announcement premium. For instance, [Ai and Bansal \(2018\)](#) assume that  $\xi_1 \equiv \tilde{\xi}_2(\tilde{A})$  and use non-time separable preferences to obtain a premium.



Note that the cross-section of stocks is spanned by both  $\Phi$  and  $\mathbf{M}$  at date 2 and by  $\Phi$  only at date 1 (because asset-specific uncertainty,  $\tilde{\epsilon}$ , is only resolved at date 3). Under the additional assumption that the market portfolio is equally weighted,  $\mathbf{M} \equiv \mathbf{1}/N$ , the cross-sectional dispersion of betas at dates 1 and 2 are related according to:

$$\text{Var}[\beta_2] = \frac{\tau_1^{-2} \bar{\Phi}^2}{\underbrace{(\tau_1^{-1} \bar{\Phi}^2 + \tau_\epsilon^{-1}/N)^2}_{\in(0,1)}} \text{Var}[\beta_1]. \quad (13)$$

**Corollary 2.** *There is a compression in beta post-announcement.*

This compression arises because all betas are shrunk towards market beta (which is 1):

$$(1 + \delta)(\beta_2 - \mathbf{1}) = \beta_1 - \mathbf{1}, \quad (14)$$

with  $\delta = \tau_1/(\tau_\epsilon \bar{\Phi} N) > 0$ , after the announcement is made.

Resolution of uncertainty is a simple mechanism that can explain why most of the equity premium is realized on days when announcements are made (e.g., Savor and Wilson (2013)) and why betas are more compressed on these occasions (Andersen et al. (2021); Bodilsen et al. (2021)). Interestingly, a higher premium (Corollary 1) or a compression in beta (Corollary 2) both cause a steepening of the SML. It is possible that these facts individually or together explain why the CAPM works better on public announcements (e.g., Savor and Wilson (2014)), but this question requires a context in which the CAPM does not always hold. Furthermore, the current puzzle in the literature is that the premium in fact builds up prior to the announcement (Lucca and Moench, 2015), a pattern that has disappeared since 2015 (Kurov, Wolfe, and Gilbert, 2021b).

In the next section we examine how this mechanism operates at high frequency (continuous time) in the presence of speculation (private information). In this context the CAPM does not always hold; this allows us to tell which of Corollary 1 or 2 leads to a better-performing CAPM, why there is a pre-announcement drift, and why it has disappeared (and may reappear).

### 3 Speculation and resolution of uncertainty

We introduce a continuous-time, multiple-stocks extension of He and Wang (1995). We present a new solution method, which makes this model not more difficult to solve than one with a single stock. Finally, we formulate expected returns in terms of market betas.

#### 3.1 Model

Consider the model of Section 2 with the following, three modifications. First, time is now continuous and trading takes place continuously over  $[0, T)$ , where  $T$  denotes the horizon date at which

stocks pay out  $\tilde{\mathbf{D}}$  (formerly date 3). We think of  $T$  as a trading day. The announcement  $\tilde{A}$  about the common factor  $\tilde{F}$  is made at date  $\tau \in (0, T)$  (formerly date 2).

Second, each investor  $i \in [0, 1]$  in the continuum now receives private information about the common factor,  $\tilde{F}$ , in the form of a flow of private signals,  $(\tilde{V}_t^i)_{t \geq 0}$ :

$$d\tilde{V}_t^i = \tilde{F}dt + \tau_v^{-1/2}dB_t^i, \quad \tilde{V}_0^i = \tilde{F} + \tilde{\epsilon}^i, \quad (15)$$

with  $\tilde{\epsilon}^i$  an “idiosyncratic” Gaussian noise with mean zero and variance  $\tau_v^{-1}$  and  $B^i$  an “idiosyncratic” Brownian noise. By “idiosyncratic” we mean that there is one such Brownian per investor  $i$  and that these Brownians are sufficiently independent for the Strong Law of Large Numbers to hold across investors (Duffie and Sun, 2007). In the analysis we will often set  $\tau_v \equiv 0$  and  $\tau_v > 0$ , meaning that investors only receive an initial private signal at date 0.<sup>3</sup>

Third, to prevent equilibrium prices from fully revealing investors’ private information, we assume that an unmodeled group of agents trade for liquidity needs and/or for non-informational reasons.<sup>4</sup> Liquidity traders have inelastic demands of  $\tilde{\mathbf{m}}_t$  shares at date  $t$ , a  $N \times 1$  vector that evolves according to the multivariate OU process:

$$d\tilde{\mathbf{m}}_t = -\mathbf{b}\tilde{\mathbf{m}}_tdt + \tau_m^{-1/2}d\mathbf{B}_{m,t}, \quad \tilde{\mathbf{m}}_0 \sim \mathcal{N}(\mathbf{0}, \tau_m^{-1} \mathbf{I}) \quad (16)$$

with  $\mathbf{B}_m$  a  $N$ -dimensional Brownian motion,  $\mathbf{b}$  a constant  $N \times N$  matrix and  $\tau_m$  a constant scalar, which represents supply precision and which is assumed identical across stocks; the remainder,  $\mathbf{M} - \tilde{\mathbf{m}}_t$ , is available for trade to informed investors at date  $t$ . This is the usual noise trading story adopted in the literature (e.g., He and Wang, 1995). We assume right away that supply shocks are IID, i.e.,  $\mathbf{b} \equiv \mathbf{0}$ . For each stock  $\mathbf{b}$  can be viewed as the fraction of the demand of noise traders who revert their trades over a time interval  $dt$ . Thus,  $\mathbf{b} \equiv \mathbf{0}$  means that, unlike informed traders, noise traders are not high-frequency traders. This assumption is appropriate for the purpose of explaining intraday patterns.

Importantly, in this framework an equilibrium does not exist without the following, key assumption. On the announcement date  $\tau$  noise traders submit an independent bulk order of size:

$$\Delta\tilde{\mathbf{m}}_\tau \sim \mathcal{N}(0, \tau_M^{-1/2} \mathbf{I}). \quad (17)$$

In other words, noise traders react (irrationally) to the announcement, with identical precision  $\tau_M$

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<sup>3</sup>We assume the flow of signals is continuous as the law of large numbers ensures idiosyncratic components vanish in the aggregate. As a result, discreteness in private signals does not create discontinuities in asset prices (Cujean, 2020), whereas these discontinuities are the main focus of this paper.

<sup>4</sup>This creates noise in the supply of assets, prevents prices from revealing  $\tilde{F}$  (Grossman and Stiglitz, 1980) and investors from refusing to trade (Milgrom and Stokey, 1982). There are different ways to endogenize liquidity trading: private investment opportunities (Wang, 1994), investor specific endowment shocks, or income shocks (Farboodi and Veldkamp, 2017). These alternatives would unnecessarily complicate the analysis, without bringing additional economic insights.

across stocks. As Wachter and Zhu (2021) emphasize an announcement conveys a *discrete* amount of news, in contrast to any other form of information in this model. In this model it must come with a similarly discrete amount of noise. Suppose not. Then the announcement outcome,  $\tilde{A}$ , a single number, would entirely drive announcement returns on all stocks; the covariance matrix of announcement returns would be non-invertible and portfolios undefined. The theoretical importance of discrete noise suggests it plays a significant role in the data around announcements, too.

### 3.2 New steps in the solution method

Commonly observable information at date  $t$  includes the history of prices and the announcement (when made) and thus satisfies:

$$\mathcal{F}_t^c \equiv \begin{cases} \sigma(\tilde{\mathbf{P}}_s, s \leq t), & t < \tau \\ \sigma(\tilde{\mathbf{P}}_s, s \leq t) \vee \sigma(\tilde{A}), & t \geq \tau \end{cases}. \quad (18)$$

This information set corresponds to the data set of an empiricist who observes all public realizations of the model. An investor  $i$  observes the empiricist's data set along with her own flow of private information at each date  $t$ :

$$\mathcal{F}_t^i \equiv \mathcal{F}_t^c \vee \sigma(\tilde{V}_s^i, s \leq t). \quad (19)$$

We adopt the following convention. Conditional on these information sets we denote the expectation of a random variable or process  $\tilde{\mathbf{x}}$ . with a hat and index it by the relevant set,  $\hat{\mathbf{x}}_t^i = \mathbb{E}[\tilde{\mathbf{x}} | \mathcal{F}_t^i]$  and  $\hat{\mathbf{x}}_t^c = \mathbb{E}[\tilde{\mathbf{x}} | \mathcal{F}_t^c]$ . Because the (instantaneous) precision of information in Eq. (15) is identical for all investors their forecasts have identical precision at all dates  $t$ :

$$\tau_t \equiv \text{Var}[\tilde{F} | \mathcal{F}_t^i]^{-1}. \quad (20)$$

Similarly, we denote the empiricist's precision at date  $t$  by  $\tau_t^c \equiv \text{Var}[\tilde{F} | \mathcal{F}_t^c]^{-1}$ .

The first steps of the solution method are standard and follow He and Wang (1995), albeit extended to multiple stocks and continuous trading; we gather these steps in the next two propositions and relegate details to the appendix. We focus on a linear equilibrium:

$$\tilde{\mathbf{P}}_t = \mathbf{1}D + \boldsymbol{\xi}_{0,t}\mathbf{M} + \boldsymbol{\xi}_t\tilde{\mathbf{m}}_t + \boldsymbol{\Phi} \left( \lambda_t\tilde{F} + (1 - \lambda_t)\hat{F}_t^c \right), \quad (21)$$

where the  $N \times N$  matrices  $\boldsymbol{\xi}_0$  and  $\boldsymbol{\xi}$  and the scalar  $\lambda$  are deterministic price coefficients to be determined in equilibrium. We also conjecture that  $\boldsymbol{\xi}_0$  and  $\boldsymbol{\xi}$  are symmetric at all times.

This conjecture allows us to determine how investors and the empiricist update their views about  $\tilde{F}$  and noise traders' demand,  $\tilde{\mathbf{m}}$ , by means of Kalman filtering. Let  $\Delta^i \equiv \hat{F}^i - \hat{F}^c$  denote how investors and the empiricist disagree about the common factor  $\tilde{F}$ ; accordingly, define the  $(N + 1) \times 1$  vector  $\boldsymbol{\Psi}^i \equiv (\Delta^i \quad (\hat{\mathbf{m}}^i)')'$ . The next proposition shows that this vector entirely

determines how an investor  $i$ 's perceives her investment opportunity set.

**Proposition 1.** *Price changes (dollar returns) are Markovian in  $\Psi^i$  under  $\mathcal{F}^i$  at all times. Specifically, for  $t \in (0, \tau) \cup (\tau, T)$  (in between announcements) they evolve continuously as:*

$$d \begin{pmatrix} \tilde{\mathbf{P}}_t \\ \Psi_t^i \end{pmatrix} = \begin{pmatrix} \dot{\xi}_{0,t} \mathbf{M} \\ \mathbf{0} \end{pmatrix} dt + \begin{pmatrix} \mathbf{A}_{\mathbf{P},t} \\ \mathbf{A}_{\Psi,t} \end{pmatrix} \Psi_t^i dt + \begin{pmatrix} \mathbf{B}_{\mathbf{P},t} \\ \mathbf{B}_{\Psi,t} \end{pmatrix} d\hat{\mathbf{B}}_t^i, \quad (22)$$

and jump at the announcement date,  $\tau$ , according to:

$$\Delta \begin{pmatrix} \tilde{\mathbf{P}}_\tau \\ \Psi_\tau^i \end{pmatrix} = \begin{pmatrix} (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{a}_{\mathbf{P}} \\ \mathbf{a}_{\Psi} \end{pmatrix} \Psi_{\tau-}^i + \begin{pmatrix} \mathbf{b}_{\mathbf{P}} \\ \mathbf{b}_{\Psi} \end{pmatrix} \tilde{\mathbf{y}}_\tau^i, \quad (23)$$

and with prices jumping at the liquidation date,  $T$ , according to:

$$\Delta \begin{pmatrix} \tilde{\mathbf{P}}_T \\ \Psi_T^i \end{pmatrix} = \begin{pmatrix} -\xi_{0,T-} \mathbf{M} \\ \mathbf{0} \end{pmatrix} + \begin{pmatrix} \mathbf{a}_{T-} \\ -1 \\ \mathbf{0} \end{pmatrix} \Psi_{T-}^i + \begin{pmatrix} \Phi & \mathbf{0} \\ 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \tilde{\mathbf{y}}_T^i + \begin{pmatrix} \mathbf{I} \\ \mathbf{0} \end{pmatrix} \tilde{\epsilon}, \quad (24)$$

where  $\hat{\mathbf{B}}^i$  is a  $(N+1)$ -dimensional Brownian with respect to  $\mathcal{F}^i$ , and  $\tilde{\mathbf{y}}_\tau^i |_{\mathcal{F}_{\tau-}^i} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Upsilon})$  and  $\tilde{\mathbf{y}}_T^i |_{\mathcal{F}_{T-}^i} \sim \mathcal{N}(\mathbf{0}, \mathbf{T}_{T-})$ , and where  $\mathbf{a}$  is a  $N \times (N+1)$  deterministic matrix that satisfies:

$$\mathbf{a}_t \equiv \begin{pmatrix} \Phi(1 - \lambda_t) & -\xi_t \end{pmatrix} \quad (25)$$

at any date  $t$ . The  $N \times (N+1)$ -matrices  $\mathbf{a}_{\mathbf{P}}$ ,  $\mathbf{b}_{\mathbf{P}}$ ,  $\mathbf{A}_{\mathbf{P}}$  and  $\mathbf{B}_{\mathbf{P}}$  and the  $(N+1) \times (N+1)$ -matrices  $\mathbf{a}_{\Psi}$ ,  $\mathbf{b}_{\Psi}$ ,  $\mathbf{A}_{\Psi}$ ,  $\mathbf{B}_{\Psi}$ ,  $\mathbf{\Upsilon}$  and  $\mathbf{T}_{T-}$  are deterministic and given in the appendix.

Given her perception of stock returns investor  $i$  decides on her investment strategy. She faces the following investment problem at date  $t$ :

$$J(W^i, \Psi^i, t) = \max_{\mathbf{w}} \mathbb{E} \left[ -\exp \left( -\gamma \tilde{W}_T^i \right) \middle| \mathcal{F}_t^i \right] \quad (26)$$

$$\text{s.t. } \tilde{W}_T^i = W_0^i + \int_0^T \mathbf{w}'_t d\tilde{\mathbf{P}}_t^{\text{cont.}} + \mathbf{w}'_{\tau-} \Delta \tilde{\mathbf{P}}_\tau + \mathbf{w}'_{T-} \Delta \tilde{\mathbf{P}}_T, \quad (27)$$

where  $J$  denotes investor  $i$ 's value function and  $\mathbf{w}$  denotes the  $N \times 1$  vector of number of shares she invests in each risky asset. This investment problem is solved piecewise over the two subregions  $[0, \tau)$  and  $[\tau, T)$  along the lines of the next proposition.

**Proposition 2.** *The value function as defined in Eq. (26) takes the form:*

$$J(W, \Psi, t) \equiv -\exp \left( -\gamma W - s(t) - \frac{1}{2} \left( \Psi^\top \mathbf{U}(t) + \mathbf{U}(t)^\top \Psi + \Psi^\top \mathbf{V}(t) \Psi \right) \right), \quad (28)$$

where  $s$  is a scalar,  $\mathbf{U}$  an  $N + 1$ -vector and  $\mathbf{V}$  an  $(N + 1) \times (N + 1)$ -matrix, all of which are deterministic. For  $t \in (0, \tau) \cup (\tau, T)$  (in between announcements) they satisfy:

$$\dot{\mathbf{U}} = (\mathbf{B}_P \mathbf{B}'_\Psi \mathbf{V} - \mathbf{A}_P)' (\mathbf{B}_P \mathbf{B}'_P)^{-1} \dot{\xi}_0 \mathbf{M} \quad (29)$$

$$+ (\mathbf{B}_\Psi (\mathbf{I} - \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{B}_P) \mathbf{B}'_\Psi \mathbf{V} + \mathbf{B}_\Psi \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P - \mathbf{A}_\Psi)' \mathbf{U}$$

$$\dot{\mathbf{V}} = -\mathbf{A}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P + \mathbf{V} (\mathbf{B}_\Psi \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P - \mathbf{A}_\Psi) \quad (30)$$

$$+ (\mathbf{B}_\Psi \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P - \mathbf{A}_\Psi)' \mathbf{V} + \mathbf{V} \mathbf{B}_\Psi (\mathbf{I} - \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{B}_P) \mathbf{B}'_\Psi \mathbf{V},$$

with boundary conditions upon the announcement:

$$\mathbf{U}_{\tau-} = (\mathbf{I} + \mathbf{a}_\Psi)' (\mathbf{I} - \mathbf{V}_\tau \mathbf{b}_\Psi \mathbf{S}_\tau \mathbf{b}'_\Psi) \mathbf{U}_\tau \quad (31)$$

$$+ (\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi))' (\mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_P)^{-1} ((\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M} - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{U}_\tau)$$

$$\mathbf{V}_{\tau-} = (\mathbf{I} + \mathbf{a}_\Psi)' (\mathbf{I} - \mathbf{V}_\tau \mathbf{b}_\Psi \mathbf{S}_\tau \mathbf{b}'_\Psi) \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \quad (32)$$

$$+ (\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi))' (\mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_P)^{-1} (\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi)),$$

where  $\mathbf{S}_\tau \equiv (\mathbf{b}'_\Psi \mathbf{V}_\tau \mathbf{b}_\Psi + \mathbf{\Upsilon}^{-1})^{-1}$  and boundary conditions at the liquidation date:

$$\mathbf{U}_{T-} = -\tau_\epsilon \left( \mathbf{a}'_{T-} \xi_{0,T-} \mathbf{M} - \frac{\tau_\epsilon}{\tau_{T-} + \tau_\epsilon} \mathbf{a}'_{T-} \Phi \Phi' \xi_{0,T-} \mathbf{M} \right), \quad (33)$$

$$\mathbf{V}_{T-} = \tau_\epsilon \left( \mathbf{a}'_{T-} \mathbf{a}_{T-} - \frac{\tau_\epsilon}{\tau_{T-} + \tau_\epsilon} \mathbf{a}'_{T-} \Phi \Phi' \mathbf{a}_{T-} \right). \quad (34)$$

Equations for the coefficient  $s$  are omitted for brevity as they do not intervene in equilibrium.

The literature usually stops at this stage and solves these equations after markets have been cleared. However, in this multiple-stocks economy and even though matrices are conjectured symmetric, this system involves  $N(N + 1)$  ODEs to be solved piecewise. In contrast and in a new step, we now show that conjecturing the form of risk premia allows one to boil down this system of equations to just 4, irrespective of the number,  $N$ , of stocks. That is, an equilibrium with  $N$  stocks is not more difficult to solve than one with a single stock.

We start by writing consensus views regarding the common factor at date  $t$  as:

$$\bar{\mathbb{E}}_t[\tilde{F}] \equiv \int_0^1 \hat{F}_t^i di = \alpha_t \tilde{F} + (1 - \alpha_t) \hat{F}_t^c, \quad (35)$$

where  $\alpha_t \equiv (\tau_t - \tau_t^c)/\tau_t$  denotes the weight Bayesian investors assign to their private information. Thus consensus views track fundamentals based on the weight that is statistically optimal. However, as emphasized in Cespa and Vives (2012), the weight  $\lambda$  that prices assign to  $\tilde{F}$  in Eq. (21) need not coincide with  $\alpha$ . Define the (relative) wedge between the two as:

$$\ell \equiv (\alpha - \lambda)/\alpha, \quad (36)$$

a coefficient that plays a central role in the analysis. We then use the wedge  $\ell$  to decompose the matrix  $\mathbf{a}$  defined in Eq. (25) into two matrices:

$$\mathbf{b}_t \equiv \begin{pmatrix} (1 - \ell_t)(1 - \alpha_t)\Phi & -\xi_t \end{pmatrix} \quad \text{and} \quad \mathbf{c}_t \equiv \begin{pmatrix} \ell_t\Phi & \mathbf{0} \end{pmatrix}, \quad (37)$$

which have the property that:

$$\mathbf{a}_t = \mathbf{b}_t + \mathbf{c}_t \quad (38)$$

and that, in the special case when  $\lambda \equiv \alpha$  (prices and Bayes rule agree),  $\mathbf{b}_t \equiv \mathbf{a}_t$  and  $\mathbf{c}_t \equiv \mathbf{0}$ .

On non-announcement dates  $t \in [0, \tau) \cup (\tau, T)$  the premium conditional on  $\mathcal{F}_t$  writes:

$$\mathbb{E}[\mathrm{d}\tilde{\mathbf{P}}_t | \mathcal{F}_t] = \mathbf{A}_P \Psi_t \mathrm{d}t + \dot{\xi}_{0,t} \mathbf{M} \mathrm{d}t. \quad (39)$$

On the announcement returns jump and the premium immediately prior to it writes:

$$\mathbb{E}[\Delta \tilde{\mathbf{P}}_t | \mathcal{F}_{\tau-}] = \mathbf{a}_P \Psi_{\tau-} + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M}. \quad (40)$$

We now use the two matrices  $\mathbf{a}$  and  $\mathbf{b}$  to “guess” the form of this premium in equilibrium.

**Conjecture 1.** *We conjecture and verify later that:*

*A The loading of the risk premium on  $\Psi$  on non-announcement dates  $t$  satisfies*

$$\mathbf{A}_P \equiv \Sigma_P ((\Pi_{1,t} \Phi \Phi' + \Pi_{2,t} \mathbf{I}) \mathbf{b}_t + \Pi_{3,t} \mathbf{c}_t), \quad (41)$$

where  $\Sigma_P \equiv \tau_m^{-1/2} \xi - (1 - \lambda)(\tau^c)^{-1} \tau_P^{1/2} \Phi \Phi'$  is the diffusion matrix of prices, and

$$\mathbf{a}_P \equiv \mathbf{b}_P \underbrace{\begin{pmatrix} \pi_1^A \Phi' \\ \pi_1 \Phi \Phi' + \pi_2 \mathbf{I} \end{pmatrix}}_{\equiv \pi_b} \mathbf{b}_{\tau-} + \mathbf{b}_P \underbrace{\begin{pmatrix} \pi_2^A \Phi' \\ \pi_3 \Phi \Phi' \end{pmatrix}}_{\equiv \pi_c} \mathbf{c}_{\tau-}, \quad (42)$$

immediately prior to the announcement. The coefficients  $\Pi$ . and  $\pi$ . are unknown market prices of risk (to be defined in equilibrium); they satisfy the relations:

$$(1 - \ell)(\Pi_1 + \Pi_2) + \ell \Pi_3 \equiv -\tau_P^{1/2} \quad (43)$$

$$\mathbf{b}_P ((1 - \ell_{t-}) \pi_b + \ell_{t-} \pi_c) \Phi \equiv \mathbf{b}_P \underbrace{\begin{pmatrix} 1 \\ -\tau_M^{-1/2} \tau_G^{1/2} \xi_t \Phi \end{pmatrix}}_{\equiv \mathbf{y}}, \quad (44)$$

where  $\tau_G^{1/2}$  and  $\tau_P^{1/2}$  denote the change in the price-signal noise ratio (the speed at which

prices reveal information) on non-announcement dates and at date  $\tau$ , respectively:

$$-\tau_P^{1/2} \Phi \equiv \tau_m^{1/2} \frac{d}{dt} \lambda_t \xi_t^{-1} \Phi \quad (45)$$

$$-\tau_G^{1/2} \Phi \equiv \tau_M^{1/2} (\lambda_\tau \xi_\tau^{-1} - \lambda_{\tau-} \xi_{\tau-}^{-1}) \Phi. \quad (46)$$

*B The unconditional part of the risk premium on non-announcement dates satisfies*

$$\dot{\xi}_0 \mathbf{M} \equiv -\Sigma_P (\Pi_{1,t} \Phi \Phi' + \Pi_{2,t} \mathbf{I}) \xi_{0,t} \mathbf{M} \quad (47)$$

and immediately prior to the announcement:

$$(\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M} \equiv -\mathbf{b}_P \pi_b \xi_{0,\tau-} \mathbf{M}. \quad (48)$$

This conjecture conveys the same economic insight as the background model. Just like Corollary 1, Eq. (44) shows that the resolution of uncertainty the announcement brings about commands a premium. A difference, however, is that the announcement may also cause prices to reveal additional information, which commands an additional premium and is the meaning of Eq. (44). In contrast, on non-announcement dates there is a single channel through which there is a premium—prices partially reveal investors' private information as per Eq. (43). By the law of large numbers private signals are not directly priced but only through their informational effect on prices, unlike the public announcement. Conjecture 1 allows one to collapse the system of matrix equations of Proposition 2 to just 4 ODEs.

**Proposition 3.** *Under Conjecture 1 the value function matrices  $\mathbf{U}$  and  $\mathbf{V}$  take the form:*

$$\mathbf{U}_t \equiv \underbrace{(\mathbf{b}'_t (u_{1,t} \Phi \Phi' + u_{2,t} \mathbf{I}) + u_{3,t} \mathbf{c}'_t)}_{\equiv \mathbf{u}_t} \xi_{0,t} \mathbf{M} \quad (49)$$

$$\mathbf{V}_t \equiv \underbrace{\mathbf{b}'_t (v_{1,t} \Phi \Phi' + v_{2,t} \mathbf{I})}_{\equiv \mathbf{v}_t} \mathbf{b}_t + v_{3,t} (\mathbf{b}'_t \mathbf{c}_t + \mathbf{c}'_t \mathbf{b}_t) + v_{4,t} \mathbf{c}'_t \mathbf{c}_t, \quad (50)$$

at all times, where  $u$ . and  $v$ . are scalar coefficients. Define the sum  $v \equiv v_1 + v_2$ , and the following combinations of  $v$ ,  $v_3$  and  $v_4$  based on the wedge  $\ell$ :

$$\bar{v}_1 \equiv (1 - \ell)v + \ell v_3 \quad (51)$$

$$\bar{v}_2 \equiv (1 - \ell)v_3 + \ell v_4 \quad (52)$$

$$\bar{v} \equiv (1 - \ell)\bar{v}_2 + \ell \bar{v}_1. \quad (53)$$

For  $\ell$  given the value function can be equivalently described in terms of  $v_2$  and of  $v$ ,  $\bar{v}_1$  and  $\bar{v}$  (or

any choice of 3 among the  $v$ .'s and the  $\bar{v}$ .'s), which in between announcement satisfy:

$$v' = (\Pi_1 + \Pi_2) \left( 2\tau_P^{1/2} \frac{\alpha\bar{v}_1 - v}{\tau^c} - (\Pi_1 + \Pi_2) \right) + \frac{\tau_v}{\tau^2} \bar{v}_1^2 \quad (54)$$

$$v_2' = -\Pi_2^2 \quad (55)$$

$$\bar{v}_1' = \tau_P \frac{\bar{v}_1}{\tau} + \tau_P^{1/2} (\Pi_1 + \Pi_2) \left( \frac{\alpha\bar{v} - \bar{v}_1}{\tau^c} + 1 \right) + \frac{\tau_v}{\tau^2} \bar{v}\bar{v}_1 \quad (56)$$

$$\bar{v}' = \frac{\tau_v}{\tau^2} \bar{v}^2 + \tau_P \left( 2\frac{\bar{v}}{\tau} - 1 \right). \quad (57)$$

The boundary conditions for these ODEs upon the announcement are given by:

$$v_{\tau-} = v + \left( \frac{v - \alpha\bar{v}_1}{1 - \alpha} \right)^2 (1 - \pi_b)^2 \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' + \Phi' \pi_b' \mathbf{b}'_{\mathbf{P}} ((\mathbf{b}_{\mathbf{P}}\mathbf{S}\mathbf{b}'_{\mathbf{P}})^{-1} - v) \mathbf{b}_{\mathbf{P}} \pi_b \Phi \quad (58)$$

$$- 2\frac{v - \alpha\bar{v}_1}{1 - \alpha} (1 - \pi_b) \bar{\mathbf{x}}\mathbf{S}\mathbf{b}'_{\mathbf{P}} (\mathbf{b}_{\mathbf{P}}\mathbf{S}\mathbf{b}'_{\mathbf{P}})^{-1} \mathbf{b}_{\mathbf{P}} \pi_b \Phi$$

$$\frac{\bar{v}_{1,\tau-}}{\tau_{\tau-}} = \frac{\bar{v}_1}{\tau} + \left( 1 + \frac{\alpha\bar{v} - \bar{v}_1}{\tau^c} \right) \left( \Phi' \pi_b' \mathbf{b}'_{\mathbf{P}} (\mathbf{b}_{\mathbf{P}}\mathbf{S}\mathbf{b}'_{\mathbf{P}})^{-1} \mathbf{b}_{\mathbf{P}} \mathbf{S}\bar{\mathbf{x}}' + \frac{\alpha\bar{v}_1 - v}{1 - \alpha} (1 - \pi_b) \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' \right) \quad (59)$$

$$\frac{\bar{v}_{\tau-}}{\tau_{\tau-}^2} = \tau_{\tau-}^{-1} - \tau^{-1} + \frac{\bar{v}}{\tau^2} + \left( 1 + \frac{\alpha\bar{v} - \bar{v}_1}{\tau^c} \right)^2 \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' \quad (60)$$

$$v_{2,\tau-} = v_{2,\tau} + \pi_2^2 \tau_M / \xi_2^2. \quad (61)$$

where  $\pi_b \equiv \Phi' \mathbf{b}_{\mathbf{P}} \pi_b \Phi$ , and  $\bar{\mathbf{x}}$  and  $\mathbf{Q}$  are matrices defined in the appendix, and all coefficients on the right are evaluated at date  $\tau$  (omitted for brevity). The boundary conditions at the horizon date satisfy:

$$v_{T-} = \bar{v}_{1,T-} = \bar{v}_{T-} = \frac{\tau_\epsilon \tau_{T-}}{\tau_{T-} + \tau_\epsilon}. \quad (62)$$

Finally, the solution of the coefficients  $u$ . satisfies at all dates:

$$u_{k,t} \equiv -v_{k,t}, \quad k = 1, 2, 3. \quad (63)$$

Using the result of Proposition 3 we can write an investor's demand as:

$$\mathbf{w}^i = \frac{\Sigma_{\mathbf{P}}^{-1}}{\gamma} \left( \underbrace{\Sigma_{\mathbf{P}}^{-1} \mathbb{E}[d\tilde{\mathbf{P}}_t | \mathcal{F}_t^i] / dt}_{\text{myopic demand}} - \underbrace{\mathbf{1}^* \mathbf{B}'_{\Psi} ((\mathbf{v}\mathbf{b} + v_3\mathbf{c})' (\mathbf{b}\Psi^i - \xi_0\mathbf{M}) + (v_3\mathbf{b} + v_4\mathbf{c})' \mathbf{c}\Psi^i)}_{\text{hedging demand}} \right) \quad (64)$$

where

$$\mathbf{1}^* \equiv \begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix}. \quad (65)$$

The first term is the customary myopic mean-variance demand and the second term corresponds to a hedging demand. Interestingly, the hedging demand shows that an investor trades on the wedge



$\ell$  (see Appendix A.3 for details).

The last step is to clear the market, verify Conjecture 1 and obtain equations for the price coefficients. The market-clearing condition at date  $t$  is:

$$\int_0^1 \mathbf{w}_t^i di = \mathbf{M} - \tilde{\mathbf{m}}_t. \quad (66)$$

From this condition we obtain the following proposition.

**Proposition 4.** *In equilibrium the matrix coefficients  $\boldsymbol{\xi}$  and  $\boldsymbol{\xi}_0$  take the form:*

$$\boldsymbol{\xi}_t \equiv \boldsymbol{\xi}_{0,t} \equiv \xi_{1,t} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \xi_{2,t} \mathbf{I}, \quad (67)$$

where  $\xi_1$  and  $\xi_2$  are scalar coefficients with  $\xi_{2,t} = -\gamma/v_{2,t} = -\gamma\tau_\epsilon$ , which implies  $\Pi_2 \equiv 0$  and  $\pi_2 \equiv 0$ ; accordingly, let  $\Pi_1 + \Pi_2 = \Pi_1 \equiv \Pi$  and  $\boldsymbol{\xi} \equiv \xi_1 + \xi_2$ , then  $\boldsymbol{\xi}$  satisfies the ODEs:

$$\boldsymbol{\xi}' = -\Sigma_{\mathbf{P}} \Pi \boldsymbol{\xi}, \quad (68)$$

where the equilibrium market price of risk,  $\Pi$ , is given by:

$$\Pi = \tau_P^{1/2} \frac{\alpha \bar{v}_1 - v}{\tau^c} - \Sigma_{\mathbf{P}} \frac{\xi v + \gamma}{\xi}, \quad (69)$$

and where  $\Sigma_{\mathbf{P}}$  denotes the diffusion of a long only “value portfolio” and satisfies:

$$\Sigma_{\mathbf{P}} \equiv \boldsymbol{\Phi}' \Sigma_{\mathbf{P}} \boldsymbol{\Phi} = \tau_m^{-1/2} \boldsymbol{\xi} - (1 - \alpha(1 - \ell)) \tau_P^{1/2} / \tau^c. \quad (70)$$

Substituting Eq. (69) into the relation in Eq. (43) gives an algebraic equation for the equilibrium coefficient  $\tau_P^{1/2}$ , or equivalently an ODE for the wedge  $\ell$  (after substituting Eq. (68) in it):

$$0 = \Sigma_{\mathbf{P}} \frac{\bar{v}_1 \boldsymbol{\xi} + \gamma(1 - \ell)}{\xi} - \tau_P^{1/2} \left( 1 + \frac{\alpha \bar{v} - \bar{v}_1}{\tau^c} \right). \quad (71)$$

These two ODEs have boundary conditions upon the announcement (analogously, the second equation below is either an algebraic equation for  $\tau_G^{1/2}$  or a boundary condition for the wedge  $\ell$ ):

$$\xi_{\tau-} = \xi_\tau + \xi_{\tau-} \pi_b \quad (72)$$

$$0 = \sigma_{\mathbf{P}} \frac{\bar{v}_{1,\tau} \xi_{\tau-} \tau_{\tau-} / \tau_\tau + \gamma(1 - \ell_{\tau-})}{\xi_{\tau-}} + \tau_{\tau-} \left( 1 + \frac{\alpha_\tau \bar{v}_\tau - \bar{v}_{1,\tau}}{\tau_\tau^c} \right) \boldsymbol{\Phi}' \mathbf{b}_{\mathbf{P}} \mathbf{S} \bar{\mathbf{X}}', \quad (73)$$

with  $y \equiv \boldsymbol{\Phi}' \mathbf{b}_{\mathbf{P}} \mathbf{y} = -\sqrt{\tau_G / \tau_M} \boldsymbol{\xi} + (1 - \lambda)(\tau_A + \tau_G) / \tau^c$ , and the equilibrium premium,  $\pi_b$ , is:

$$\pi_b = \tau_\tau \frac{v_\tau - \alpha_\tau \bar{v}_{1,\tau}}{\tau_\tau^c} \boldsymbol{\Phi}' \mathbf{b}_{\mathbf{P}} \mathbf{S} \bar{\mathbf{X}}' \xi_\tau / \xi_{\tau-} - \sigma_{\mathbf{P}} \frac{\gamma + v_\tau \xi_\tau}{\xi_{\tau-}} \quad (74)$$

and where  $\sigma_{\mathbf{P}} \equiv \Phi' \mathbf{b}_{\mathbf{P}} \mathbf{S} \mathbf{b}_{\mathbf{P}}' \Phi$  satisfies:

$$\sigma_{\mathbf{P}}^{-1} = v_{\tau} + \frac{\left( \frac{v_{\tau} - 2\alpha_{\tau} \bar{v}_{1,\tau} + \bar{v}_{\tau} \alpha_{\tau}^2}{(1-\alpha_{\tau})^2} \frac{\tau_{\tau} - \tau_{\tau-}}{\tau_{\tau}} + \tau_{\tau-} - 2 \frac{v_{\tau} - \alpha_{\tau} \bar{v}_{1,\tau}}{1-\alpha_{\tau}} y \right) \frac{\tau_{\tau} \tau_M}{\xi_{\tau}^2} - \left( \frac{v_{\tau} - \alpha_{\tau} \bar{v}_{1,\tau}}{1-\alpha_{\tau}} \right)^2 \tau_A}{\left( \frac{v_{\tau} - 2\alpha_{\tau} \bar{v}_{1,\tau} + \bar{v}_{\tau} \alpha_{\tau}^2}{(1-\alpha_{\tau})^2} - \tau_{\tau} \right) \tau_A + \tau_{\tau}^2 - \frac{1-\lambda_{\tau}}{1-\alpha_{\tau}} \left( \frac{1-\lambda_{\tau}}{1-\alpha_{\tau}} \frac{\tau_{\tau} - \tau_{\tau-}}{\tau_{\tau}} - 2y \right) \frac{\tau_{\tau} \tau_M}{\xi_{\tau}^2}}. \quad (75)$$

The boundary conditions at the liquidation date are:

$$\xi_{T-} = -\gamma v_{T-}^{-1} \equiv -\gamma(\tau_{T-}^{-1} + \tau_{\epsilon}^{-1}), \quad (76)$$

$$\ell_{T-} = 0. \quad (77)$$

The system of ODEs in Proposition 3 and 4 along with common precision (see Eq. (130)):

$$\tau_t^c = \tau_0^c + \int_0^t \tau_{P,s} ds + (\tau_A + \tau_G) \mathbf{1}_{t \geq \tau} \quad (78)$$

form an autonomous system whose dimension is invariant in the number of stocks. This system is to be solved backwards for  $\tau_{T-}^c$  given. The last step in closing the equilibrium is to determine  $\tau_0^c$ , which is an endogenous number: at date 0 the empiricist observes a first realization of the vector of prices and updates her precision according to

$$\tau_0^c = \tau_F + \tau_m \left( \frac{\lambda_0}{\xi_0} \right)^2. \quad (79)$$

Thus, we can solve the ODEs numerically by iterating (typically through a “shooting method”) until  $\tau_{T-}^c$  and  $\tau_0^c$  are consistent with one another.

### 3.3 Beta formulation

Last we use Conjecture 1 to formulate expected returns in terms of market betas. Let  $\boldsymbol{\mu}_t \equiv \mathbb{E}[\mathbf{d}\tilde{\mathbf{P}}_t]$  denote the vector of unconditional expected excess returns across the entire cross section of stocks. To make results as transparent as possible we make the following assumption.<sup>5</sup>

**Assumption 1.** *The market portfolio is equally weighted, meaning that  $\mathbf{M} \equiv \mathbf{1}/N$ .*

It is helpful to start at the horizon date,  $T$ , where the CAPM holds in the eyes of investors but not from the perspective of the empiricist. In particular, investors’ betas satisfy:

$$\boldsymbol{\beta}_{T-} = \frac{1}{\sigma_{\mathbf{M},T-}^2} (\tau_{T-}^{-1} \bar{\Phi} \Phi' + \tau_{\epsilon}^{-1} / N \mathbf{1}), \quad (80)$$

<sup>5</sup>This assumption simplifies computations but is not without loss of generality (see Andrei, Cujean, and Wilson (2021)).

where  $\sigma_{\mathbf{M}}^2$  denotes the variance of market returns under investors' information:

$$\sigma_{\mathbf{M},T-}^2 = \tau_{T-}^{-1} \bar{\Phi}^2 + \tau_{\epsilon}^{-1}/N \quad \text{and} \quad \hat{\sigma}_{\mathbf{M},T-}^2 = (\tau_{T-}^c)^{-1} \bar{\Phi}^2 + \tau_{\epsilon}^{-1}/N, \quad (81)$$

and where we denote with a hat the analogue under empiricist's information. The two vectors of betas are related through:

$$\hat{\boldsymbol{\beta}}_{T-} - \mathbf{1} = \frac{\sigma_{\mathbf{M},T-}^2}{\hat{\sigma}_{\mathbf{M},T-}^2} \frac{\tau_{T-}}{\tau_{T-}^c} (\boldsymbol{\beta}_{T-} - \mathbf{1}). \quad (82)$$

Denote by  $\boldsymbol{\mu}_{T-} \equiv \mathbb{E}[\Delta \tilde{\mathbf{P}}_{T-}]$  the vector of expected returns on the cross section of stocks and by  $\mu_{\mathbf{M},T-} \equiv \mathbb{E}[\mathbf{M}' \Delta \tilde{\mathbf{P}}_{T-}]$  expected market returns. Conditioning down we find the CAPM holds for investors at the horizon date,  $\boldsymbol{\mu}_{T-} = \boldsymbol{\beta}_{T-} \mu_{\mathbf{M},T-}$ , which is because the problem is static at  $T$  (hedging demands drop out). However, because there is an informational gap between investors and the empiricist, the CAPM fails for the empiricist (Andrei et al., 2021):

$$\boldsymbol{\mu}_{T-} = \left( 1 - \frac{\hat{\sigma}_{\mathbf{M},T-}^2}{\sigma_{\mathbf{M},T-}^2} \frac{\tau_{T-}^c}{\tau_{T-}} \right) \mu_{\mathbf{M},T-} \mathbf{1} + \frac{\hat{\sigma}_{\mathbf{M},T-}^2}{\sigma_{\mathbf{M},T-}^2} \frac{\tau_{T-}^c}{\tau_{T-}} \mu_{\mathbf{M},T-} \hat{\boldsymbol{\beta}}_{T-}, \quad (83)$$

which follows from substituting empiricist's betas in Eq. (82).

However, central to this paper, *noise* trading implies the CAPM fails both for investors and the empiricist at any other date, in contrast to the background model of Section 2 in which the CAPM holds at all dates. As is customary in this type of models, risk aversion and noise  $\tau_M$  (or  $\tau_m$ ) do not matter separately but only as a combination:

$$n_j \equiv \gamma / \tau_j^{1/2}, \quad j = m, M \quad (84)$$

which we refer to as “market noise” and captures aggregate variation in returns. The following proposition shows that this parameter determines the extent of the CAPM failure.

**Proposition 5.** *Let  $\mu_{\mathbf{M}}$  and  $\sigma_{\mathbf{M}}$  denote market expected return and volatility, respectively. At any date strictly prior to the liquidation date and for the whole cross section of stocks the time pattern of investors' betas,  $\boldsymbol{\beta}$ , is entirely parametrized by  $\sigma_{\mathbf{M}}$  according to:*

$$\boldsymbol{\beta}(\sigma_{\mathbf{M}}; \tau) \equiv \frac{1}{\sigma_{\mathbf{M}}^2} ((\sigma_{\mathbf{M}}^2 - n^2 \tau_{\epsilon}/N) / \bar{\Phi} \boldsymbol{\Phi} + n^2 \tau_{\epsilon}/N \mathbf{1}), \quad (85)$$

and the time pattern of expected returns,  $\boldsymbol{\mu}$ , is parametrized by  $\mu_{\mathbf{M}}$  and  $\sigma_{\mathbf{M}}$  according to:

$$\boldsymbol{\mu}(\mu_{\mathbf{M}}, \sigma_{\mathbf{M}}; \tau) \equiv \underbrace{-\frac{n^2 \tau_{\epsilon}/N \mu_{\mathbf{M}}}{\sigma_{\mathbf{M}}^2 - n^2 \tau_{\epsilon}/N}}_{\text{intercept}} \mathbf{1} + \underbrace{\frac{\sigma_{\mathbf{M}}^2 \mu_{\mathbf{M}}}{\sigma_{\mathbf{M}}^2 - n^2 \tau_{\epsilon}/N}}_{\text{slope}} \boldsymbol{\beta}(\sigma_{\mathbf{M}}; \tau). \quad (86)$$

At any date in between announcements,  $t \in (0, \tau) \cup (\tau, T)$ ,  $\tau \equiv \tau_m$  and:

$$\mu_{\mathbf{M},t} \equiv -\sigma_{\mathbf{P}} \xi_t \Pi_t \bar{\Phi}^2 \quad (87)$$

$$\sigma_{\mathbf{M},t}^2 \equiv ((\sigma_{\mathbf{P}}^2 - n_m^2 \tau_\epsilon^2) \bar{\Phi}^2 + n_m^2 \tau_\epsilon^2 / N), \quad (88)$$

with  $\hat{\beta} \equiv \beta$  (empiricist's and investors' beta coincide). At the announcement,  $\tau \equiv \tau_M$  and:

$$\mu_{\mathbf{M},\tau-} \equiv -\xi_{\tau-} \pi_b \bar{\Phi}^2 \quad (89)$$

$$\sigma_{\mathbf{M},\tau-}^2 \equiv (\sigma_{\mathbf{P},\tau}^2 - n_M^2 \tau_\epsilon^2) \bar{\Phi}^2 + n_M^2 \tau_\epsilon^2 / N, \quad (90)$$

where  $\sigma_{\mathbf{P},\tau}^2 \equiv \tau_{\tau-}^{-1} \left( \tau_G^{1/2} \tau_M^{-1/2} \xi_\tau - (1 - \lambda_\tau)^2 \tau_\tau / \tau_\tau^c \right)^2 - (1 - \lambda_\tau) \tau_\tau / (\tau_\tau^c)^2 + \tau_M^{-1} \xi_\tau^2$ . Empiricist's and investors' betas differ immediately prior to the announcement and satisfy the relation:

$$\hat{\beta}_{\tau-} - \mathbf{1} = \frac{\sigma_{\mathbf{M},\tau-}^2 \hat{\sigma}_{\mathbf{M},\tau-}^2 - n_M^2 \tau_\epsilon^2 / N}{\hat{\sigma}_{\mathbf{M},\tau-}^2 \sigma_{\mathbf{M},\tau-}^2 - n_M^2 \tau_\epsilon^2 / N} (\beta_{\tau-} - \mathbf{1}). \quad (91)$$

We use the beta formulation of Proposition 5 to examine the implications of the announcement for beta dispersion and the slope of the Securities Market Line (SML) across different equilibria.

## 4 Asset-pricing implications of switching the drift on and off

### 4.1 Constructing (at least) two equilibria

We start by constructing two equilibria, a “low  $\tau_G$  equilibrium” and a “high  $\tau_G$  equilibrium,” which we refer to as *E1* and *E2*, respectively. In doing so we make two simplifying assumptions, the first of which we highlight below.

**Assumption 2.** *Investors only receive a private signal at date  $t = 0$ , i.e.,  $\tau_V > 0$  but  $\tau_v \equiv 0$ .*

Under this assumption there always exists an equilibrium in which the wedge  $\ell$  is zero post-announcement; it is also the equilibrium that obtains absent residual uncertainty in payoffs (and which, in this case, is unique); it is described in the next lemma.

**Lemma 1.** *Under Assumption 2 an equilibrium in which:*

$$\ell_t \equiv 0 \quad (92)$$

$$\xi_t \equiv \xi_{T-} \quad (93)$$

*always exists with  $v_t \equiv \bar{v}_{1,t} \equiv \bar{v}_t \equiv v_{T-}$  and  $\tau_P \equiv 0$  at all times  $t \geq \tau$ .*

Since the debate in the literature concerns pre-announcement returns we exploit this result to simplify boundary conditions at the announcement.

**Assumption 3.** *We select the equilibrium of Lemma 1 post-announcement.*

The boundary conditions for the wedge,  $\ell$ , and the liquidity discount,  $\xi$ , now simplify to:

$$\ell_{t-} = \frac{1 - \alpha_{t-}}{\alpha_{t-}} \left( \frac{v_t \xi_t^2 - \xi_t \tau_G^{1/2} \tau_M^{1/2} + \tau_M}{v_t \xi_t^2 (1 + \tau_G / \tau_{t-}) + \tau_M} - 1 \right) \quad (94)$$

$$\xi_t - \xi_{t-} = \underbrace{\gamma(\tau_{t-}^{-1} - \tau_t^{-1})}_{\text{background}} + \underbrace{\gamma \frac{\alpha_{t-}}{(1 - \alpha_{t-}) \tau_{t-}} \ell_{t-}}_{\text{dynamic effect}}. \quad (95)$$

Unless  $\tau_A \equiv 0$  (the announcement is uninformative) or  $\tau_\epsilon^{-1} \equiv 0$  (there is no residual uncertainty), Eq. (73) implies  $\tau_G \equiv 0$  is not an equilibrium. That is, the announcement necessarily causes prices to reveal information,  $\tau_G < 0$ . Hence, although the wedge is zero post-announcement, Eq. (94) implies that it is strictly negative,  $\ell < 0$ , immediately prior to the announcement, which increases the announcement premium relative to the background model of Section 2 (by Eq. (95)). Note that a wedge  $\ell \neq 0$  cannot arise in a static context; it is a dynamic phenomenon that arises because investors trade on short-term price swings—not only on fundamental value—causing prices to load more or less aggressively on fundamentals relative to Bayes’ rule (Cespa and Vives, 2012). We elaborate on the way this mechanism operates in this model in Section 4.2.

Under Assumption 3, Eq. (73) is a cubic equation in  $\tau_G$  (for  $\tau_{t-}^c$  given), and may thus have multiple solutions. We focus on constructing two equilibria that are associated with different solutions for  $\tau_G$  and that we denote by *E1* (low  $\tau_G$ ) and *E2* (high  $\tau_G$ ). Since it is difficult to characterize under which circumstances *E1* and *E2* exist, we proceed with an approximation. Note that the announcement is summarized by exactly two parameters, its informativeness,  $\tau_A$ , and noise traders’ reaction to it as parametrized by market noise  $n_M$ . When this noise is “small” we have the following result.

**Lemma 2.** *For “small market noise”,  $n_M$ , and for  $\tau_{t-}^c$  given, multiple equilibria exist provided that  $\tau_A < \frac{c}{8\tau_\epsilon}$  and that:*

$$\frac{c\sqrt{\tau_\epsilon(4\tau_A\tau_\epsilon(5c - 2\tau_A\tau_\epsilon) - d + c^2)}}{2^{3/2}(\tau_A\tau_\epsilon + c)^{3/2}(\tau_{t-}^c + \tau_V + \tau_\epsilon)} \leq n_M \leq \frac{c\sqrt{\tau_\epsilon(4\tau_A\tau_\epsilon(5c - 2\tau_A\tau_\epsilon) + d + c^2)}}{2^{3/2}(\tau_A\tau_\epsilon + c)^{3/2}(\tau_{t-}^c + \tau_V + \tau_\epsilon)}, \quad (96)$$

where  $c \equiv \tau_V(\tau_{t-}^c + \tau_V)$  and  $d = c^{1/2}(c - 8\tau_A\tau_\epsilon)^{3/2}$ .

A complication in determining the region of equilibrium multiplicity is that  $\tau_{t-}^c$  is endogenous, resulting from pre-announcement trading and depends itself on  $n_M$  and  $\tau_A$ . In fact the next corollary shows that solving the equilibrium pre-announcement can also be done for  $\tau^c$  given.

**Corollary 3.** *Under Assumption 2 the coefficients  $(v, \alpha\bar{v}_1, \xi, Y)$  as functions of common variance,  $o^c \equiv 1/\tau^c$ , form an autonomous system of ODEs, which pre-announcement satisfies:*

$$\frac{d}{do^c} v = \left( \frac{v + \gamma/\xi}{\alpha\bar{v}_1 + \gamma Y} - \frac{v - \alpha\bar{v}_1}{k_{\tau-} - \alpha\bar{v}_1} \right) \frac{d}{do^c} \alpha\bar{v}_1 \quad (97)$$

$$\frac{d}{do^c} \xi = -\frac{\xi}{\alpha\bar{v}_1 + \gamma Y} \frac{d}{do^c} \alpha\bar{v}_1 \quad (98)$$

$$\frac{d}{do^c} \alpha\bar{v}_1 = -(k_{\tau-} - \alpha\bar{v}_1)^2 \left( \frac{v - \alpha\bar{v}_1}{k_{\tau-} - \alpha\bar{v}_1} + \frac{v + \gamma/\xi}{\alpha\bar{v}_1 + \gamma Y} \right) \quad (99)$$

$$\frac{d}{do^c} Y = \frac{\tau_m^{1/2}}{o^c} \left( Y - \frac{k_{\tau-} + \gamma Y}{\alpha\bar{v}_1 + \gamma Y} \xi^{-1} \right). \quad (100)$$

with  $Y \equiv \lambda/\xi$  the price signal-noise ratio and  $k_j \equiv \alpha_j(\alpha_j\bar{v}_j + \tau_j^c)$  for  $j = \tau-, T-$  a constant. Immediately prior to the announcement,  $Y_{\tau-}$  and  $\xi_{\tau-}$  behave according to Eqs. (94) and (95), and the coefficients of the value function satisfy:

$$v_{\tau-} = \xi_{\tau-}/\xi_{\tau-} \left( v_{\tau} + v_{\tau} \frac{\xi_{t-} - \xi_{\tau}}{\xi_{t-}} + \frac{v_{\tau} - \alpha_{\tau}\bar{v}_{1,\tau}}{k_{T-} - \alpha_{\tau}\bar{v}_{1,\tau}} (\alpha_{\tau}\bar{v}_{1,\tau} - \alpha_{\tau-}\bar{v}_{1,\tau-}) \right) \quad (101)$$

$$\alpha_{\tau-}\bar{v}_{1,\tau-} = \alpha_{\tau}\bar{v}_{1,\tau} + \xi_{\tau}/\xi_{\tau-} \left( \frac{v_{\tau} - \alpha_{\tau}\bar{v}_{1,\tau}}{v_{\tau} - \alpha_{\tau}\bar{v}_{1,\tau} + \frac{\tau_Y\tau_M}{\xi_{\tau}^2\tau_A} y} - \frac{\pi_b}{1 - \pi_b} \right) (\alpha\bar{v}_{1,\tau} + \gamma Y_{\tau-}) \quad (102)$$

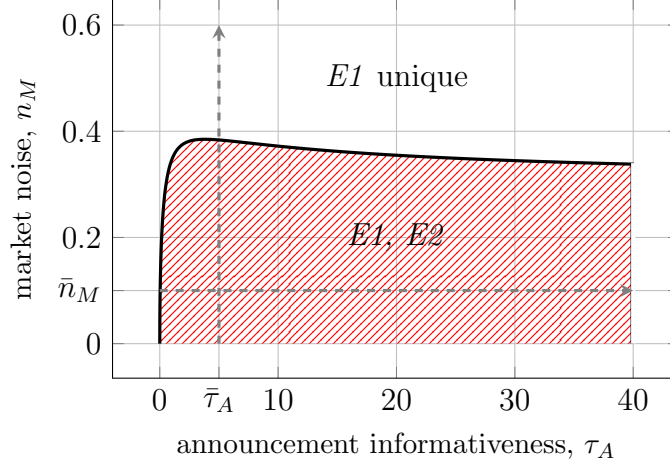
$$\alpha_{\tau}^2\bar{v}_{\tau} = \alpha_{\tau}^2\bar{v}_{1,\tau} + \alpha_{\tau}\alpha_{\tau-}(\tau_A + \tau_G) - \frac{k_{T-} - \alpha_{\tau}\bar{v}_{1,\tau}}{v_{\tau} - \alpha_{\tau}\bar{v}_{1,\tau} + \frac{\tau_Y\tau_M}{\xi_{\tau}^2\tau_A} y} (\alpha_{\tau}\bar{v}_{1,\tau} + \gamma Y_{\tau-}), \quad (103)$$

Once this system is solved for the equilibrium coefficients as functions of  $o^c$ , we conclude the equilibrium computation by solving Eq. (78) for  $\tau_c$ . At this stage, proceeding numerically we are in a position to illustrate the region in the  $(\tau_A, n_M)$ -space in which at least two equilibria exist, which we do in Figure 1. Interestingly, Figure 1 shows that for a given level of market noise two equilibria exist irrespective of announcement informativeness, but the converse is not true. Namely, for a given level of announcement informativeness,  $E2$  disappears as market noise rises. Hence, the key aspect in the announcement triggering equilibrium multiplicity is market noise, as opposed to its informativeness.

## 4.2 Asset-pricing patterns across equilibria

The purpose of this section is to examine how asset-pricing implications differ across the two equilibria,  $E1$  and  $E2$ , which we constructed in Section 4.1. We find they differ in two respects (Predictions 1 and 3 below):

**Prediction 1.** *A strong drift comes with high price informativeness.* Although there is a pre-announcement drift in both equilibria informed speculation is aggressive and thus price informativeness and the drift are substantially higher in  $E2$  than they are in  $E1$ .



**Figure 1: Multiplicity regions in the informativeness-noise space.** This figure illustrates the region in which  $E1$  and  $E2$  exist (shaded red area) and that in which  $E1$  is unique for different combinations of announcement informativeness,  $\tau_A$ , and market noise,  $n_M$ . The dashed arrows show for a given level of market noise both equilibria exist irrespective of announcement informativeness but the converse is not true. The calibration is  $\tau_V = 1.4$ ,  $\tau_\epsilon = 1.1$ ,  $\tau_F = 1$ ,  $\gamma = 0.5$ ,  $\tau_m = 1/4$ ,  $\tau = 0.7$  and  $T = 1$ .

**Prediction 2.** *Conditioning on “good” and “bad” news is key* in linking price informativeness to the drift, because the drift is (strongly) asymmetric across the two.

**Prediction 3.** *Beta dispersion close to and at the announcement moves in opposite directions across equilibria*, with betas sharply moving apart in  $E1$ , although beta compression is systematic post-announcement (reminiscent of Corollary 2 in the background model).

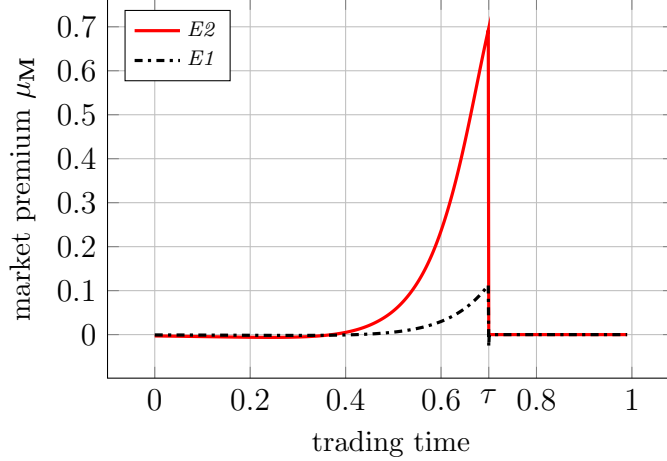
**Prediction 4.** *The CAPM works “too well”* or on the contrary the SML is downward sloping, which is entirely explained by dispersion in beta and in value.

We now elaborate on each of these predictions.

**The drift results from informed speculation (Prediction 1).** Let  $dR_{\mathbf{M},t}^e \equiv \mathbf{M}'d\tilde{\mathbf{P}}_t$  denote excess returns on the market (recall that the riskfree rate is normalized to zero), and  $\mu_{\mathbf{M},t} \equiv \mathbb{E}[dR_{\mathbf{M},t}^e]$  its unconditional expected return at date  $t$ :

$$\mu_{\mathbf{M},t} = \underbrace{-\bar{\Phi}^2 \xi_t^{-1} \sigma_{\mathbf{P}} \Pi_t dt}_{\text{effect of PI}} - \bar{\Phi}^2 \gamma \xi_{\tau-}^{-1} \left( \tau_{t-}^{-1} - \tau_t^{-1} + \underbrace{\frac{\alpha_{t-}}{(1 - \alpha_{t-}) \tau_{t-}} \ell_{t-}}_{\text{effect of PI}} \right) \mathbf{1}_{t=\tau}. \quad (104)$$

The main difference between  $E1$  and  $E2$  is the extent of informed speculation and thus the magnitude of  $\mu_{\mathbf{M}}$ . Note first that in the absence of speculation the “diffusive part” of the premium  $\Pi_t \equiv 0$  associated with resolution of uncertainty through prices is zero at all times, and the premium entirely accrues in a lump when the announcement resolves uncertainty, as in the background model (Section 2). In contrast with speculation an early premium can arise—a pre-announcement



**Figure 2: Switching the drift on and off.** This figure illustrates the market premium  $\mu_M$  across  $E1$  (black, dashdotted line) and  $E2$  (solid, red line) (see Lemma 1 and Corollary 3, respectively) over the day. The calibration is  $\tau_V = 1.4$ ,  $\tau_\epsilon = 1.1$ ,  $\tau_M = 10$ ,  $\tau_A = 1.5$ ,  $\bar{\Phi} = 0.5$ ,  $N = 5$ ,  $\tau_F = 1$ ,  $\gamma = 0.5$ ,  $\tau_m = 1/4$ ,  $\tau = 0.7$  and  $T = 1$ .

drift—because prices reveal investors’ private information,  $\tau_P \neq 0$ . The “lump” in the premium at the announcement is also higher because the announcement now affects price informativeness too. Figure 2 illustrates expected (dollar) returns across the two equilibria.

The indirect effect of price informativeness on premia is driven, to a large extent, by the wedge  $\ell$ . Eq. (104) makes this clear at the announcement but the same mechanism holds pre-announcement. Formally, we can rewrite investors’ demand in Eq. (64) as follows.

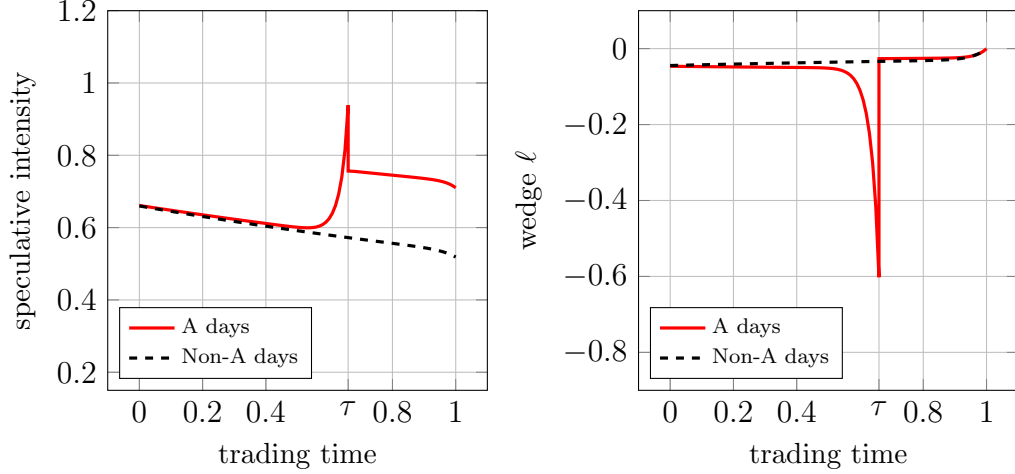
**Lemma 3.** *In equilibrium investors’ demand satisfies:*

$$\mathbf{w}^i = \underbrace{-(1 - \ell)(1 - \alpha)/\xi\Delta^i\bar{\Phi}}_{\text{speculative intensity}} + \mathbf{M} - \tilde{\mathbf{m}}. \quad (105)$$

Suppose that  $\ell < 0$ , meaning that prices reveal more fundamental information than Bayes’ rule can justify. Then investors speculate more aggressively than they would in an economy in which prices reflect the statistically optimal amount of fundamental information (and vice versa when  $\ell > 0$ ). Investors believe (rationally) that all other investors will trade ahead of the announcement, speculating on their own private piece of information. Speculation causes prices to reveal more information,  $\ell < 0$ , than Bayes’ rule can justify, leading in turn to more intensive speculation; more intensive speculation feeds back into prices and further broadens the wedge,  $\ell$ , ultimately resulting in a premium building ahead of the announcement at exponential speed. In  $E2$  this force is strong whereas in  $E1$  it is weak. Figure 3 illustrates this feedback effect between the wedge  $\ell$  and speculative intensities.

Overall, a strong pre-announcement drift comes with high price informativeness, and this constitutes a first way of telling the two equilibria apart. However, in linking price informativeness to





**Figure 3: Speculative intensity and the wedge  $\ell$  in  $E2$ .** The left-hand and right-hand plot illustrate speculative intensity (see Lemma 3) and the wedge  $\ell$  in  $E2$  (see Corollary 3) as a function of time (over the day). The black, dashed line assumes there is no public announcement over the day (Non-A days), whereas the solid, red line assumes an announcement takes place (A-days) at time  $\tau$ . The calibration is  $\tau_F = 0.5/0.35$ ,  $\tau_V = 1/2$ ,  $\tau_\epsilon = 0.8$ ,  $\gamma = 0.8$ ,  $T = 1$ ,  $\tau = 2/3$ ,  $\tau_m = 1/4$ ,  $\tau_M = 3$ ,  $\tau_A = 1.6$ ,  $\bar{\Phi} = 0.15$  and  $N = 20$ .

the drift it is key to condition on good and bad announcement outcomes, as we now explain.

**Asymmetry across good and bad news (Prediction 2).** The mechanism of Section 2 (uncertainty resolution commands a premium) is nondirectional—there is a premium on announcements whether the outcome  $\tilde{A}$  is good or bad. Suppose now that we condition pre-announcement returns based on whether the outcome  $\tilde{A}$  is good or bad, then we should observe an asymmetry between the two; the next proposition formalizes this intuition and Figure 4 provides an illustration.

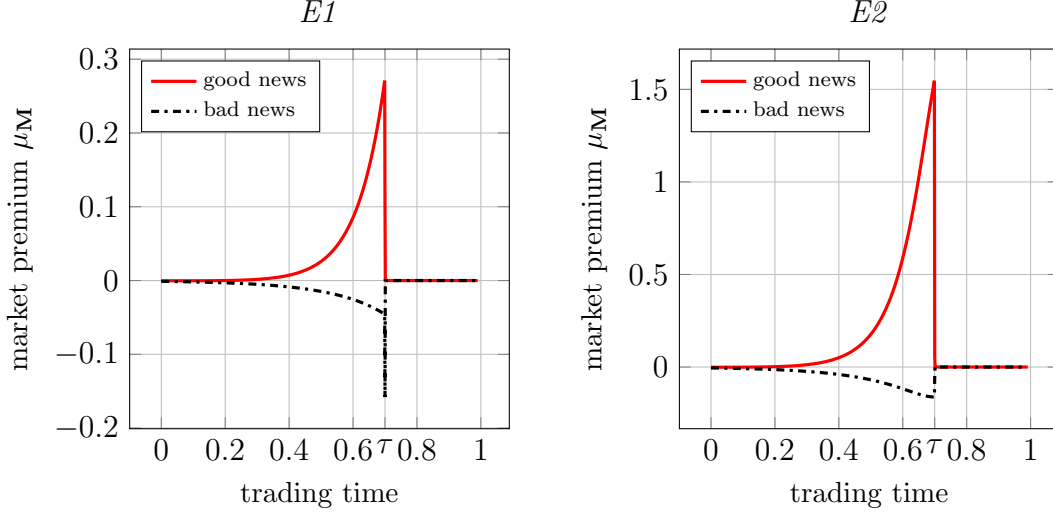
**Proposition 6.** *Conditioning expected excess market returns on “good” ( $\text{sign}(\tilde{A}) = +1$ ) and bad ( $\text{sign}(\tilde{A}) = -1$ ) public announcements, returns now satisfy the relation:*

$$\mathbb{E} \left[ dR_{M,t}^e \mid \text{sign}(\tilde{A}) \right] = \left( \mu_{M,t} - \Sigma_{\mathbf{P}}(\lambda_t \Pi_t + \tau_P^{1/2}) \text{sign}(\tilde{A}) \frac{1}{\tau_t^c} \sqrt{\frac{2}{\pi} \frac{\tau_A \tau_F}{\tau_A + \tau_F}} (1 - \mathbf{1}_{t=\tau}) \bar{\Phi} \right) dt \quad (106)$$

$$+ \mathbf{1}_{t=\tau} \left( \mu_{M,t-} + \text{sign}(\tilde{A}) \frac{1}{\tau_t^c} \sqrt{\frac{2}{\pi} \frac{\tau_A \tau_F}{\tau_A + \tau_F}} (y - \lambda_{t-} \pi_b + 1 - \lambda_t) \bar{\Phi} \right).$$

Figure 4 shows that the asymmetry in drift across good and bad news can be substantial. In particular, pre-announcement returns are highly informative on good news but exhibit little informational content on bad news. We conclude that examining price informativeness without conditioning on news direction will lead the empiricist to understate the extent of informativeness in pre-announcement returns.

**Beta dispersion moves in opposite directions pre-announcement (Prediction 3).** We first emphasize that a great advantage of observing data at high frequency is that quadratic variations



**Figure 4: Asymmetric drift across good and bad news.** This figure illustrates the market premium  $\mu_M$  conditioning on good ( $\text{sign}(\tilde{A}) = +1$ , solid, red line) and bad ( $\text{sign}(\tilde{A}) = -1$ , black, dashdotted line) news over the day. The left-hand (right-hand) panel makes this comparison in  $E1$  ( $E2$ ). The calibration is  $\tau_V = 1.4$ ,  $\tau_\epsilon = 1.1$ ,  $\tau_M = 10$ ,  $\tau_A = 1.5$ ,  $\bar{\Phi} = 0.5$ ,  $N = 5$ ,  $\tau_F = 1$ ,  $\gamma = 0.5$ ,  $\tau_m = 1/4$ ,  $\tau = 0.7$  and  $T = 1$ .

are observable. This means that the econometrician and investors must agree on instantaneous covariances. In particular, the information gap between investors and the empiricist becomes negligible as the sampling frequency increases—the law of total variance becomes an identity:

$$\mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_t^c] = \mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_t^i] + \underbrace{\mathbb{V}\left[\mathbb{E}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_t^i] \middle| \mathcal{F}_t^c\right]}_{O(dt^2) \rightarrow 0}. \quad (107)$$

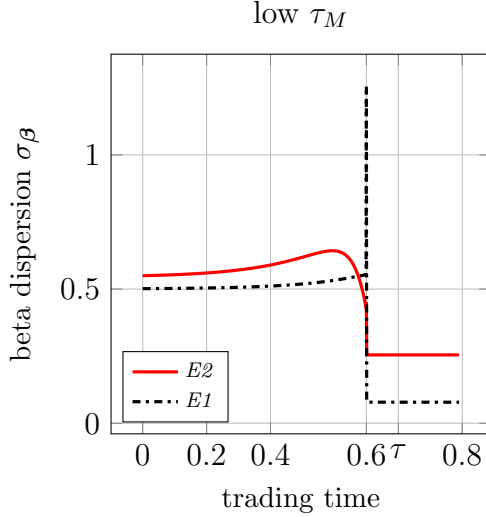
As a result empiricist and investors agree on betas,  $\hat{\beta} \equiv \beta$ , and thus measuring betas based on realized returns is theoretically appropriate (in between announcements but not at announcements).<sup>6</sup>

We first use Eq. (85) in Proposition 5 to compute the cross-sectional dispersion of betas:

$$\sigma_\beta \equiv \frac{1}{\sigma_M^2} |\sigma_M^2 - n^2/\tau_\epsilon/N| \frac{\sigma_\Phi}{\bar{\Phi}}. \quad (108)$$

This expression shows that the only source of cross-sectional variation in betas is the dispersion of loadings,  $\sigma_\Phi \equiv \sqrt{1 - \bar{\Phi}^2}$ , on the common factor, and absent market noise,  $n \equiv 0$ , the two exactly coincide. Market noise being the origin of the CAPM failure in this model it also determines the behavior of beta dispersion. Figure 5 illustrates beta dispersion and shows that if there is little market noise around the announcement beta dispersion reacts in opposite directions across the two equilibria. Specifically, in  $E1$  betas become suddenly more dispersed towards the announcement,

<sup>6</sup>On the announcement, however, the jump implied by the discreteness of the announcement regenerates the information gap between investors and, as a result, their betas differ (according to Eq. (91)). We leave this issue out of the analysis.



**Figure 5: Beta dispersion across equilibria.** This figure illustrates cross-sectional dispersion of investors’ beta  $\sigma_\beta$  (see Proposition 5) across  $E1$  (black, dashdotted line) and  $E2$  (solid, red line) (see Lemma 1 and Corollary 3, respectively) over the day. The calibration is  $\tau_V = 1.4$ ,  $\tau_\epsilon = 1.1$ ,  $\tau_M = 10$ ,  $\tau_A = 1.5$ ,  $\bar{\Phi} = 0.5$ ,  $N = 5$ ,  $\tau_F = 1$ ,  $\gamma = 0.5$ ,  $\tau_m = 1/4$ ,  $\tau = 0.7$  and  $T = 1$ .

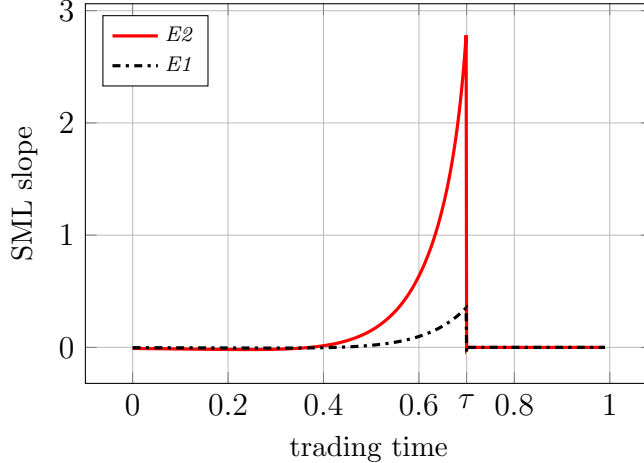
whereas they become suddenly more compressed in  $E2$ . In both cases, consistent with Corollary 2, they become more compressed after the announcement.

The possibility of increased dispersion does not arise in the background model because it does not incorporate market noise. Interestingly, low market noise is a necessary condition for both equilibria to prevail (see Figure 1). Hence if both equilibria are to prevail they will be associated with opposite patterns in beta dispersion, and thus another way to tell these two equilibria apart is to examine the respective pattern in beta dispersion. The pattern in beta dispersion in turn has implications for SML slope, and the way it may be responsible for a better-performing CAPM.

**The CAPM works “too well” (Prediction 4).** Proposition 5 shows that the CAPM fails at all times both for investors and the empiricist except at the close. This failure is a direct consequence of market noise, without which the CAPM would hold (see Section 2). The form this failure takes in the beta-return relation of Eq. (86) is simple: the SML has a nonzero intercept and a slope that differs from the market premium,  $\mu_M$ .<sup>7</sup> Note also that the way the beta-return relation evolves over time is parametrized by just two numbers,  $\mu_M$  and  $\sigma_M$ .

There are two possible scenarios (depending on the sign of  $\sigma_M^2 - n^2\tau_\epsilon/N$ ). If this expression is positive the slope is greater than  $\mu_M$  and the CAPM works “too well”; if it is negative, the relation is downward-sloping, even though the market premium is always positive. Under low market noise we expect the former scenario to prevail, namely the SML slope to exceed the market premium.

<sup>7</sup>The CAPM fails this way due to the assumption of a diagonal covariance matrix of noise in Eqs. (16) and (17). Relaxing this assumption could be interesting as the SML no longer plots on a line, for instance.



**Figure 6: SML slope across equilibria.** This figure illustrates the slope of SML as measured by investors (see Proposition 5) across  $E1$  (black, dashdotted line) and  $E2$  (solid, red line) over the day. The calibration is  $\tau_V = 1.4$ ,  $\tau_\epsilon = 1.1$ ,  $\tau_M = 10$ ,  $\tau_A = 1.5$ ,  $\bar{\Phi} = 0.5$ ,  $N = 5$ ,  $\tau_F = 1$ ,  $\gamma = 0.5$ ,  $\tau_m = 1/4$ ,  $\tau = 0.7$  and  $T = 1$ .

Figure 6 illustrates this outcome, with the SML slope appearing as a scaled-up version of the market premium in Figure 2.

This “excess slope” may have two origins in the model. Formally, we can rewrite this slope as:

$$\text{SML Slope} = \mu_M \frac{\sigma_\Phi}{\bar{\Phi}\sigma_\beta}, \quad (109)$$

In words, the extent to which the slope “overshoots” market premium is determined by the ratio of dispersion in  $\Phi$  to dispersion in betas. [Andrei, Cujean, and Fournier \(2019\)](#) show that  $\Phi$  can be thought of as market-to-book ratios. Therefore, dispersion in book-to-market ratios and/or compression in betas together explain excess slope. Note that in  $E1$  betas become more dispersed pre-announcement, which is a force that reduces slope. However, since the slope is higher than  $\mu_M$  in  $E1$  it follows that  $\sigma_\Phi/(\bar{\Phi}\sigma_\beta) > 1$  and thus excess slope is entirely explained by dispersion in market-to-book ratios. In  $E2$  betas become gradually more compressed and thus excess slope is also explained by beta compression.

## 5 Empirical tests

In this section we create two subsamples according to the type of announcements, those followed by a press conference (PC) and those that are not (non-PC). We then attempt to link the difference in asset-pricing patterns across the two subsamples to a shift from equilibrium  $E2$  to  $E1$ , exploiting the four predictions of Section 4. We start with a description of the data and proceed with each prediction in separate subsections.

## 5.1 Data description and sample selection

The analysis focuses on intraday stock returns around FOMC announcements between January 2001 and December 2022.<sup>8</sup> The FOMC holds eight regularly scheduled meetings during the year and policy decisions are published in a statement after each meeting. From 2001 to March 2011, statements were published at 2:15pm, from April 2011 to January 2013 at 12:30pm (eight announcements) or 2:15pm (seven announcements), and since March 2013 at 2:00pm.<sup>9</sup> Because there is substantial intraday variation in returns we drop 12:30pm announcements to make meaningful comparisons across announcements that take place at the same time of the day. We end up with a sample of 167 announcement days.

In April 2011 the Fed adopted a new communication policy, introducing a press conference (PC) intended to improve forward guidance following announcements. The literature documents that asset-pricing patterns now concentrate on such “PC days,” highlighting the importance of this particular subsample (e.g., [Boguth et al. \(2019\)](#); [Bodilsen et al. \(2021\)](#)); accordingly, we create a subsample “PC” that only contains announcements followed by a press conference (PC) and a corresponding subsample “non-PC.” The PC sample spans the period that starts in April 2011 and runs through December 2022 (55 announcements) and the non-PC sample runs from January 2001 through June 2018 (112 announcements). Most announcements (82 out of 112) in the non-PC sample take place prior to April 2011. Yet, there is a time overlap across the two subsamples between 2011 and 2018 as press conferences took place every second announcement over this period and only became systematic from 2018 onwards.<sup>10</sup>

We collect intraday stock price data from the New York Stock Exchange (NYSE) Trade and Quote (TAQ) database. We restrict the stock universe to S&P 500 stocks that are traded on the NYSE. The data-cleaning procedure follows [Barndorff-Nielsen, Reinhard Hansen, Lunde, and Shephard \(2009\)](#) and only considers transactions that are settled via one exchange (NYSE in our case). Because not all of the S&P 500 stocks are traded on the NYSE, our daily average sample size consists of 370 stocks.

On each trading day, we sample stock prices every five minutes and apply the previous-tick method: at the end of every 5-minute interval, we choose the most recent transaction price for each stock. To avoid potential issues arising at the market opening, we exclude the first five minutes of the trading day so that the first interval starts at 09:34 am.<sup>11</sup> Finally, we use a value-

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<sup>8</sup>We select data starting in 2001 to ensure sufficient trading volume so as to avoid problems caused by stale prices.

<sup>9</sup>Since the actual publication time sometimes differs slightly from 2:15 p.m., we use the Bloomberg or Dow Jones Newswires timestamp provided by [Lucca and Moench \(2015\)](#) as a more precise announcement time.

<sup>10</sup>Alternatively, one could form two subsamples by simply separating the whole sample at the introduction of PCs, and then have a pre-PC subsample and a post-PC subsample. We have decided against this, since existing literature also differentiates between non-PC and PC days and find different asset pricing patterns across them (e.g., [Boguth et al. \(2019\)](#); [Bodilsen et al. \(2021\)](#))

<sup>11</sup>This starting time is chosen so that we can cleanly separate the pre-announcement and post-announcement periods. Last prices before the announcement time are at 13:59 or 14:14, respectively.

weighted portfolio of all stocks present in the sample as a market proxy. For the intraday analysis of the CAPM, we create beta-sorted portfolios and follow [Bodilsen et al. \(2021\)](#) closely. A detailed procedure is described in Section 5.3.

## 5.2 Pre-announcement returns across good and bad news (Prediction 1 and 2)

In the model the pre-announcement drift reflects information (Prediction 1), and the relation between the drift and return informativeness is substantially stronger upon good news (Prediction 2). Taken together these two predictions imply that conditioning on good and bad news is key in examining the extent to which pre-announcement returns are informative.

### 5.2.1 Drift across good and bad news

We start by examining pre-announcement returns across good and bad news. We define good and bad news based on the return over the day, assigning each announcement to one of the following two groups: “good news” (PN) correspond to announcement days with a daily return that falls in the upper 25% quantile of the return distribution of all announcement days in the subsamples and “bad news” (NN) are defined similarly for the lower 25% quantile.<sup>12</sup>

Figure 7 depicts cumulative market returns for the non-PC and PC samples, and Table 1 provides a more detailed overview with summary statistics. There are three main facts. First, there is a strong asymmetry in the pre-announcement drift across days with good and bad news in the non-PC sample: the average pre-announcement return on PN days is 54.5 bps, whereas it is close to zero (4.5 bps) on NN days. Second, in the PC sample, pre-announcement returns exhibit virtually no drift, both across PN and NN days: there is an average positive drift of 11.5 bps on PN days and a drift of 8.5 bps on NN days, and none are statistically different from zero. Third, the announcement premium is larger in the non-PC sample compared to the PC sample (14.5 bps vs. 2 bps, see Table 1).<sup>13</sup> These findings suggest that returns on non-PC days are associated with the *E2* equilibrium outcome whereas returns on PC days are consistent with the *E1* equilibrium outcome.

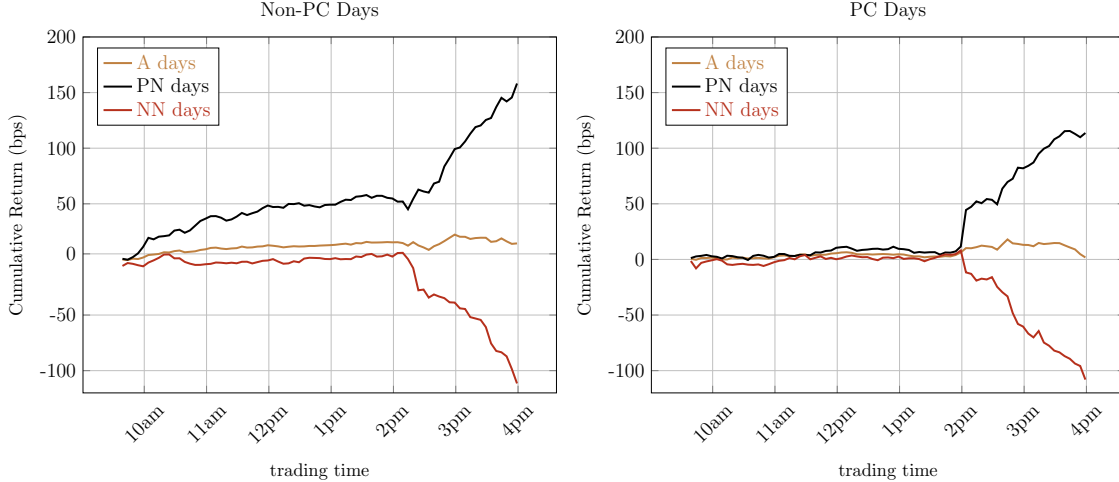
### 5.2.2 Informativeness across good and bad news

We now examine the extent to which pre-announcement returns are informative. To measure informativeness before the announcement, we adapt the measure of informed trading of [Weller \(2017\)](#),

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<sup>12</sup>We use full day returns instead of post-announcement returns because we want to consider potential anticipations before the announcement is made.

<sup>13</sup>The absence of the drift in the PC sample is consistent with the conclusion of [Boguth et al. \(2019\)](#) and [Kurov et al. \(2021a\)](#) that the average drift before all A days has vanished since the introduction of PCs.



**Figure 7: Cumulative returns on announcement days.** This figure shows the average intraday cumulative market return. The time period of the non-PC sample runs from January 2001 through June 2018. The time period of the PC sample runs from April 2011 through December 2022. Positive news (PN) days are announcement days with a daily return that falls in the upper 75% quantile of the daily return distribution, and negative news (NN) days returns in the lower 25% quantile.

which is close to the model’s definition of price informativeness. We define:

$$\omega = \frac{R^{pre}}{R^{post}}, \quad (110)$$

where  $R^{pre}$  denotes the return between the market open and the announcement time and  $R^{post}$  is the return post-announcement (including the announcement). The statistic  $\omega$  has an intuitive interpretation: positive values indicate that the market was correct in anticipating the outcome of the announcement, whereas negative values indicate the opposite. We calculate  $\omega$  for all announcement days and report in Table 1 the relative frequency of  $\omega$  being positive, which we denote by  $\Omega$ . By construction,  $\Omega$  takes values between 0 and 1. If  $\Omega=50\%$ , prices are informative only by chance (coin toss). If  $\Omega > 50\%$ , prices are informative in the sense that an external observer can learn about the announcement outcome from observing returns in the run-up to the announcement. Values that are below 50% indicate that markets are systematically wrong in anticipating the announcement outcome.

If PC days correspond to  $E1$  and non-PC days to  $E2$ , the model implies the drift on non-PC days in Figure 7 should be associated with intense pre-announcement speculation. Table 1 confirms this prediction, showing that the strong drift ahead of PN days in the non-PC sample is associated with high price informativeness: in 82% of cases, the market correctly anticipates the outcome of the announcement. On NN days, however,  $\Omega$  is substantially lower (50%). In the PC sample, we observe a similar asymmetry between PN and NN days, but informativeness is much weaker. When taking all announcements together,  $\Omega$  is slightly above 50% in the non-PC

	A-Days	PN Days	NN Days
<b>Non-PC Days</b>			
Return (bps)	14.42 (1.35)	158.15*** (9.21)	-111.48*** (-8.64)
Pre-A Drift (bps)	15.66*** (3.44)	54.60*** (5.20)	4.50 (0.50)
$\Omega$	52.68	82.14	50.00
Observations	112	28	28
<b>PC Days</b>			
Return (bps)	1.78 (0.15)	113.67*** (7.26)	-108.06*** (-8.85)
Pre-A Drift (bps)	6.30 (1.42)	11.60 (1.42)	8.03 (0.71)
$\Omega$	32.73	57.14	42.86
Observations	55	14	14

**Table 1: Returns and price informativeness on announcement days.** This table shows the average pre-announcement returns, average returns and price informativeness  $\Omega$  on FOMC announcement days.  $\Omega$  is the relative frequency of  $\omega$  being positive, indicating a correct anticipation of the announcement outcome of the FOMC announcement. The time period of the non-PC sample runs from January 2001 through June 2018. The time period of the PC sample runs from April 2011 (when press conferences were introduced) through December 2022. Positive news (PN) days are announcement days with a daily returns that fall in the upper 75% quantile of the daily return distribution of the respective sample, and negative news (NN) days in the lower 25% quantile. The t-statistics are shown in brackets. \*\*\*, \*\*, \* indicate significance at the 10%, 5% and the 1% level, respectively.

sample and substantially below 50% in the PC sample, which is consistent with the literature finding that pre- and post-announcement returns are unrelated (e.g., [Laarits \(2022\)](#)); in the PC sample betting in the direction opposite to pre-announcement returns would have done better at predicting the announcement outcome than the market prior to the announcement. These findings support the model’s predictions that the drift is strongly associated to price informativeness and that conditioning on good and bad news is key in identifying informative pre-announcement returns.

### 5.2.3 Alternative explanations for the asymmetry in the drift

In the model speculation is associated with an upward (downward) pre-announcement drift ahead of positive (negative) news. Because prices are informative they partially resolve uncertainty and thus command a premium pre-announcement. This premium adds to the existing positive return on



PN days, but partly offsets the negative drift on NN days, which ultimately creates an asymmetry in pre-announcement returns. This prediction of the model is consistent with what we find in the data. Nevertheless, the observed asymmetry in the drift could have other origins than those implied by the model, which we will now test and discuss in this section. We start by controlling for two plausible, alternative stories. We then control for factors that are known to explain a strong drift.

**Difficulty in short-selling.** Since speculation on bad news requires selling short, a natural explanation for the asymmetry is that short-selling is costly. We create a proxy for short-selling costs using option prices, which we obtain from OptionMetrics.<sup>14</sup> Following Lamont and Thaler (2003) we create a synthetic short position in the market index by writing a call at the bid, buying a put at the ask and borrowing money in the amount of the strike price.<sup>15</sup> We use European at-the-money (ATM) put and call option pairs on the S&P 500 index (SPX) with same strike price and maturity to remain as close as possible to put-call parity. We then define implicit short-selling costs as the percentage deviation of the synthetic short from the S&P 500 index one day before the announcements. We consider all available ATM put and call pairs for different maturities at each date and calculate a weighted average of short-selling costs, taking the relative open interest of the respective option pairs as the weight.

Since there is a strong asymmetry in the pre-announcement drift in the non-PC sample, it may be that short-selling costs were particularly high prior to 2011, for instance. To examine whether this is the case, we regress pre-announcement returns on our measure of short-selling costs for positive news (Table 2) and negative news (Table 3) separately. The two tables show that short-selling costs do not have a significant impact on PN days nor on NN days on the pre-announcement drift, which suggests that short-selling costs are an unlikely driver of the asymmetry.

**Asymmetry in signal.** Another possibility is that there is an asymmetry in the signal itself. Among possible asymmetries the Fed’s put comes to mind.<sup>16</sup> There is evidence that the Fed tends to conduct accommodating monetary policies following periods of low market returns, but does not systematically tighten policies in good times. That is, there exists a Fed put without a corresponding Fed call, which introduces an asymmetry in signals across positive and negative news. From a trader’s perspective, negative returns over the past FOMC cycle (Cieslak and Vissing-Jorgensen, 2020) signal a Fed put.<sup>17</sup> To control for the Fed put, we create a dummy variable that takes value 1 if market excess return is negative over the previous FOMC cycle and 0 otherwise.

Regressing pre-announcement returns on this dummy, Table 3 shows that negative returns over the past cycle in fact raise pre-announcement returns upon bad news (see regression (2) in the

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<sup>14</sup>There is evidence that put-call parity violations are positively related to short-selling costs (Lamont and Thaler (2003); Ofek, Richardson, and Whitelaw (2004), Evans, Geczy, Musto, and Reed (2008) Atmaz and Basak (2019).)

<sup>15</sup>If the underlying pays dividends, the present value of the dividends must be additionally borrowed

<sup>16</sup>See Cieslak and Vissing-Jorgensen (2020) for a detailed study on this matter.

<sup>17</sup>A FOMC cycle is the period between two FOMC announcements, which usually lasts six weeks.

table), which suggests that the market may incorrectly anticipate a Fed put on NN days. Whereas past negative returns also lead to a higher pre-announcement drift on PN days (see Table 2), which would indicate that the market “correctly” speculates on the Fed put, the effect is statistically weak. We conclude that the Fed put could be a driver of the asymmetry in the drift, but perhaps in an unexpected way (as the market seems to incorrectly speculate on it).

**Other relevant variables.** Another possibility, which we do not consider in our model, is that the asymmetry comes from the “type” of news disclosed. The literature makes a distinction between a “monetary policy channel” and an “information channel” (e.g., [Gürkaynak, Sack, and Swanson \(2005\)](#), [Matheson and Stavrev \(2014\)](#), [Cieslak and Schrimpf \(2019\)](#), [Kroencke et al. \(2021\)](#) or [Laarits \(2022\)](#)). They identify the former channel with positive comovement between stock and bond returns and the latter with negative comovement.<sup>18</sup> To examine whether the asymmetry in the drift can be explained by an asymmetry between the two channels, we create a dummy variable *MP-channel* that takes value of 1 if treasury bond returns and stock returns move in the same direction on the announcement day (“monetary mechanism”) and 0 if they move in opposite directions (“information channel”).<sup>19</sup> Regression (5) in Table 2 shows that the pre-announcement drift on PN days is on average 51.29 bps weaker if the announcement affects the stock market through the monetary channel. This suggests that non-monetary announcements could be in part responsible for the asymmetry.

We also verify that the market properly anticipates the news, that is PN and NN days are not merely positive or negative surprises. We follow [Kuttner \(2001\)](#) and evaluate the effect of a surprise change in the Fed funds target rate. Specifically, we compute surprise changes from 30-day Federal funds futures data, which we include as a control in the regression (*unexp*  $\Delta FF$ ). The effect is significant on PN days and insignificant on NN days, although one should apply some caution in interpreting this result. First, the intercept on PN days remains strongly significant and positive, meaning that surprises likely play a minor role in explaining the drift on good news. Second, given the zero lower bound on rates and that rates have been low in the last decade significant changes may only correspond to interest rate hikes. Given our further finding in regression (5) that the drift is substantially higher on non-monetary news, the positive effect of surprises on PN days is difficult to interpret.

Finally, we control for measures of risk. In particular, we use the “risk shift” (RS) measure of [Kroencke et al. \(2021\)](#), which is intended to measure the effect of the announcement on risk

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<sup>18</sup>The former concerns monetary news and their effect on asset prices, e.g., unexpected hikes (drops) in target rates reduce (raise) asset prices, which is associated with positive comovement between stock and bond returns; the latter captures non-monetary news that affect expectations of economic outlooks or risk premia, and tends to be associated with a negative covariance between stocks and bonds ([Cieslak and Schrimpf, 2019](#)).

<sup>19</sup>We use the return on a portfolio consisting of an ETF of 1-3 year treasury bonds (ticker: SHY) and an ETF of 7-10 year treasury bonds (ticker: IEF) as a measure of bond returns. The data is retrieved from CRSP. Since time series for these ETFs start in July 2002, we use changes of treasury yields with different maturities obtained from [s](#) as a measure for bond returns for the period January 2001-July 2002.

appetite (high risk shift indicates that markets goes in “risk-on” mode).<sup>20</sup> Additionally, we control for the level of the VIX at the open of the announcement day, as [Lucca and Moench \(2015\)](#) find that higher levels of VIX are associated with a higher pre-announcement drift. We find that higher risk shifts have a significantly negative effect on the pre-announcement drift on PN days. We interpret this result as risk shifts being highest if news are positive and come as a surprise (there is no anticipation in the run-up and thus no drift). Consistent with [Lucca and Moench \(2015\)](#) we find that higher VIX has a positive effect on the pre-announcement return on both PN and NN days, although the magnitude is statistically stronger and twice as large on NN days.

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<sup>20</sup>We obtain the data on RS for 2006-2019 directly from Tim Kroenckes website. For the 2001-2005 period, VIX is used as the only risky asset to calculate RS.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	16.81 (20.27)	32.11*** (11.53)	-4.54 (22.11)	42.58*** (8.32)	64.69*** (12.65)	62.39*** (11.33)	24.72 (26.53)
<i>ShortSellingCosts</i>	0.67 (0.44)						-0.02 (0.80)
<i>FedPutSignal</i>		16.30 (16.57)					-5.08 (20.48)
<i>VIX</i>			1.87* (1.06)				1.90 (1.39)
<i>unexp ΔFF (bps)</i>				3.84** (1.75)			2.51 (3.27)
<i>MP-Channel</i>					-51.29*** (14.15)		-37.60 (27.51)
<i>RiskShift</i>						-21.68** (9.59)	-7.42 (15.55)
R-squared	0.10	0.02	0.13	0.11	0.24	0.13	0.42
Observations	40	42	42	42	42	38	38

**Table 2: Drivers of pre-Announcement returns on PN Days** This table shows the regression estimates of the pre-announcement market return on PN days on various potential drivers. The pre-announcement drift is calculated between the open and the time when the announcement is made. *unexp ΔFF (bps)* is the unexpected change of the Federal funds target rate, which is calculated with the approach of Kuttner (2001). *MP-channel* is a dummy variable which is 1 if the returns on the announcement day of a portfolio of treasury securities and the market return have the same sign, and 0 otherwise. *FedPutSignal* is a dummy variable which assumes the value 1 if the previous FOMC cycle return was negative, and 0 otherwise. *VIX* is the CBOE implied volatility index at the market open of the announcement day. *ShortSellingCosts* obtained from a synthetic short position by using ATM European Index options on the S&P500 and *RiskShift* is the measure for change in risk appetite introduced by Kroencke et al. (2021). The considered time horizon starts in January 2001 and ends in December 2022, except of the regressions that include *RS*, as the data is only available until 2019 and regressions that include *ShortSellingCosts*, as this data is only available until 2021 (at the time of writing). Heteroskedasticity-robust standard errors are shown in brackets. \*\*, \*\*\*, \* indicate significance at the 10%, 5% and the 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	-4.00 (13.63)	-9.66 (6.15)	-45.67*** (16.34)	5.22 (7.83)	9.41 (10.53)	5.04 (10.94)	-69.46* (40.91)
<i>ShortSellingCosts</i>	0.18 (0.40)						0.00 (0.34)
<i>FedPutSignal</i>		42.93*** (15.56)					27.19 (20.53)
<i>VIX</i>			2.37*** (0.83)				2.47 (1.76)
<i>uncexp ΔFF (bps)</i>				0.92 (2.97)			2.18 (2.72)
<i>MP-Channel</i>					-9.80 (13.00)		15.75 (20.00)
<i>RisksShift</i>						-4.40 (7.60)	-12.37 (13.06)
R-squared	0.02	0.21	0.28	0.00	0.01	0.01	0.48
Observations	38	42	42	42	42	34	34

**Table 3: Drivers of pre-Announcement returns on NN Days** This table shows the regression estimates of the pre-announcement market return on PN days on various potential drivers. The pre-announcement drift is calculated between the open and the time when the announcement is made. *uncexp ΔFF (bps)* is the unexpected change of the Federal funds target rate, which is calculated with the approach of Kuttner (2001). *MP-channel* is a dummy variable which is 1 if the returns on the announcement day of a portfolio of treasury securities and the market return have the same sign, and 0 otherwise. *FedPutSignal* is a dummy variable which assumes the value 1 if the previous FOMC cycle return was negative, and 0 otherwise. *VIX* is the CBOE implied volatility index at the market open of the announcement day. *ShortSellingCosts* obtained from a synthetic short position by using ATM European Index options on the S&P500 and *RisksShift* is the measure for change in risk appetite introduced by Kroencke et al. (2021). The considered time horizon starts in January 2001 and ends in December 2022, except of the regressions that include *RS*, as the data is only available until 2019 and regressions that include *ShortSellingCosts*, as this data is only available until 2021 (at the time of writing). Heteroskedasticity-robust standard errors are shown in brackets. \*\*\*, \*\*, \* indicate significance at the 10%, 5% and the 1% level, respectively.

### 5.3 The CAPM on FOMC announcements (Prediction 3 and 4)

The model predicts that the CAPM works “too well” on announcement days under the low market noise scenario and that dispersion in value and potentially beta compression can explain this fact. Is this true in the data? We now attempt to link predictions across equilibria to empirical findings across the PC and non-PC sample. We start by testing Prediction 3 regarding beta dispersion.

#### 5.3.1 Beta dispersion across subsamples (Prediction 3)

Under the low-noise scenario Prediction 3 indicates that beta dispersion moves in opposite directions across equilibria around announcements, but drops systematically in the post-announcement window (beta compression). To estimate beta we follow Andersen et al. (2021) and Bodilsen et al. (2021) closely. Specifically, on each trading day we estimate the intraday beta at the stock level:

$$\beta_i = \frac{\sum_{i=1}^{n_c} R_i R_m}{\sum_{i=1}^{n_v} R_m^2}, \quad (111)$$

where  $R_m$  is the market return and  $R_i$  is the return on stock  $i$ . Since covariances tend to be downward biased at high frequencies due to asynchronous trading—the so-called “Epps effect” (Epps, 1979)—we use a 10-minute frequency for the covariation part ( $n_c=38$ ) and a 5-minute frequency for the variation part ( $n_v=75$ ). This way we attenuate measurement issues associated with asynchronous trading and microstructure noise. Each day, realized betas on individual stocks are used to form ten value-weighted beta-sorted portfolios, which we index by  $p = \{1, 2, \dots, 10\}$ . In a third step, we estimate betas for each beta-sorted portfolio using a 5-minute frequency for both the covariation and the variation part of the beta formulation in Eq. (111).<sup>21</sup>

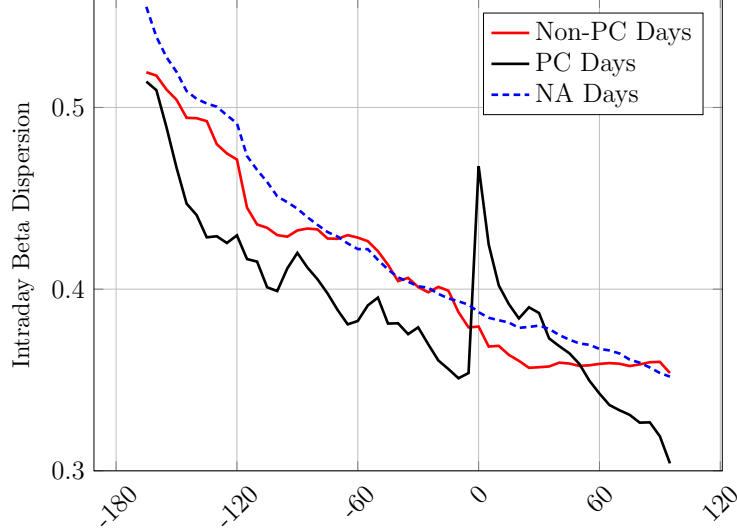
We follow Andersen et al. (2021) in estimating beta dispersion, and define:

$$\sigma_\beta = \sqrt{\frac{1}{10} \sum_{p=1}^{10} (\beta_p - 1)^2}. \quad (112)$$

We then plot beta dispersion in Figure 8 on announcement (A) days over 90-minutes rolling windows on PC-days, non-PC days and non-announcement (NA) days separately.<sup>22</sup> We include NA days because they serve as benchmark to assess whether betas become relatively more or less dispersed throughout the trading day. Beta compression increases over the trading day, consistent with the pattern documented by Andersen et al. (2021) and Bodilsen et al. (2021) and which holds on any day, whether or not an announcement takes place. A key difference across subsamples, however, is that, at the announcement, there is a spike in beta dispersion in the PC sample, whereas betas

<sup>21</sup>The complications arising from asynchronous trading and microstructure noise is a minor issue if we use portfolios instead of stocks. Therefore, we estimate covariances at a 5-min frequency.

<sup>22</sup>Following Andersen et al. (2021), we restrict the NA sample to Wednesdays without FOMC announcements.



**Figure 8: Intraday beta dispersion.** This figure shows the average intraday beta dispersion on announcement and non-announcement (NA) days of S&P500 stocks which are traded on the NYSE. We use ten beta-sorted portfolios and estimate intraday beta dispersion over 90-min rolling windows. The non-PCs sample consists of FOMC announcement days without press conferences (PCs) between January 2001-June 2018. The time period of the PC sample runs from April 2011 through December 2022. NA days consists of all Wednesdays without an announcement and span the period from 2001-2022.

become gradually more compressed in the non-PC sample.<sup>23</sup> Moreover, following the spike at the announcement, there is a sharp drop in beta dispersion in the post-announcement window on PC days, whereas it remains flat towards the end of the trading day in the non-PC sample. Importantly, these findings match the patterns in beta dispersion across equilibria  $E1$  and  $E2$  under the low noise scenario as shown in Figure 5.

### 5.3.2 The CAPM works “too well” (Prediction 4)

One of the main asset-pricing facts about FOMC announcements is that the CAPM works better on these occasions (Savor and Wilson, 2014). The model indicates that there are two scenarios for the CAPM on announcement days: (i) it either works “too well” on announcements, meaning that the SML slope exceeds the market premium, or (ii) the SML slope and the market premium have opposite signs; accordingly we define excess slope as the difference between the SML slope and market returns and refer to “positive excess slope” when SML slope is above (below) the corresponding positive (negative) market return. If excess slope is 0, then the CAPM works spot on. In contrast, a negative excess slope indicates that the SML is “too flat.” To examine how well the CAPM works in the data, we run Fama and MacBeth (1973) regressions at the daily level. We

<sup>23</sup>The spike in beta dispersion was not found by Bodilsen et al. (2021). This is most likely coming from the fact that we do not truncate the stock returns, as we are particularly interested in both the diffusion and the jumps in returns.

	A-Days	PC Days	Non-PC Days
Intercept	-13.01 (-1.63)	-22.59 (-1.67)	-8.22 (-0.83)
Slope	26.97* (1.72)	25.17 (1.10)	27.87 (1.35)
Excess Slope	46.00*** (6.59)	49.92*** (4.23)	44.03*** (5.07)
Avrg Beta Disp	0.37	0.34	0.39
Avrg Value Disp	0.28	0.29	0.28
Observations	129	43	86

**Table 4: Excess Slope of the SML.** This table shows the average daily intercept and slope of the SML estimate with the [Fama and MacBeth \(1973\)](#) approach using day-specific betas which are estimated intraday, the difference between the slope and the market return (excess slope), the average beta dispersion and the average dispersion in value (book-to-market ratios). The non-PCs sample consists of FOMC announcement days without press conferences (PCs) between January 2001-June 2018. The time period of the PC sample runs from April 2011 through December 2022. All measures are in basis points (bps). T-statistics are shown in brackets. \*\*\*, \*\*, \* indicate significance at the 10%, 5% and the 1% level, respectively.

form day-specific portfolio betas estimated intraday along the lines of Section 5.3.1. We find that for 38 out of 167 announcements, the SML slope and the market return have opposite signs, which corresponds to the second scenario. We now wish to verify whether the first scenario holds within the remaining 129 announcement days to which we restrict the analysis. The results are summarized in Table 4.

Table 4 shows that Prediction 4 is verified on announcement days and within both the PC and non-PC sample, meaning that conditional on the SML slope and market premium having the same sign excess slope is positive or equivalently the CAPM works too well across all announcements. In particular, excess slope is economically and strongly statistically significant on all announcement days, with a slightly stronger magnitude on PC days. We conclude that the first scenario holds within the remaining 129 announcement days.

The model further predicts that positive excess slope (in the first scenario) originates from beta dispersion,  $\sigma_\beta$ , or dispersion in fundamental value,  $\sigma_\Phi$ , or both (see Eq. (109)). Specifically, which of more compressed betas or more dispersed values or both lead to a steeper SML depends on which of the two equilibria  $E1$  or  $E2$  prevails. To investigate this matter we need a measure of dispersion in value and thus a measure of fundamental value. [Andrei et al. \(2019\)](#) show that the vector  $\Phi$  is equivalent to market-to-book ratios, and we adopt their approach by constructing book-to-market



	A Days	PC Days	non-PC Days
	(1)	(2)	(3)
Constant	-17.60 (-0.77)	-50.10 (-1.31)	-12.06 (-0.43)
Ratio	80.17*** (2.61)	111.15** (2.41)	75.79* (1.83)
R-squared	0.20	0.19	0.21
Observations	129	43	86

**Table 5: Excess slope and dispersion in beta and value.** This table shows the coefficient estimates of a linear regression of the excess slope of the security market line (SML) relative to market return on the ratio between dispersion in beta and dispersion in value (B/M ratios). The non-PCs sample consists of FOMC announcement without press conferences (PCs) between January 2001-June 2018. The time period of the PC sample runs from April 2011 through December 2022. Heteroskedasticity robust t-statistics are shown in brackets. \*\*\*, \*\*, \* indicate significance at the 10%, 5% and the 1% level, respectively.

(B/M) ratios at the stock level following [Bali, Engle, and Murray \(2016\)](#) closely. We then calculate the value-weighted average B/M of each beta-sorted portfolio one day prior to the announcement, compute its standard deviation  $\sigma_{B/M}$  and scale it by the absolute value of the mean B/M ratio:

$$\frac{\sigma_{\Phi}}{\bar{\Phi}} \approx \frac{\sigma_{B/M}}{|B/M|}. \quad (113)$$

Because we assume fundamentals are driven by a single common factor in the model beta dispersion itself is entirely driven by dispersion in value (see Eq. (108)). The correlation between the two across announcement days is 0.23 in the non-PC sample and 0.58 in the PC sample. Therefore, collinearity is potentially an issue when controlling jointly for the two. To circumvent this issue we use Eq. (109), which shows that the ratio of the two:

$$\text{Ratio} \equiv \frac{\sigma_{\Phi}/\bar{\Phi}}{\sigma_{\beta}} \quad (114)$$

determines entirely the extent of excess slope. We then regress excess slope on this ratio and report the results in Table 5. This ratio is highly statistically significant on all announcement days, but relatively weaker on non-PC days. However, in all subsamples the ratio explains around 20% of variation in excess slope. Equally importantly, in all subsamples the average excess slope, which without controlling for the ratio is strongly statistically different from 0, becomes statistically insignificant, which is consistent with the model's predictions and more specifically with Eq. (108).

### 5.3.3 Market noise and the introduction of PCs

The findings of the previous subsections suggest that asset-pricing implications differ across the PC and non-PC samples the same way they do across equilibria  $E1$  and  $E2$ . Furthermore, in Section 4.1 we have shown that  $E2$  disappears as market noise rises. Therefore, it is possible that the apparent shift from  $E2$  to  $E1$  is in fact unrelated to the introduction of PCs but rather that market noise rose in the period following their introduction. Note that the purpose of PCs is to improve forward guidance, which in the model would correspond to an increase in the announcement informativeness,  $\tau_A$ . This is, however, different from the noise in market reaction to the announcement, as measured by  $n_M$  in the model. Section 4.1 shows that the extent of noise  $n_M$  is the dimension determining the existence of  $E2$ . We now examine the evolution of noise in market reaction to announcements over our sample period, looking specifically for notable changes across the pre- and post-2011 period.

In the model market reaction to the announcement represents a discontinuity (“a jump”) in returns. This jump contains fundamental information and noise. Furthermore, in the model fundamental information remains unchanged throughout the trading day (it is a permanent component) whereas noise gradually reverts in the post-announcement period (it is a transitory component). To construct a proxy for market noise,  $n_M$ , we need to capture the transitory part of market reaction. In a first step, we identify jumps following the announcement based on the approach in Mancini (2001, 2009). In particular, we define a jump as an increment in returns that exceeds the threshold level:

$$v_{t_M} = 3\sqrt{BV_{t,n}^M}n^{-0.47}, \quad (115)$$

where  $n$  is the number of time increments within one trading day  $t$  and  $BV_{t,n}^M$  denotes the bipower variation (Barndorff-Nielsen and Shephard, 2004) of the market portfolio  $M$  on day  $t$ .<sup>24</sup> In the model, the jump occurs immediately at the announcement time, but in the data the market reaction to the announcement may lag a few minutes after the announcement. Figure 9 illustrates the frequency of jumps occurrence over an announcement day. Unsurprisingly the vast majority of jumps occur right at the announcement. Yet, since there is a nontrivial number of jumps occurring within minutes following the announcement, we define market reaction (jump) as the return over the 30-minute window following the announcement.

Now, to disentangle fundamental information from noise in jumps, we follow the idea of Boguth, Grégoire, and Martineau (2022) and run the following regressions:

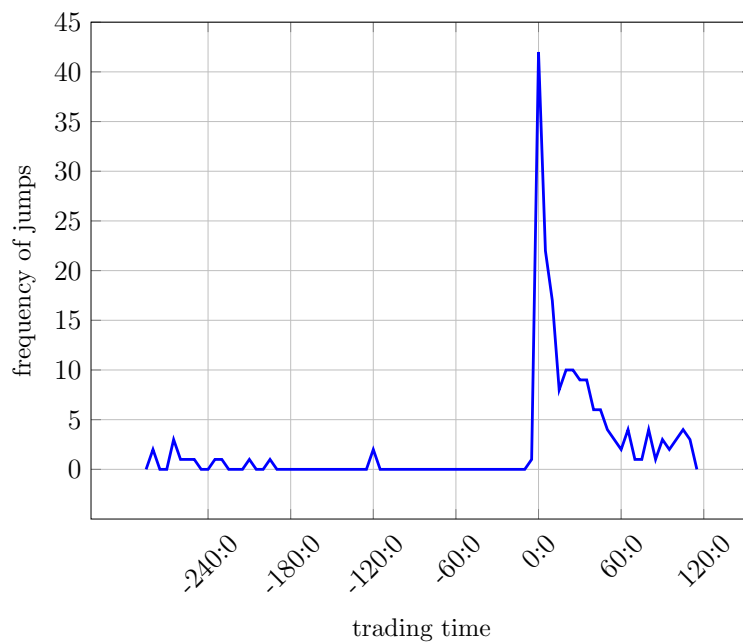
$$R(t_{open}, (t+1)_{close}) = \alpha + \beta R(t_{open}, t_{announcement-1min}) + \epsilon_t, \quad (116)$$

$$R(t_{open}, (t+1)_{close}) = \alpha + \beta R(t_{open}, t_{announcement+30min}) + \epsilon_t. \quad (117)$$

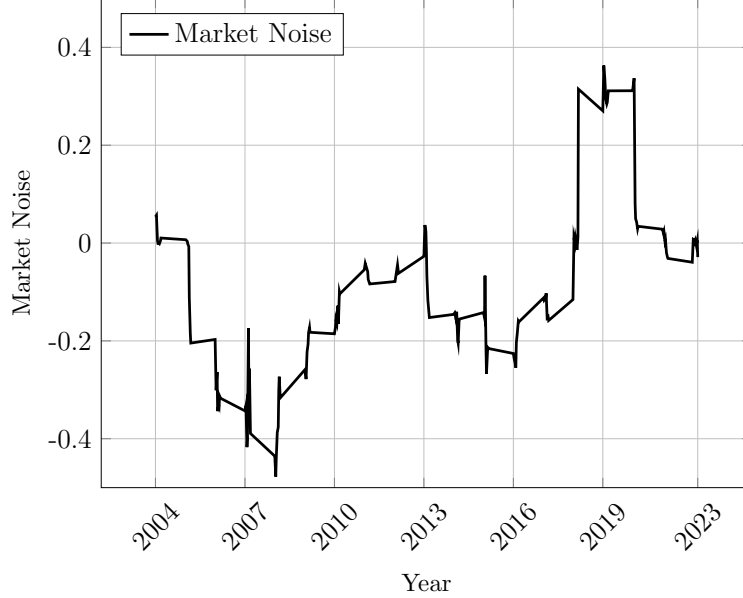
In a first regression we regress the return over the trading day and the following day on the

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<sup>24</sup>Here, one trading day lasts 6h25min, which results in  $n = \frac{6h25min}{5min} = 77$



**Figure 9: Jumps of the market return.** This figure shows the total number of jumps over the trading day across announcement days of the return on a market portfolio consisting of S&P 500 firms which are traded on the NYSE. A return is considered as a jump if it exceeds the threshold  $v_{t_n} = 3\sqrt{BV_{t,n}^m n^{-0.47}}$ , where  $BV_{t,n}^m$  is the bipower variation (Barndorff-Nielsen and Shephard (2004)) that estimates the diffusive part of the quadratic variation of the market portfolio  $m$  on day  $t$ . The considered time period runs from January 2001 through December 2022.



**Figure 10: Market noise over time.** This figure shows the evolution of the noise in the market reaction to the announcement between 2001-2022. Market noise is measured as  $1 - \delta R^2$ , where  $\Delta R^2$  is the difference between the  $R_{pre}^2$  and the  $R_{post}^2$ .  $R_{pre}^2$  is the fraction of the variance in the market return from the open at the announcement day to the close of the subsequent day which is explained by the pre-announcement return.  $R_{post}^2$  is the fraction that is explained by the cumulative return from the open to 30 minutes post-announcement. market noise is estimated over 3-year rolling windows (24 announcements.)

pre-announcement return and record its R-squared which we denote by  $R_{pre}^2$ . We use the return post-announcement as dependent variable to control for the fundamental part of the price; we extend it to the next trading day to account for any delayed reversion of the noise in the jump. In a second regression we regress the same dependent variable on the return between the open and 30 minutes after the announcement and record the R-squared which we denote by  $R_{post}^2$ . Finally, we measure noise as the difference between the two R-squared:

$$\text{market noise} \equiv -(R_{post}^2 - R_{pre}^2) = -\Delta R^2 \approx n_M. \quad (118)$$

Intuitively, when market noise  $\equiv -1$  the market reaction is fully informative; in this case the variance of the return on the announcement day and the next day is fully explained by the jump in the return at the announcement. In the other extreme case when market noise  $\equiv 1$ , the market reaction is pure noise. The evolution of the resulting measure of market noise estimated over three-years rolling windows (24 announcements) is plotted in Figure 10.

There are two main patterns in Figure 10. First, market noise drops substantially in the first part of the sample. Second, since the financial crisis of 2008 market noise is trending upwards, with a substantial rise between 2018 and 2020. Mapping this evolution into Figure 1 suggests that the disappearance of  $E2$  is merely coinciding with the introduction of PCs but that its genuine cause

is the rise of market noise that also coincides with their introduction.

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# A Appendix

## A.1 Proof of Proposition 1

In this appendix we determine how an investor  $i$  perceives her investment opportunity set conditional on her information. We slice the time interval  $[0, T)$  into two subperiods,  $[0, \tau)$  and  $[\tau, T)$ . We first determine how beliefs are updated over  $(0, \tau)$  and  $(\tau, T)$  and then turn to how they are updated at date  $\tau$ . Removing commonly observable terms, prices are informationally equivalent to:

$$\tilde{\mathbf{P}}_t^a = \boldsymbol{\xi}_t \tilde{\mathbf{m}}_t + \Phi \lambda_t \tilde{F}. \quad (119)$$

We stack observables (excluding public announcements), both private and public, in a vector  $\mathbf{s}_t^i = (\tilde{V}_t^i \quad (\tilde{\mathbf{P}}_t^a)')'$  and unobservables in a vector  $\mathbf{z}_t = (\tilde{F} \quad \tilde{\mathbf{m}}_t)'$ . Applying Ito's lemma to the sufficient price statistic, their dynamics satisfy:

$$d\mathbf{s}_t^i = \underbrace{\begin{pmatrix} 1 & \mathbf{0} \\ \lambda_t' \Phi & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix}}_{\equiv \mathbf{A}_{sz}} \mathbf{z}_t dt + \underbrace{\begin{pmatrix} \tau_v^{-1/2} & \mathbf{0} \\ \mathbf{0} & \tau_m^{-1/2} \boldsymbol{\xi}_t \end{pmatrix}}_{\equiv \mathbf{b}_s} d \underbrace{\begin{pmatrix} B_t^i \\ \mathbf{B}_{m,t} \end{pmatrix}}_{\equiv \mathbf{B}_t}, \quad (120)$$

and

$$d\mathbf{z}_t = \underbrace{\begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & -\mathbf{b} \end{pmatrix}}_{\equiv \mathbf{A}_{zz}} \mathbf{z}_t dt + \underbrace{\begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \tau_m^{-1/2} \mathbf{I} \end{pmatrix}}_{\equiv \mathbf{b}_z} d\mathbf{B}_t, \quad (121)$$

where  $\mathbf{0}$  denotes a vector or matrix of zeros of conformable size. Kalman filtering equations (e.g., [Lipster and Shiryaev \(2001\)](#)) imply that investor  $i$ 's expectations about  $\mathbf{z}_t$  are updated according to:

$$d\hat{\mathbf{z}}_t^i = \mathbf{A}_{zz} \hat{\mathbf{z}}_t^i dt + (\mathbf{T}_t \mathbf{A}'_{sz} + \mathbf{b}_z \mathbf{b}'_s) (\mathbf{b}_s \mathbf{b}'_s)^{-1/2} d\hat{\mathbf{B}}_t^i, \quad (122)$$

where  $\hat{\mathbf{B}}^i$  is a  $N + 1$ -dimensional  $\hat{\mathbb{P}}^i$ -Brownian motion and  $\mathbf{T}_t \equiv \text{Var}[\mathbf{z}_t | \mathcal{F}_t^i]$  denotes the posterior covariance matrix. Using observational equivalence, which implies that

$$\tilde{\mathbf{m}}_t - \hat{\mathbf{m}}_t^i = -\boldsymbol{\xi}_t^{-1} \Phi \lambda_t (\tilde{F} - \hat{F}_t^i), \quad (123)$$

we can rewrite this matrix as:

$$\mathbf{T}_t = \tau_t^{-1} \begin{pmatrix} 1 & -\lambda_t \Phi' \xi_t^{-1} \\ -\lambda_t \xi_t^{-1} \Phi & \underbrace{\lambda_t^2 \xi_t^{-1} \Phi \Phi' \xi_t^{-1}}_{\equiv \Omega_t} \end{pmatrix}. \quad (124)$$

Rewriting the (incremental) price signal-to-noise ratio  $\tau_{P,t}$  defined in Eq. (45) as (here assuming that  $\mathbf{b} \neq \mathbf{0}$  for generality):

$$-\tau_{P,t}^{1/2} \Phi = \tau_m^{1/2} \left( \lambda_t' \xi_t^{-1} - \lambda_t \xi_t^{-1} \dot{\xi}_t \xi_t^{-1} + \mathbf{b} \lambda_t \xi_t^{-1} \right) \Phi, \quad (125)$$

we can in turn rewrite the filter dynamics, which hold over the two subperiods (pre- and post-announcement) as:

$$d\widehat{\mathbf{z}}_t^i = \mathbf{A}_{zz} \widehat{\mathbf{z}}_t^i dt + \begin{pmatrix} \tau_v^{1/2} \tau_t^{-1} & -\tau_t^{-1} \tau_{P,t}^{1/2} \Phi' \\ -\lambda_t \tau_t^{-1} \tau_v^{1/2} \xi_t^{-1} \Phi & \lambda_t \tau_t^{-1} \tau_{P,t}^{1/2} \xi_t^{-1} \Phi \Phi' + \tau_m^{-1/2} \mathbf{I} \end{pmatrix} d\widehat{\mathbf{B}}_t^i, \quad t \in (0, \tau) \cup (\tau, T). \quad (126)$$

The Kalman filtering equation for  $\mathbf{T}_t$  satisfies:

$$\dot{\mathbf{T}}_t = \mathbf{A}_{zz} \mathbf{T}_t + \mathbf{T}_t \mathbf{A}'_{zz} + \mathbf{b}_z \mathbf{b}'_z - (\mathbf{T}_t \mathbf{A}'_{sz} + \mathbf{b}_z \mathbf{b}'_s) (\mathbf{b}_s \mathbf{b}'_s)^{-1} (\mathbf{A}_{sz} \mathbf{T}_t + \mathbf{b}_s \mathbf{b}'_z). \quad (127)$$

Using the simplification in Eq. (124) and the definition of price informativeness gives an ODE for  $\tau_t$  over each subinterval:

$$\tau_t' = \tau_{P,t} + \tau_v, \quad t \in (0, \tau) \cup (\tau, T). \quad (128)$$

Repeating these steps under  $\mathcal{F}^c$  (simply set  $\tau_v \equiv 0$ ) we obtain:

$$d\widehat{\mathbf{z}}_t^c = \mathbf{A}_{zz} \widehat{\mathbf{z}}_t^c dt + \begin{pmatrix} -(\tau_t^c)^{-1} \tau_{P,t}^{1/2} \Phi' \\ \lambda_t (\tau_t^c)^{-1} \tau_{P,t}^{1/2} \xi_t^{-1} \Phi \Phi' + \tau_m^{-1/2} \mathbf{I} \end{pmatrix} d\widehat{\mathbf{B}}_{m,t}^c \quad t \in (0, \tau) \cup (\tau, T) \quad (129)$$

with  $\widehat{\mathbf{B}}^c$  a  $\widehat{\mathbb{P}}^c$ -Brownian motion and

$$(\tau_t^c)' = \tau_{P,t} \quad t \in (0, \tau) \cup (\tau, T). \quad (130)$$

We now determine how beliefs are updated upon the announcement, that is at date  $\tau$ . The announcement creates a discontinuity in the empiricist's information set,  $\mathcal{F}^c$ , as she not only observes an additional bulk of information (the announcement) but prices also move

discontinuously to reflect this information (by  $\Delta\tilde{\mathbf{P}}_\tau \equiv \tilde{\mathbf{P}}_\tau - \tilde{\mathbf{P}}_{\tau-}$ ):

$$\mathcal{F}_\tau^c = \mathcal{F}_{\tau-}^c \vee \sigma\left(\{\tilde{\mathbf{P}}_\tau, A_\tau\}\right). \quad (131)$$

More specifically, the sufficient statistic for prices immediately after the announcement jumps to reflect the lump in noise traders' demand:

$$\tilde{\mathbf{P}}_\tau^a = \Phi\lambda_\tau\tilde{F} + \boldsymbol{\xi}_\tau(\tilde{\mathbf{m}}_{\tau-} + \Delta\tilde{\mathbf{m}}_\tau), \quad (132)$$

compared to its level right before the announcement:

$$\tilde{\mathbf{P}}_{\tau-}^a = \Phi\lambda_{\tau-}\tilde{F} + \boldsymbol{\xi}_{\tau-}\tilde{\mathbf{m}}_{\tau-}, \quad (133)$$

and thus the sufficient statistic for the price jumps according to:

$$\Delta\tilde{\mathbf{P}}_\tau^a = \Phi(\lambda_\tau - \lambda_{\tau-})\tilde{F} + (\boldsymbol{\xi}_\tau - \boldsymbol{\xi}_{\tau-})\tilde{\mathbf{m}}_{\tau-} + \boldsymbol{\xi}_\tau\Delta\tilde{\mathbf{m}}_\tau. \quad (134)$$

We gather the sufficient price statistic and the announcement signal in the following vector of observables:

$$\mathbf{y}_\tau \equiv \begin{pmatrix} \tilde{A}_\tau \\ \tilde{\mathbf{P}}_\tau^a \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \mathbf{0} \\ \lambda_\tau\Phi & \boldsymbol{\xi}_\tau \end{pmatrix}}_{\equiv \mathbf{h}_\tau} \mathbf{z}_\tau + \tilde{\boldsymbol{\epsilon}}_\tau, \quad \tilde{\boldsymbol{\epsilon}}_\tau \sim \mathcal{N}\left(\mathbf{0}, \underbrace{\begin{pmatrix} \tau_A^{-1/2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}}_{\equiv \mathbf{r}}\right), \quad (135)$$

with the vector of hidden states jumping to:

$$\mathbf{z}_\tau = \mathbf{z}_{\tau-} + \underbrace{\begin{pmatrix} 0 \\ \mathbf{I} \end{pmatrix}}_{\equiv \mathbf{b}} \Delta\tilde{\mathbf{m}}_\tau. \quad (136)$$

Discrete-time Kalman filtering equations (e.g., [Jazwinski, 1970](#), Theorem 7.1) imply that the empiricist's views and standard error jump according to:

$$\Delta\hat{\mathbf{z}}_\tau^c = \mathbf{k}_\tau^c(\mathbf{y}_\tau - \mathbf{h}_\tau\hat{\mathbf{z}}_{\tau-}^c), \quad (137)$$

$$\Delta\mathbf{T}_\tau^c = -\mathbf{k}_\tau^c\mathbf{h}_\tau\mathbf{T}_{\tau-}^c + \tau_M^{-1}(\mathbf{I} - \mathbf{k}_\tau^c\mathbf{h}_\tau)\mathbf{b}\mathbf{b}', \quad (138)$$

where  $\mathbf{k}^c$  denotes the filter gain and satisfies:

$$\mathbf{k}_\tau^c = (\mathbf{T}_{\tau-}^c + \tau_M^{-1}\mathbf{b}\mathbf{b}')\mathbf{h}_\tau'(\mathbf{h}_\tau(\mathbf{T}_{\tau-}^c + \tau_M^{-1}\mathbf{b}\mathbf{b}')\mathbf{h}_\tau' + \mathbf{r})^{-1}. \quad (139)$$

Note that since  $\tilde{\mathbf{P}}_{\tau-}^a \in \mathcal{F}_{\tau-}^c$  observational equivalence still implies:

$$\mathbf{T}_{\tau-}^c = (\tau_{\tau-}^c)^{-1} \boldsymbol{\Omega}_{\tau-}. \quad (140)$$

Substituting the definition of  $\tau_G^{1/2}$  in Eq. (46) and simplifying the Kalman filtering equations yields the updating rule for the empiricist's precision:

$$\Delta\tau_{\tau}^c \equiv \tau_{\tau}^c - \tau_{\tau-}^c = \tau_G + \tau_A, \quad (141)$$

and the updating rule for her conditional expectations:

$$\Delta\hat{\mathbf{z}}_{\tau}^c = \mathbf{k}_{\tau}^c \begin{pmatrix} \tilde{A}_{\tau} - \hat{F}_{\tau-}^c \\ \tilde{\mathbf{P}}_{\tau}^a - \boldsymbol{\Phi} \lambda_{\tau} \hat{F}_{\tau-}^c - \boldsymbol{\xi}_{\tau} \hat{m}_{\tau-}^c \end{pmatrix}, \quad (142)$$

where the filter gain simplifies to:

$$\mathbf{k}_{\tau}^c = (\tau_{\tau}^c)^{-1} \begin{pmatrix} \tau_A & -\tau_M^{1/2} \tau_G^{1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_{\tau}^{-1} \\ -\tau_A \lambda_{\tau} \boldsymbol{\xi}_{\tau}^{-1} \boldsymbol{\Phi} & (\tau_M^{1/2} \tau_G^{1/2} \lambda_{\tau} \boldsymbol{\xi}_{\tau}^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_{\tau}^c \mathbf{I}) \boldsymbol{\xi}_{\tau}^{-1} \end{pmatrix}. \quad (143)$$

Finally, since the jump in filtration is caused by public information only, the updating rule for investors has the same form as that of the empiricist expect that the filter gain is based on their own precision. Specifically, the updating rule for investors is:

$$\Delta\tau_{\tau} = \tau_G + \tau_A \equiv \Delta\tau_{\tau}^c, \quad (144)$$

$$\Delta\hat{\mathbf{z}}_{\tau}^i = \mathbf{k}_{\tau} \begin{pmatrix} \tilde{A}_{\tau} - \hat{F}_{\tau-}^i \\ \tilde{\mathbf{P}}_{\tau}^a - \boldsymbol{\Phi} \lambda_{\tau} \hat{F}_{\tau-}^i - \boldsymbol{\xi}_{\tau} \hat{m}_{\tau-}^i \end{pmatrix}, \quad (145)$$

where the filter gain for investors satisfies:

$$\mathbf{k}_{\tau} = \tau_{\tau}^{-1} \begin{pmatrix} \tau_A & -\tau_M^{1/2} \tau_G^{1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_{\tau}^{-1} \\ -\tau_A \lambda_{\tau} \boldsymbol{\xi}_{\tau}^{-1} \boldsymbol{\Phi} & (\tau_M^{1/2} \tau_G^{1/2} \lambda_{\tau} \boldsymbol{\xi}_{\tau}^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_{\tau} \mathbf{I}) \boldsymbol{\xi}_{\tau}^{-1} \end{pmatrix}. \quad (146)$$

This completes the description of how investors and the empiricist update their beliefs.

We now determine how investors perceive their investment opportunity set, which requires expressing the dynamics of  $\hat{F}_t^c$  under  $\mathcal{F}^i$ . From the definition of filter innovations, we have:

$$d\hat{\mathbf{B}}_{m,t}^i = \tau_m^{1/2} \boldsymbol{\xi}_t^{-1} \left( d\tilde{\mathbf{P}}_t^a - \begin{pmatrix} \lambda_t' \boldsymbol{\Phi} & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix} \hat{\mathbf{z}}_t^i dt \right), \quad (147)$$

$$d\hat{\mathbf{B}}_{m,t}^c = \tau_m^{1/2} \boldsymbol{\xi}_t^{-1} \left( d\tilde{\mathbf{P}}_t^a - \begin{pmatrix} \lambda_t' \boldsymbol{\Phi} & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix} \hat{\mathbf{z}}_t^c dt \right). \quad (148)$$

Comparing these two equations we obtain a relation between  $\widehat{\mathbf{B}}_m^i$  and  $\widehat{\mathbf{B}}_{m,t}^c$ :

$$d\widehat{\mathbf{B}}_{m,t}^c = d\widehat{\mathbf{B}}_{m,t}^i + \tau_m^{1/2} \boldsymbol{\xi}_t^{-1} \begin{pmatrix} \lambda_t' \boldsymbol{\Phi} & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix} \begin{pmatrix} \widehat{F}_t^i - \widehat{F}_t^c \\ \widehat{\mathbf{m}}_t^i - \widehat{\mathbf{m}}_t^c \end{pmatrix} dt \quad (149)$$

$$= d\widehat{\mathbf{B}}_{m,t}^i + \tau_m^{1/2} \boldsymbol{\xi}_t^{-1} \begin{pmatrix} \lambda_t' \boldsymbol{\Phi} & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ -\boldsymbol{\xi}_t^{-1} \boldsymbol{\Phi} \lambda_t \end{pmatrix} \underbrace{(\widehat{F}_t^i - \widehat{F}_t^c)}_{\equiv \Delta_t^i} dt \quad (150)$$

$$= d\widehat{\mathbf{B}}_{m,t}^i - \tau_{P,t}^{1/2} \Delta_t^i \boldsymbol{\Phi} dt \quad (151)$$

where the second line exploits observational equivalence and where the third equality is rewritten in terms of investor  $i$ 's informational advantage relative to the empiricist,  $\Delta^i$ . We will now show that the state variables that are investment-relevant to investor  $i$  is the  $(N+1) \times 1$  vector  $\boldsymbol{\Psi}^i \equiv (\Delta^i \ (\widehat{\mathbf{m}}^i)')'$ . Since  $\Delta^i$  defines the change of measure between the empiricist and investors and that it follows a Gaussian process, Example 3.a in [Lipster and Shiryaev \(2001\)](#), p.233 implies that the change of measure is a true martingale and thus Girsanov's theorem applies. Using this change of measure and the updating rule above we can write the dynamics of  $\boldsymbol{\Psi}^i$  under  $\mathcal{F}^i$  over the two subperiods  $t \in (0, \tau) \cup (\tau, T)$  as:

$$d\boldsymbol{\Psi}_t^i = \underbrace{\begin{pmatrix} -\tau_{P,t}/\tau_t^c & \mathbf{0} \\ \mathbf{0} & -\mathbf{b} \end{pmatrix}}_{\equiv \mathbf{A}_{\boldsymbol{\Psi},t}} \boldsymbol{\Psi}_t^i dt + \underbrace{\begin{pmatrix} \tau_v^{1/2} \tau_t^{-1} & -\tau_{P,t}^{1/2} (\tau_t^{-1} - (\tau_t^c)^{-1}) \boldsymbol{\Phi}' \\ -\lambda_t \tau_t^{-1} \tau_v^{1/2} \boldsymbol{\xi}_t^{-1} \boldsymbol{\Phi} & \lambda_t \tau_t^{-1} \tau_{P,t}^{1/2} \boldsymbol{\xi}_t^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_m^{-1/2} \mathbf{I} \end{pmatrix}}_{\equiv \mathbf{B}_{\boldsymbol{\Psi},t}} d\widehat{\mathbf{B}}_t^i. \quad (152)$$

Applying Ito's lemma to the price shows that its dynamics are Markovian in  $\boldsymbol{\Psi}^i$  under  $\mathcal{F}^i$ :

$$d\widetilde{\mathbf{P}}_t = \dot{\boldsymbol{\xi}}_{0,t} \mathbf{M} dt + \underbrace{\begin{pmatrix} \boldsymbol{\Phi} (\lambda_t + (1 - \lambda_t) (\tau_t^c)^{-1} \tau_{P,t}) & \dot{\boldsymbol{\xi}}_t - \boldsymbol{\xi}_t \mathbf{b} \end{pmatrix}}_{\equiv \mathbf{A}_{\mathbf{P},t}} \boldsymbol{\Psi}_t^i dt \quad (153)$$

$$+ \underbrace{\begin{pmatrix} \mathbf{0} & \tau_m^{-1/2} \boldsymbol{\xi}_t - (1 - \lambda_t) (\tau_t^c)^{-1} \tau_{P,t}^{1/2} \boldsymbol{\Phi} \boldsymbol{\Phi}' \end{pmatrix}}_{\equiv \mathbf{B}_{\mathbf{P},t}} d\widehat{\mathbf{B}}_t^i, \quad t \in (0, \tau) \cup (\tau, T).$$

It remains to show that changes in prices on the announcement are also Markovian in  $\boldsymbol{\Psi}^i$ , that is both at date  $\tau$  and at the liquidation date,  $T$ . As we did for continuous dynamics we need to determine how an investor  $i$  perceives the change in empiricist's beliefs  $\Delta \widehat{F}_\tau^c$  on the

announcement date  $\tau$ . Using the updating rule above we have:

$$\Delta \widehat{\mathbf{z}}_\tau^c = \mathbf{k}_\tau^c (\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^c) = \mathbf{k}_\tau^c (\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^i) + \mathbf{k}_\tau^c \mathbf{h}_\tau (\widehat{\mathbf{z}}_{\tau-}^i - \widehat{\mathbf{z}}_{\tau-}^c) \quad (154)$$

$$= \mathbf{k}_\tau^c (\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^i) + \underbrace{\mathbf{k}_\tau^c \mathbf{h}_\tau \begin{pmatrix} 1 \\ -\boldsymbol{\xi}_{\tau-}^{-1} \boldsymbol{\Phi} \lambda_{\tau-} \end{pmatrix}}_{\equiv \boldsymbol{\omega}_{\tau-}} \Delta_{\tau-}^i. \quad (155)$$

Taking the first entry of this vector, substituting the expressions for  $\mathbf{h}$  and the filter gain  $\mathbf{k}^c$  above and simplifying gives:

$$\Delta \widehat{F}_\tau^c = \begin{pmatrix} \tau_A / \tau_\tau^c & -\tau_M^{1/2} \tau_G^{1/2} / \tau_\tau^c \boldsymbol{\Phi}' \boldsymbol{\xi}_\tau^{-1} \end{pmatrix} (\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^i) + (\tau_A + \tau_G) / \tau_\tau^c \Delta_{\tau-}^i. \quad (156)$$

Based on the updating rule for  $\widehat{\mathbf{z}}_\tau^i$  we obtained on announcement, we conclude that the vector of investment-relevant variables  $\boldsymbol{\Psi}^i$  moves discontinuously by:

$$\begin{aligned} \Delta \boldsymbol{\Psi}_\tau^i &= \underbrace{\begin{pmatrix} -(\tau_A + \tau_G) / \tau_\tau^c & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}}_{\equiv \mathbf{a}_\Psi} \boldsymbol{\Psi}_{\tau-}^i \\ &+ \underbrace{\begin{pmatrix} \tau_A (\tau_\tau^{-1} - (\tau_\tau^c)^{-1}) & -(\tau_\tau^{-1} - (\tau_\tau^c)^{-1}) \tau_G^{1/2} \tau_M^{1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_\tau^{-1} \\ -\tau_\tau^{-1} \tau_A \lambda_\tau \boldsymbol{\xi}_\tau^{-1} \boldsymbol{\Phi} & \boldsymbol{\xi}_\tau^{-1} + \tau_\tau^{-1} \lambda_\tau \tau_G^{1/2} \tau_M^{1/2} \boldsymbol{\xi}_\tau^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' \boldsymbol{\xi}_\tau^{-1} \end{pmatrix}}_{\equiv \mathbf{b}_\Psi} \underbrace{(\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^i)}_{\equiv \widetilde{\mathbf{y}}_\tau^i}, \end{aligned} \quad (157)$$

where under  $\mathcal{F}_{\tau-}^i$  the innovation in  $\Delta \boldsymbol{\Psi}^i$  is Gaussian  $\widetilde{\mathbf{y}}_\tau^i | \mathcal{F}_{\tau-}^i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Upsilon})$  with the conditional covariance matrix of standard errors given by:

$$\boldsymbol{\Upsilon} \equiv \text{Var} [\mathbf{y}_\tau - \mathbf{h}_\tau \widehat{\mathbf{z}}_{\tau-}^i | \mathcal{F}_{\tau-}^i] \quad (158)$$

$$= \text{Var} [\mathbf{h}_\tau (\mathbf{z}_\tau - \widehat{\mathbf{z}}_{\tau-}^i) + \widetilde{\boldsymbol{\epsilon}}_\tau | \mathcal{F}_{\tau-}^i] \quad (159)$$

$$= \mathbf{h}_\tau \text{Var} [\mathbf{z}_\tau | \mathcal{F}_{\tau-}^i] \mathbf{h}_\tau' + \begin{pmatrix} \tau_A^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \quad (160)$$

$$= \mathbf{h}_\tau \left( \mathbf{T}_{\tau-} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \tau_M^{-1} \mathbf{I} \end{pmatrix} \right) \mathbf{h}_\tau' + \begin{pmatrix} \tau_A^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \quad (161)$$

$$= \begin{pmatrix} \tau_{\tau-}^{-1} + \tau_A^{-1} & -\tau_{\tau-}^{-1} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_\tau \\ -\tau_{\tau-}^{-1} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_\tau \boldsymbol{\Phi} & \tau_M^{-1} \boldsymbol{\xi}_\tau (\mathbf{I} + \tau_{\tau-}^{-1} \tau_G \boldsymbol{\Phi} \boldsymbol{\Phi}') \boldsymbol{\xi}_\tau \end{pmatrix}, \quad (162)$$

where the last line follows from using  $\mathbf{T}_{\tau-} = \tau_{\tau-}^{-1} \boldsymbol{\Omega}_{\tau-}$ , substituting  $\mathbf{h}_\tau$  and simplifying using

the definition of  $\tau_G$ . Also we will keep in mind for later use that filter gain satisfies:

$$\mathbf{k}_\tau = (\mathbf{T}_{\tau-} + \mathbf{b}\Sigma_m\mathbf{b}')\mathbf{h}'_\tau\Upsilon^{-1}. \quad (163)$$

We now want to express the jump in prices on announcements in terms of  $\Delta\Psi^i$ . Since  $\tilde{\mathbf{P}}_\tau \in \mathcal{F}_\tau^i$  we have:

$$\tilde{\mathbf{P}}_\tau = \mathbf{1}D + \xi_{0,\tau}\mathbf{M} + \xi_\tau\hat{\mathbf{m}}_\tau^i + \Phi\left(\hat{F}_\tau^c + \lambda_\tau\Delta_\tau^i\right). \quad (164)$$

Similarly, because  $\tilde{\mathbf{P}}_{\tau-} \in \mathcal{F}_{\tau-}^i$  it equally holds that:

$$\tilde{\mathbf{P}}_{\tau-} = \mathbf{1}D + \xi_{0,\tau}\mathbf{M} + \xi_{\tau-}\hat{\mathbf{m}}_{\tau-}^i + \Phi\left(\hat{F}_{\tau-}^c + \lambda_{\tau-}\Delta_{\tau-}^i\right). \quad (165)$$

Subtracting one from the other and simplifying yields:

$$\begin{aligned} \Delta\tilde{\mathbf{P}}_\tau &= (\xi_{0,\tau} - \xi_{0,\tau-})\mathbf{M} + \left( \Phi(\lambda_\tau - \lambda_{\tau-} + (\tau_A + \tau_G)/\tau_\tau^c) \quad \xi_\tau - \xi_{\tau-} \right) \Psi_{\tau-}^i \\ &\quad + \left( \Phi\lambda_\tau \quad \xi_\tau \right) \Delta\Psi_\tau^i + \Phi\left( \tau_A/\tau_\tau^c \quad -\tau_M^{1/2}\tau_G^{1/2}/\tau_\tau^c\Phi'\xi_\tau^{-1} \right) \tilde{\mathbf{y}}_\tau^i \end{aligned} \quad (166)$$

Then substituting  $\mathbf{a}_\Psi$  and  $\mathbf{b}_\Psi$  we defined above and simplifying gives:

$$\begin{aligned} \Delta\tilde{\mathbf{P}}_\tau &= (\xi_{0,\tau} - \xi_{0,\tau-})\mathbf{M} + \underbrace{\left( \Phi(\lambda_\tau - \lambda_{\tau-} + (1 - \lambda_\tau)(\tau_A + \tau_G)/\tau_\tau^c) \quad \xi_\tau - \xi_{\tau-} \right)}_{\equiv \mathbf{a}_\Psi} \Psi_{\tau-}^i \\ &\quad + \underbrace{\left( (1 - \lambda_\tau)\tau_A/\tau_\tau^c\Phi \quad \mathbf{I} - (1 - \lambda_\tau)\tau_M^{1/2}\tau_G^{1/2}/\tau_\tau^c\Phi\Phi'\xi_\tau^{-1} \right)}_{\equiv \mathbf{b}_\Psi} \tilde{\mathbf{y}}_\tau^i, \end{aligned} \quad (167)$$

which confirms that the jump in prices on announcements is also Markovian in  $\Psi^i$ . Finally, at the liquidation date,  $T$ , the vector of payoffs,  $\tilde{\mathbf{D}}$ , is revealed, and the vector  $\Psi^i$  jumps according to:

$$\Delta\Psi_{T-}^i = - \begin{pmatrix} 1 \\ \mathbf{0} \end{pmatrix} \Psi_{T-}^i + \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \underbrace{\begin{pmatrix} \tilde{F} - \hat{F}_{T-}^i \\ \tilde{\mathbf{m}} - \hat{\mathbf{m}}_{T-}^i \end{pmatrix}}_{\equiv \tilde{\mathbf{y}}_T^i}, \quad (168)$$

where  $\tilde{\mathbf{y}}_T^i | \mathcal{F}_{T-}^i \sim \mathcal{N}(\mathbf{0}, \mathbf{T}_{T-})$ . The associated price discontinuity satisfies:

$$\Delta\tilde{\mathbf{P}}_T = \tilde{\mathbf{D}} - \tilde{\mathbf{P}}_{T-} \quad (169)$$

$$= \Phi(1 - \lambda_{T-})\Delta_T + \tilde{\boldsymbol{\epsilon}} - \xi_{0,T-}\mathbf{M} - \xi_{T-}\tilde{\mathbf{m}}_T. \quad (170)$$



Since  $\tilde{\mathbf{P}}_{T-} \in \mathcal{F}_{T-}^i$  this can be written in the eyes of investor  $i$  as:

$$\Delta \tilde{\mathbf{P}}_T = \mathbf{a}_{T-} \Psi_{T-}^i - \xi_{0,T-} \mathbf{M} + \begin{pmatrix} \Phi & \mathbf{0} \end{pmatrix} \tilde{\mathbf{y}}_T^i + \tilde{\epsilon}. \quad (171)$$

This shows that conditional on  $\mathcal{F}_{T-}^i$  we have:

$$\Delta \tilde{\mathbf{P}}_T \Big|_{\mathcal{F}_{T-}^i} \sim \mathcal{N} \left( \mathbf{a}_{T-} \Psi_{T-}^i - \xi_{0,T-} \mathbf{M}, \underbrace{(\tau_{T-})^{-1} \Phi \Phi' + \tau_\epsilon^{-1} \mathbf{I}}_{\equiv \Sigma} \right), \quad (172)$$

where we recognize investors' covariance matrix  $\Sigma$  as given in [Andrei et al. \(2021\)](#). This confirms that the vector  $\Psi^i$  is the only state variable that is relevant for investor  $i$ 's investment decision, which completes the description of how an investor  $i$  views this economy.

## A.2 Proof of Proposition 2

In this appendix we derive a system of matrix Riccati equations that the coefficients of investor  $i$ 's must satisfy, towards determining her investment decision. Within each subregion  $[0, \tau)$  and  $[\tau, T)$  the value function must solve the following Hamilton-Jacobi-Bellman equation:

$$0 = \max_{\mathbf{w}} \left\{ J_W \mathbf{w}' (\dot{\xi}_0 \mathbf{M} + \mathbf{A}_P \Psi) + \frac{1}{2} J_{WW} \mathbf{w}' \mathbf{B}_P \mathbf{B}_P' \mathbf{w} + \mathbf{w}' \mathbf{B}_P \mathbf{B}_\Psi' \mathbf{J}_{W\Psi} \right\} \quad (173)$$

$$+ J_t + \mathbf{J}'_\Psi \mathbf{A}_\Psi \Psi + \frac{1}{2} \text{tr}(\mathbf{B}_\Psi \mathbf{B}_\Psi' \mathbf{J}_{\Psi\Psi}).$$

From the first-order condition we get the optimal investment policy at time  $t \in (0, \tau) \cup (\tau, T)$ :

$$\mathbf{w}_t = -\frac{J_W}{J_{WW}} (\mathbf{B}_P \mathbf{B}_P')^{-1} (\dot{\xi}_0 \mathbf{M} + \mathbf{A}_P \Psi) - \frac{1}{J_{WW}} (\mathbf{B}_P \mathbf{B}_P')^{-1} \mathbf{B}_P \mathbf{B}_\Psi' \mathbf{J}_{W\Psi}, \quad (174)$$

which, when plugged back in the HJB equation, gives the following PDE, which holds at any time  $t \in (0, \tau) \cup (\tau, T)$ :

$$0 = J_t + \mathbf{J}'_\Psi \mathbf{A}_\Psi \Psi + \frac{1}{2} \text{tr}(\mathbf{B}_\Psi \mathbf{B}_\Psi' \mathbf{J}_{\Psi\Psi}) \quad (175)$$

$$- \frac{1}{2J_{WW}} \left( J_W (\dot{\xi}_0 \mathbf{M} + \mathbf{A}_P \Psi) + \mathbf{B}_P \mathbf{B}_\Psi' \mathbf{J}_{W\Psi} \right)' (\mathbf{B}_P \mathbf{B}_P')^{-1} \left( J_W (\dot{\xi}_0 \mathbf{M} + \mathbf{A}_P \Psi) + \mathbf{B}_P \mathbf{B}_\Psi' \mathbf{J}_{W\Psi} \right).$$

We now conjecture that the solution to this PDE is of the form in Eq. (28). In addition, we conjecture and will verify that  $\mathbf{V}$  is a symmetric matrix. For brevity, since the scalar  $s$  does not enter into the determination of equilibrium prices, we will only focus on  $\mathbf{U}$  and  $\mathbf{V}$ . Separating variables produces the two matrix Riccati equations in Eqs. (29)–(30)

that hold over each subinterval. Next we verify that the conjecture in Eq. (28) holds on the announcement date,  $\tau$ , which in turn will provide boundary conditions for the matrix Riccati equations above in between subintervals. On the announcement date an investors' wealth jumps to:

$$W_\tau = W_{\tau-} + \mathbf{w}'_{\tau-} \Delta \tilde{\mathbf{P}}_\tau, \quad (176)$$

and their expectations jump by  $\Delta \Psi$  (as shown above). Immediately prior to the announcement investors choose their portfolio so as to maximize:

$$\max_{\mathbf{w}} \mathbb{E} \left[ J(\tau, W + \mathbf{w}' \Delta \tilde{\mathbf{P}}, \Psi + \Delta \Psi) \middle| \mathcal{F}_{\tau-}^i \right], \quad (177)$$

which in turn tells us how they speculate on the outcome of the announcement. Substituting the conjectured functional form in Eq. (28) and simplifying we can write expected utility as:

$$\mathbb{E} \left[ J(\tau, W + \mathbf{w}' \Delta \tilde{\mathbf{P}}, \Psi + \Delta \Psi) \middle| \mathcal{F}_{\tau-}^i \right] = \quad (178)$$

$$- e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) - \gamma \mathbf{w}' (\mathbf{a}_\mathbf{P} \Psi + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M})}$$

$$\mathbb{E} \left[ e^{-\frac{1}{2} \left( (\gamma \mathbf{b}'_{\mathbf{P}} \mathbf{w} + \mathbf{b}'_{\Psi} \mathbf{U}_\tau + \mathbf{b}'_{\Psi} \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) \tilde{\mathbf{y}}_\tau + \tilde{\mathbf{y}}'_\tau (\gamma \mathbf{b}'_{\mathbf{P}} \mathbf{w} + \mathbf{b}'_{\Psi} \mathbf{U}_\tau + \mathbf{b}'_{\Psi} \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) + \tilde{\mathbf{y}}'_\tau \mathbf{b}'_{\Psi} \mathbf{V}_\tau \mathbf{b}_\Psi \tilde{\mathbf{y}}_\tau \right)} \middle| \mathcal{F}_{\tau-}^i \right]$$

$$\equiv - e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) - \gamma \mathbf{w}' (\mathbf{a}_\mathbf{P} \Psi + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M})}$$

$$\int_{\mathbb{R}^{N+1}} e^{-\frac{1}{2} (\mathbf{b}' \mathbf{y} + \mathbf{y}' \mathbf{b} + \mathbf{y}' \mathbf{a} \mathbf{y})} (2\pi)^{-\frac{N+1}{2}} |\Upsilon|^{-1/2} e^{-\frac{1}{2} \mathbf{y}' \Upsilon^{-1} \mathbf{y}} d\mathbf{y} \quad (179)$$

$$= - e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) - \gamma \mathbf{w}' (\mathbf{a}_\mathbf{P} \Psi + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M})}$$

$$|\Upsilon \mathbf{a} + \mathbf{I}|^{-1/2} e^{\frac{1}{2} \mathbf{b}' (\mathbf{a} + \Upsilon^{-1})^{-1} \mathbf{b}}, \quad (180)$$

where the coefficients  $\mathbf{a}$  and  $\mathbf{b}$  in the second equality are a redefinition of the expression multiplying the linear and quadratic part in  $\mathbf{y}$ , respectively. Plugging these coefficients back expected utility is given explicitly by:

$$\mathbb{E} \left[ J(\tau, W + \mathbf{w}' \Delta \tilde{\mathbf{P}}, \Psi + \Delta \Psi) \middle| \mathcal{F}_{\tau-}^i \right] = \quad (181)$$

$$= - e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi) - \gamma \mathbf{w}' (\mathbf{a}_\mathbf{P} \Psi + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M})}$$

$$|\Upsilon \mathbf{b}'_{\Psi} \mathbf{V}_\tau \mathbf{b}_\Psi + \mathbf{I}|^{-1/2} e^{\frac{1}{2} (\gamma \mathbf{b}'_{\mathbf{P}} \mathbf{w} + \mathbf{b}'_{\Psi} \mathbf{U}_\tau + \mathbf{b}'_{\Psi} \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)' (\mathbf{b}'_{\Psi} \mathbf{V}_\tau \mathbf{b}_\Psi + \Upsilon^{-1})^{-1} (\gamma \mathbf{b}'_{\mathbf{P}} \mathbf{w} + \mathbf{b}'_{\Psi} \mathbf{U}_\tau + \mathbf{b}'_{\Psi} \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)}.$$

From the first-order condition associated with Eq. (177) we get the optimal portfolio choice on announcements:

$$\mathbf{w}_{\tau-} = \frac{1}{\gamma} (\mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_P)^{-1} \left( (\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi)) \Psi + (\xi_{0,\tau} - \xi_{0,\tau-}) \mathbf{M} - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{U}_\tau \right), \quad (182)$$

where

$$\mathbf{S}_\tau \equiv (\mathbf{b}'_\Psi \mathbf{V}_\tau \mathbf{b}_\Psi + \Upsilon^{-1})^{-1}. \quad (183)$$

Plugging back and simplifying expected utility at the optimum is:

$$\mathbb{E} \left[ J(\tau, W + \mathbf{w}' \Delta \tilde{\mathbf{P}}, \Psi + \Delta \Psi) \middle| \mathcal{F}_{\tau-}^i \right] = - |\Upsilon \mathbf{b}'_\Psi \mathbf{V}_\tau \mathbf{b}_\Psi + \mathbf{I}|^{-1/2} \quad (184)$$

$$\begin{aligned} & e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)} \\ & e^{-\frac{1}{2} \gamma^2 \mathbf{w}' \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_P \mathbf{w} + \frac{1}{2} (\mathbf{U}_\tau + \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)' \mathbf{b}_\Psi \mathbf{S}_\tau \mathbf{b}'_\Psi (\mathbf{U}_\tau + \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)} \\ & = - |\Upsilon \mathbf{b}'_\Psi \mathbf{V}_\tau \mathbf{b}_\Psi + \mathbf{I}|^{-1/2} e^{-s_\tau - \gamma W - \frac{1}{2} (\mathbf{U}'_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{U}_\tau + \Psi' (\mathbf{I} + \mathbf{a}_\Psi)' \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)} \\ & e^{-\frac{1}{2} ((\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi)) \Psi + \Delta \xi_{0,\tau} \mathbf{M} - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{U}_\tau)' (\mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_P)^{-1} ((\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi)) \Psi + \Delta \xi_{0,\tau} \mathbf{M} - \mathbf{b}_P \mathbf{S}_\tau \mathbf{b}'_\Psi \mathbf{U}_\tau)} \\ & e^{\frac{1}{2} (\mathbf{U}_\tau + \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)' \mathbf{b}_\Psi \mathbf{S}_\tau \mathbf{b}'_\Psi (\mathbf{U}_\tau + \mathbf{V}_\tau (\mathbf{I} + \mathbf{a}_\Psi) \Psi)}, \end{aligned} \quad (185)$$

where  $\Delta \xi_{0,\tau} \equiv \xi_{0,\tau} - \xi_{0,\tau-}$  (just for the purpose of putting everything on a single line), and where the first equality pre-multiplies the first-order condition by  $\mathbf{w}'$  and uses the result to simplify the expression for expected utility, and the second equality substitutes the optimal portfolio policy. Rational anticipation then requires that:

$$J(\tau-, W, \Psi) = \mathbb{E} \left[ J(\tau, W + \mathbf{w}' \Delta \tilde{\mathbf{P}}, \Psi + \Delta \Psi) \middle| \mathcal{F}_{\tau-}^i \right], \quad (186)$$

which, after separation of variables, delivers the two systems of algebraic matrix Riccati equations for  $\mathbf{U}$  and  $\mathbf{V}$  in Eqs. (31)–(32) (once more we leave out the equation for  $s$  as it does not intervene in the portfolio decision). Although expected in this continuous-discrete framework, these two algebraic matrix Riccati equations are the discrete-time counterparts to the matrix differential equations in Eqs. (29) and (30). Specifically, Eq. (29) can be rewritten as:

$$-\dot{\mathbf{U}} = (\mathbf{A}'_\Psi - \mathbf{V} \mathbf{B}_\Psi \mathbf{B}'_\Psi) \mathbf{U} + (\mathbf{A}_P - \mathbf{B}_P \mathbf{B}'_\Psi \mathbf{V})' (\mathbf{B}_P \mathbf{B}'_P)^{-1} (\dot{\xi}_0 \mathbf{M} - \mathbf{B}_P \mathbf{B}'_\Psi \mathbf{U}) \quad (187)$$

and Eq. (30) can be rewritten as:

$$-\dot{\mathbf{V}} = \mathbf{V}\mathbf{A}_\Psi + \mathbf{A}'_\Psi \mathbf{V} - \mathbf{V}\mathbf{B}_\Psi \mathbf{B}'_\Psi \mathbf{V} + (\mathbf{A}_\mathbf{P} - \mathbf{B}_\mathbf{P} \mathbf{B}'_\Psi \mathbf{V})' (\mathbf{B}_\mathbf{P} \mathbf{B}'_\mathbf{P})^{-1} (\mathbf{A}_\mathbf{P} - \mathbf{B}_\mathbf{P} \mathbf{B}'_\Psi \mathbf{V}). \quad (188)$$

Finally, we determine the boundary condition on the liquidation date (when the fundamental value is revealed). Right before the fundamental is revealed an investor  $i$  selects her last portfolio position according to the customary static portfolio optimization problem:

$$\max_{\mathbf{w}} \mathbb{E} \left[ -e^{-\gamma(W + \mathbf{w}' \Delta \tilde{\mathbf{P}}_T)} \middle| \mathcal{F}_{T-}^i \right] = \max_{\mathbf{w}} -e^{-\gamma W - \gamma \mathbf{w}' \mathbb{E}[\Delta \tilde{\mathbf{P}}_T | \mathcal{F}_{T-}^i] + \frac{\gamma^2}{2} \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w}}, \quad (189)$$

where the second expression uses Eq. (A.1). From the first-order condition to this problem the optimal, terminal portfolio is the customary mean-variance portfolio:

$$\mathbf{w}_{T-} = \frac{1}{\gamma} \boldsymbol{\Sigma}^{-1} \mathbb{E}[\Delta \tilde{\mathbf{P}}_T | \mathcal{F}_{T-}^i] = \frac{1}{\gamma} (\tau_{T-}^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_\epsilon^{-1} \mathbf{I})^{-1} (\mathbf{a}_{T-} - \boldsymbol{\Psi} - \boldsymbol{\xi}_{0,T-} \mathbf{M}). \quad (190)$$

Substituting back and simplifying using the first-order condition gives:

$$\mathbb{E} \left[ -e^{-\gamma(W + \mathbf{w}' \Delta \tilde{\mathbf{P}}_T)} \middle| \mathcal{F}_{T-}^i \right] = -e^{-\gamma W - \frac{\gamma^2}{2} \mathbf{w}' \boldsymbol{\Sigma} \mathbf{w}} \quad (191)$$

$$= -e^{-\gamma W - \frac{1}{2} (\mathbf{a}_{T-} - \boldsymbol{\Psi} - \boldsymbol{\xi}_{0,T-} \mathbf{M})' \boldsymbol{\Sigma}^{-1} (\mathbf{a}_{T-} - \boldsymbol{\Psi} - \boldsymbol{\xi}_{0,T-} \mathbf{M})}. \quad (192)$$

Rational anticipation (namely that this equals  $J(T-, W, \boldsymbol{\Psi})$ ), separation of variables, and using Woodbury matrix identity to rewrite  $\boldsymbol{\Sigma}^{-1}$  give the two terminal conditions for  $\mathbf{U}$  and  $\mathbf{V}$  in Eqs. (33)–(34) (once again leaving out that for  $s$ ).

### A.3 Proof of Proposition 3

In this appendix we show that the matrix coefficients  $\mathbf{U}$  and  $\mathbf{V}$  of Proposition 2 satisfy the form in Eq. (49). The procedure is in four steps.

1. We use the following observation to rewrite the equations of Proposition 2 for  $\mathbf{U}$  and  $\mathbf{V}$  in terms of  $\mathbf{b}$ ,  $\mathbf{c}$  and  $\mathbf{A}_\mathbf{P}$  (or  $\mathbf{a}_\mathbf{P}$ ). In particular, from the definition of  $\mathbf{A}_\mathbf{P}$  in Eq. (153) we can rewrite this coefficient as:

$$\mathbf{A}_\mathbf{P} \equiv -\frac{d}{dt} \mathbf{a} - \mathbf{a} \mathbf{A}_\Psi, \quad (193)$$

$$= -\left( (\mathbf{b} + \mathbf{c}) \mathbf{A}_\Psi + \frac{d}{dt} (\mathbf{b} + \mathbf{c}) \right), \quad (194)$$

where the second line follows from substituting the identity in Eq. (38) into the first

line; from the definition of  $\mathbf{a}_P$  in Eq. (167), the discrete counterpart to Eq. (193) is:

$$\mathbf{a}_P \equiv \mathbf{a}_{t-} - \mathbf{a}_t(\mathbf{I} + \mathbf{a}_\Psi), \quad (195)$$

$$= \mathbf{b}_{t-} + \mathbf{c}_{t-} - (\mathbf{b}_t + \mathbf{c}_t)(\mathbf{I} + \mathbf{a}_\Psi), \quad (196)$$

where the second line substitutes the identity in Eq. (38).

2. We then use the following observation to simplify the terms that multiply  $\mathbf{b}$ ,  $\mathbf{c}$  and  $\mathbf{A}_P$  (or  $\mathbf{a}_P$ ). Defining the vector

$$\mathbf{X}_{1,t} \equiv \tau_t^{-1} \begin{pmatrix} \tau_v^{1/2} \Phi & -\tau_P^{1/2} \Phi \Phi' \end{pmatrix}', \quad (197)$$

the diffusion of prices and that of state variables are related as:

$$\mathbf{B}_P = -\mathbf{a} \mathbf{B}_\Psi + \mathbf{X}'_1. \quad (198)$$

Substituting the identity in Eq. (38) in this equation and splitting terms in  $\mathbf{b}$  and in  $\mathbf{c}$  we have:

$$\mathbf{b} \mathbf{B}_\Psi = -\mathbf{B}_P + \begin{pmatrix} (1 - \ell_t) \tau_v^{1/2} \tau_t^{-1} \Phi & -(1 - \lambda_t) (\tau_t^c)^{-1} \tau_{P,t}^{1/2} \Phi \Phi' \end{pmatrix} \quad (199)$$

$$= -\mathbf{B}_P + (1 - \ell_t) \mathbf{X}'_1 - \underbrace{\ell_t \begin{pmatrix} \mathbf{0} & (\tau_t^c)^{-1} \tau_{P,t}^{1/2} \Phi \Phi' \end{pmatrix}}_{\equiv \mathbf{X}'_2}, \quad (200)$$

and

$$\mathbf{c} \mathbf{B}_\Psi = \ell_t \begin{pmatrix} \tau_v^{1/2} \tau_t^{-1} \Phi & \alpha_t (\tau_t^c)^{-1} \tau_{P,t}^{1/2} \Phi \Phi' \end{pmatrix} \quad (201)$$

$$= \ell_t (\mathbf{X}_1 + \mathbf{X}_2)', \quad (202)$$

where the second line in each of these two equations uses  $(1 - \alpha)/\tau^c = \tau^{-1}$ . Defining the vector:

$$\mathbf{x}_t \equiv \tau_t^{-1} \begin{pmatrix} \tau_A \Phi & -\tau_M^{1/2} \tau_{G,t}^{1/2} \Phi \Phi' \xi_t^{-1} \end{pmatrix}, \quad (203)$$

upon the announcement the covariance matrix of prices and that of state variables are

related as:

$$\mathbf{b}_P = -\mathbf{a}\mathbf{b}_\Psi + \mathbf{x} \quad (204)$$

$$= \mathbf{1}^* + (1 - \lambda_t) \frac{\tau_t}{\tau_t^c} \mathbf{x} \quad (205)$$

which is the discrete counterpart to Eq. (198). Using the identity in Eq. (38) and simplifying using  $(1 - \alpha)/\tau^c = \tau^{-1}$ , we can further decompose this relation into:

$$\mathbf{b}\mathbf{b}_\Psi = -\mathbf{1}^* \quad (206)$$

and

$$\mathbf{c}\mathbf{b}_\Psi = -\mathbf{b}_P + \mathbf{1}^* + \mathbf{x} \quad (207)$$

$$= -\alpha_t \ell_t \frac{\tau_t}{\tau_t^c} \mathbf{x}. \quad (208)$$

3. We then substitute the conjectured form for  $\mathbf{A}_P$  and  $\mathbf{a}_P$  of Conjecture 1 to express the equations for  $\mathbf{U}$  and  $\mathbf{V}$  just in terms of  $\mathbf{b}$  and  $\mathbf{c}$ .
4. We exploit the relation in Eqs. (43) and (44) to obtain equations for the linear combinations in Eqs. (51) and (53). Regarding boundary conditions, a key identity in simplifying the resulting equations is (defining  $\beta \equiv (\tau_t^c - \tau_{t-}^c)/\tau_t^c$  and  $\bar{\mathbf{x}} \equiv \Phi' \mathbf{x}$ ):

$$(\mathbf{S}^{-1} + \mathbf{b}'_\Psi (\mathbf{b}'\mathbf{v} + v_3 \mathbf{c}') \mathbf{b}_P) \mathbf{y} = -\beta (v - \alpha \bar{v}_1) \mathbf{b}'_P \Phi + \left( \frac{v - 2\alpha \bar{v}_1 + \alpha^2 \bar{v}}{1 - \alpha} \beta + \tau_{t-} \right) \bar{\mathbf{x}}'. \quad (209)$$

We start by applying these steps to  $\mathbf{V}$  and then repeat these steps on  $\mathbf{U}$ . For brevity in what follows we apply steps 1-3 at once, computing each term separately for each equation. We start with the matrix Riccati equation for  $\mathbf{V}$  in Eq. (30), which is easier to handle. First, step 1 and 2 allow us to write (using also the conjecture for  $\mathbf{V}$  in Eq. (49)):

$$\mathbf{V}(\mathbf{B}_\Psi \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P - \mathbf{A}_\Psi) = -(\mathbf{v}\mathbf{b} + v_3 \mathbf{c}') (\mathbf{A}_P + \mathbf{b}\mathbf{A}_\Psi) - (v_3 \mathbf{b} + v_4 \mathbf{c}') \mathbf{c}\mathbf{A}_\Psi \quad (210)$$

$$\begin{aligned} &+ (((1 - \ell)\mathbf{X}_1 - \ell\mathbf{X}_2)(\mathbf{v}\mathbf{b} + v_3 \mathbf{c}) + \ell(\mathbf{X}_1 + \mathbf{X}_2)(v_3 \mathbf{b} + v_4 \mathbf{c}))' \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P \\ &= ((v - v_3)\mathbf{b} + (v_3 - v_4)\mathbf{c})' \mathbf{c}\mathbf{A}_\Psi + (\mathbf{v}\mathbf{b} + v_3 \mathbf{c}') \frac{d}{dt} (\mathbf{b} + \mathbf{c}) \\ &+ (\mathbf{X}_1(\bar{v}_1 \mathbf{b} + \bar{v}_2 \mathbf{c}) + \ell\mathbf{X}_2((v_3 - v)\mathbf{b} + (v_4 - v_3)\mathbf{c}))' \mathbf{B}'_P (\mathbf{B}_P \mathbf{B}'_P)^{-1} \mathbf{A}_P, \end{aligned} \quad (211)$$

where the first equality substitutes Eqs. (199) and (201) and the second equality substitutes

Eq. (194). Further moving to step 3 and thus using Conjecture 1, part A the second line can be rewritten, after simplifications, as:

$$(\mathbf{X}_1(\bar{v}_1\mathbf{b} + \bar{v}_2\mathbf{c}) + \ell\mathbf{X}_2((v_3 - v)\mathbf{b} + (v_4 - v_3)\mathbf{c}))'\mathbf{B}'_{\mathbf{P}}(\mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\mathbf{P}})^{-1}\mathbf{A}_{\mathbf{P}} \quad (212)$$

$$\begin{aligned} &= (\tau^c)^{-1}\tau_P^{1/2}((\alpha\bar{v}_1 - v)\mathbf{b} + (\alpha\bar{v}_2 - v_3)\mathbf{c})'\mathbf{\Phi}\mathbf{\Phi}'\Sigma_{\mathbf{P}}^{-1}\mathbf{A}_{\mathbf{P}} \\ &= (\tau^c)^{-1}\tau_P^{1/2}((\alpha\bar{v}_1 - v)\mathbf{b} + (\alpha\bar{v}_2 - v_3)\mathbf{c})'\mathbf{\Phi}\mathbf{\Phi}'((\Pi_1 + \Pi_2)\mathbf{b} + \Pi_3\mathbf{c}), \end{aligned} \quad (213)$$

where the first equality substitutes  $\mathbf{X}_1$  and  $\mathbf{X}_2$  and simplifies. Substituting back in the expression above along with  $\mathbf{A}_{\Psi}$  we get:

$$\begin{aligned} \mathbf{V}(\mathbf{B}_{\Psi}\mathbf{B}'_{\mathbf{P}}(\mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\mathbf{P}})^{-1}\mathbf{A}_{\mathbf{P}} - \mathbf{A}_{\Psi}) &= \frac{\tau_P}{\tau^c}((v_3 - v)\mathbf{b} + (v_4 - v_3)\mathbf{c})'\mathbf{c} + (\mathbf{v}\mathbf{b} + v_3\mathbf{c})'\frac{d}{dt}(\mathbf{b} + \mathbf{c}) \\ &+ \frac{\tau_P^{1/2}}{\tau^c}((\alpha\bar{v}_1 - v)\mathbf{b} + (\alpha\bar{v}_2 - v_3)\mathbf{c})'\mathbf{\Phi}\mathbf{\Phi}'((\Pi_1 + \Pi_2)\mathbf{b} + \Pi_3\mathbf{c}). \end{aligned} \quad (214)$$

An immediate simplification in Eq. (30) then follows since differentiating Eq. (49) gives:

$$\begin{aligned} \dot{\mathbf{V}} &= \left(\frac{d}{dt}\mathbf{b}\right)'(\mathbf{v}\mathbf{b} + v_3\mathbf{c}) + (\mathbf{v}\mathbf{b} + v_3\mathbf{c})'\frac{d}{dt}\mathbf{b} + \left(\frac{d}{dt}\mathbf{c}\right)'(v_3\mathbf{b} + v_4\mathbf{c}) + (v_3\mathbf{b} + v_4\mathbf{c})'\frac{d}{dt}\mathbf{c} \\ &+ \mathbf{b}'\dot{\mathbf{v}}\mathbf{b} + v_3'(\mathbf{b}'\mathbf{c} + \mathbf{c}'\mathbf{b}) + v_4'\mathbf{c}'\mathbf{c}, \end{aligned} \quad (215)$$

where we can re-express the time derivative of  $\mathbf{c}$  as:

$$\frac{d}{dt}\mathbf{c} = \ell^{-1}\frac{d}{dt}\ell\mathbf{c}. \quad (216)$$

Conjecture 1, part A (step 3) also allows us to write the squared Sharpe ratio as:

$$\mathbf{A}'_{\mathbf{P}}\Sigma_{\mathbf{P}}^{-1}\Sigma_{\mathbf{P}}^{-1}\mathbf{A}_{\mathbf{P}} = \mathbf{b}'(((\Pi_1 + \Pi_2)^2 - \Pi_2^2)\mathbf{\Phi}\mathbf{\Phi}' + \Pi_2^2\mathbf{I})\mathbf{b} + \Pi_3(\Pi_1 + \Pi_2)(\mathbf{b}'\mathbf{c} + \mathbf{c}'\mathbf{b}) + \Pi_3^2\mathbf{c}'\mathbf{c}. \quad (217)$$

Moving on to the last term term in Eq. (30), it can be obtained by direct substitution of the relevant matrices:

$$\mathbf{B}_{\Psi}(\mathbf{I} - \mathbf{B}'_{\mathbf{P}}(\mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\mathbf{P}})^{-1}\mathbf{B}_{\mathbf{P}})\mathbf{B}'_{\Psi} = \tau_v\tau_t^{-2}\mathbf{\Omega}_t. \quad (218)$$

and by further noting that (in particular,  $\mathbf{a}_t \cdot \boldsymbol{\omega}_t = \Phi$ )

$$\mathbf{b}\boldsymbol{\omega} = (1 - \ell)\Phi \quad \text{and} \quad \mathbf{c}\boldsymbol{\omega} = \ell\Phi, \quad (219)$$

so we can write  $\mathbf{V}\boldsymbol{\omega}$  as a weighted sum of two vectors:

$$\mathbf{V}\boldsymbol{\omega} = (1 - \ell)(\mathbf{v}\mathbf{b} + v_3\mathbf{c})'\boldsymbol{\Phi} + \ell(v_3\mathbf{b} + v_4\mathbf{c})'\boldsymbol{\Phi} \quad (220)$$

$$= (\bar{v}_1\mathbf{b} + \bar{v}_2\mathbf{c})'\boldsymbol{\Phi}, \quad (221)$$

where the weight is given by the relative wedge,  $\ell$ , hence the relevance of  $\bar{v}_1$  and  $\bar{v}_2$  in Eqs. (51) and (53). This property, that the problem “wants” to aggregate in weighted sums, is also apparent in Eq. (43). This allows us to write:

$$\mathbf{V}\mathbf{B}_\Psi(\mathbf{I} - \mathbf{B}_P(\mathbf{B}_P\mathbf{B}'_P)^{-1}\mathbf{B}'_P)\mathbf{B}'_\Psi\mathbf{V} = \tau_v\tau^{-2}(\bar{v}_1^2\mathbf{b}'\boldsymbol{\Phi}\boldsymbol{\Phi}'\mathbf{b} + \bar{v}_2^2\mathbf{c}'\mathbf{c} + \bar{v}_1\bar{v}_2(\mathbf{b}'\mathbf{c} + \mathbf{c}'\mathbf{b})). \quad (222)$$

As desired, substituting these expressions back into Eq. (30) and separating terms in  $\mathbf{a}'\mathbf{a}$ ,  $\mathbf{a}'\boldsymbol{\Phi}\boldsymbol{\Phi}'\mathbf{a}$ ,  $\mathbf{b}'\mathbf{c}$ ,  $\mathbf{c}'\mathbf{b}$  and in  $\mathbf{c}'\mathbf{c}$ , gives the following system of four ODEs:

$$v'_1 = 2\frac{\tau_P^{1/2}}{\tau^c}(\Pi_1 + \Pi_2)(\alpha\bar{v}_1 - v) + \frac{\tau_v}{\tau^2}\bar{v}_1^2 - (\Pi_1 + \Pi_2)^2 + \Pi_2^2 \quad (223)$$

$$v'_2 = -\Pi_2^2 \quad (224)$$

$$v'_3 = (v - v_3)\left(\ell^{-1}\frac{d}{dt}\ell - \frac{\tau_P}{\tau^c}\right) + \frac{\tau_v}{\tau^2}\bar{v}_1\bar{v}_2 - (\Pi_1 + \Pi_2)\Pi_3 \quad (225)$$

$$+ \frac{\tau_P^{1/2}}{\tau^c}(\Pi_3(\alpha\bar{v}_1 - v) + (\Pi_1 + \Pi_2)(\alpha\bar{v}_2 - v_3))$$

$$v'_4 = 2(v_3 - v_4)\left(\ell^{-1}\frac{d}{dt}\ell - \frac{\tau_P}{\tau^c}\right) + \frac{\tau_v}{\tau^2}\bar{v}_2^2 + 2\Pi_3\frac{\tau_P^{1/2}}{\tau^c}(\alpha\bar{v}_2 - v_3) - \Pi_3^2, \quad (226)$$

over each subinterval  $(0, \tau)$  and  $(\tau, T)$ . We will see that  $v_2$  can be solved independently of the other coefficients. Thus, we obtain an ODE for the sum  $v$  by adding the two first ODEs, Eqs. (223) and (224), which gives Eq. (54). This concludes step 1-3. Under step 4 we want to further combine the ODE for  $v$  with those for  $v_3$  and  $v_4$  into two ODEs for the weighted sums  $\bar{v}_1$  and  $\bar{v}_2$ . Differentiating  $\bar{v}_1$ :

$$\bar{v}'_1 = (1 - \ell)v' + \ell v'_3 + (v_3 - v)\frac{d}{dt}\ell, \quad (227)$$

and substitute Eqs. (54) and (225) to obtain:

$$\bar{v}'_1 = -\ell\frac{\tau_P}{\tau^c}(v - v_3) + \frac{\tau_P^{1/2}}{\tau^c}(\Pi_1 + \Pi_2)(\alpha\bar{v} - \bar{v}_1) + \frac{\tau_v}{\tau^2}\bar{v}\bar{v}_1 \quad (228)$$

$$+ (\ell\Pi_3 + (1 - \ell)(\Pi_1 + \Pi_2))\left(\frac{\tau_P^{1/2}}{\tau^c}(\alpha\bar{v}_1 - v) - (\Pi_1 + \Pi_2)\right),$$



where the second line substitutes Eq. (43) from Conjecture 1 and simplifying gives Eq. (56). Repeating the operations above for  $\bar{v}_2$  we get:

$$\bar{v}'_2 = (\bar{v}_1 - \bar{v}_2) \left( \ell^{-1} \frac{d}{dt} \ell - \frac{\tau_P}{\tau^c} \right) + \frac{\tau_P^{1/2}}{\tau^c} \left( \Pi_3(\alpha \bar{v} - \bar{v}_1) + \tau_P^{1/2}(1 - \alpha)\bar{v}_2 \right) + \tau_P^{1/2} \Pi_3 + \frac{\tau_v}{\tau^2} \bar{v}_2 \bar{v}. \quad (229)$$

Finally, combining  $\bar{v}_1$  and  $\bar{v}_2$  into the weighted sum  $\bar{v}$  we obtain the ODE in Eq. (57), which uses that  $(1 - \alpha)/\tau^c = \tau^{-1}$ .

We now repeat step 1-4 on the matrix equation associated with the boundary condition for  $\mathbf{V}$  in Eq. (32). Starting with step 1-3 we compute each of the two terms in Eq. (32) separately. Using Eq. (195) under step 1 along with Eq. (49) we can first write:

$$(\mathbf{I} + \mathbf{a}_\Psi) \mathbf{V}_t = (\mathbf{b}_{t-} - \mathbf{a}_\mathbf{P})' (\mathbf{v} \mathbf{b}_t + v_3 \mathbf{c}_t) + \mathbf{c}'_{t-} (\tilde{v}_1 \mathbf{b}_t + \tilde{v}_2 \mathbf{c}_t), \quad (230)$$

where  $\tilde{v}_{1,t} \equiv \left( 1 - \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \right) v + \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} v_3$  and  $\tilde{v}_{2,t} \equiv \left( 1 - \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \right) v_3 + \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} v_4$ . Using the relations in Eqs. (204)–(207) under step 2 we can also write:

$$\mathbf{V}_t \mathbf{b}_\Psi = -(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_\mathbf{P} + \left( \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_t + \frac{v_{3,t} - \alpha_t \tilde{v}_{2,t}}{1 - \alpha_t} \mathbf{c}'_t \right) \mathbf{x}. \quad (231)$$

These two expressions in turn allow us to write the first term in Eq. (32) as:

$$\begin{aligned} & (\mathbf{I} + \mathbf{a}_\Psi) (\mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi) - \mathbf{V}_t \mathbf{b}_\Psi \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi)) \\ &= -(\mathbf{I} + \mathbf{a}_\Psi) (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) (\mathbf{a}_\mathbf{P} - \mathbf{b}_\mathbf{P} \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi)) \\ &+ (\mathbf{I} + \mathbf{a}_\Psi) \left( \begin{array}{c} (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_{t-} + (\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \mathbf{c}_{t-} \\ - \left( \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_t + \frac{v_{3,t} - \alpha_t \tilde{v}_{2,t}}{1 - \alpha_t} \mathbf{c}'_t \right) \mathbf{x} \mathbf{S} \mathbf{b}'_\Psi \left( \begin{array}{c} (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) (\mathbf{b}_{t-} - \mathbf{a}_\mathbf{P}) \\ + (\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \mathbf{c}_{t-} \end{array} \right) \end{array} \right). \end{aligned} \quad (232)$$

Focussing on the second term in the expression above and using once more the relations under step 1-2 we can further write:

$$(\mathbf{I} + \mathbf{a}_\Psi) (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) = (\mathbf{b}_{t-} - \mathbf{a}_\mathbf{P})' \mathbf{v}_t + \tilde{v}_{1,t} \mathbf{c}'_{t-} \quad (233)$$

$$(\mathbf{I} + \mathbf{a}_\Psi) (\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) = (\mathbf{b}_{t-} - \mathbf{a}_\mathbf{P})' \tilde{v}_{1,t} + \tilde{v}_{3,t} \mathbf{c}'_{t-} \quad (234)$$

$$(\mathbf{I} + \mathbf{a}_\Psi) \left( \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_t + \frac{v_{3,t} - \alpha_t \tilde{v}_{2,t}}{1 - \alpha_t} \mathbf{c}'_t \right) = (\mathbf{b}_{t-} - \mathbf{a}_\mathbf{P})' \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} + \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_{4,t}}{1 - \alpha_t} \mathbf{c}'_{t-}, \quad (235)$$

where  $\tilde{v}_{3,t} \equiv \left( 1 - \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \right) \tilde{v}_{1,t} + \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \tilde{v}_{3,t}$  and  $\tilde{v}_{4,t} \equiv \left( 1 - \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \right) \tilde{v}_{1,t} + \frac{\ell_t}{\ell_{t-}} \frac{\tau_t^c}{\tau_{t-}^c} \tilde{v}_{2,t}$ . Substituting

back into the second term of Eq. (232) and now moving to step 3, that is substituting Conjecture 1, part A, we obtain:

$$\begin{aligned}
& (\mathbf{I} + \mathbf{a}_\Psi)(\mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi) - \mathbf{V}_t \mathbf{b}_\Psi \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi)) \tag{236} \\
& = -(\mathbf{b}'_{t-}(\mathbf{I} - \pi'_b \mathbf{b}'_P) \mathbf{v}_t + \mathbf{c}'_{t-}(\tilde{v}_{1,t} - \pi'_c \mathbf{b}'_P \mathbf{v}_t))(\mathbf{a}_P - \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi)) \\
& \quad + \left( \begin{array}{c} (\mathbf{b}'_{t-}(\mathbf{I} - \mathbf{b}_P \pi_b)' \mathbf{v}_t + \mathbf{c}'_{t-}(\tilde{v}_{1,t} - \mathbf{v}_t \mathbf{b}_P \pi_c)' \mathbf{b}_{t-} \\ + (\mathbf{b}'_{t-}(\mathbf{I} - \mathbf{b}_P \pi_b)' \tilde{v}_{1,t} + \mathbf{c}'_{t-}(\tilde{v}_{3,t} - \tilde{v}_{1,t} \pi'_c \mathbf{b}'_P)) \mathbf{c}_{t-} \end{array} \right) \\
& \quad - \left( \begin{array}{c} \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_{t-}(\mathbf{I} - \mathbf{b}_P \pi_b)' \\ + \mathbf{c}'_{t-} \left( \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_{4,t}}{1 - \alpha_t} - \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \pi'_c \mathbf{b}'_P \right) \end{array} \right) \mathbf{xSb}'_\Psi \left( \begin{array}{c} (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{I} - \mathbf{b}_P \pi_b) \mathbf{b}_{t-} \\ + \left( \begin{array}{c} \mathbf{b}'_t(\tilde{v}_{1,t} - v_t \mathbf{b}_P \pi_c) \\ + \mathbf{c}'_t(\tilde{v}_{2,t} - v_{3,t} \mathbf{b}_P \pi_c) \end{array} \right) \mathbf{c}_{t-} \end{array} \right).
\end{aligned}$$

Now focus on the first term of Eq. (232), which applying step 1-3 and factoring  $\mathbf{b}_P \mathbf{S}$  allow us to rewrite as:

$$\mathbf{a}_P - \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi) = \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} (\mathbf{S}^{-1} \pi_b + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{b}_P \pi_b - \mathbf{I})) \mathbf{b}_{t-} \\ + \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \pi_c \\ - \mathbf{b}'_\Psi(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{array} \right) \mathbf{c}_{t-} \end{array} \right). \tag{237}$$

This expression also takes care of the second term in Eq. (32). We then plug everything back into Eq. (32) and separate terms into  $\mathbf{b}'_{t-} \mathbf{b}_{t-}$ ,  $\mathbf{b}'_{t-} \mathbf{c}_{t-}$ ,  $\mathbf{c}'_{t-} \mathbf{b}_{t-}$  and  $\mathbf{c}'_{t-} \mathbf{c}_{t-}$ , which produces three equations for  $\mathbf{v}_{t-}$ ,  $v_{3,t-}$  and  $v_{4,t-}$ , respectively:

$$\mathbf{v}_{t-} = (\mathbf{I} - \mathbf{b}_P \pi_b)' \left( \begin{array}{c} \mathbf{v}_t - \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \mathbf{xSb}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{I} - \mathbf{b}_P \pi_b) \\ - \mathbf{v}_t \mathbf{b}_P \mathbf{S} (\mathbf{S}^{-1} \pi_b + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{b}_P \pi_b - \mathbf{I})) \end{array} \right) \tag{238}$$

$$\begin{aligned}
& + \left( \begin{array}{c} \mathbf{S}^{-1} \pi_b + \\ \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{b}_P \pi_b - \mathbf{I}) \end{array} \right)' \mathbf{Sb}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} \mathbf{S}^{-1} \pi_b + \\ \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{b}_P \pi_b - \mathbf{I}) \end{array} \right) \\
v_{3,t-} = (1 - \pi_b) \left( \begin{array}{c} \tilde{v}_{1,t} - \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \bar{\mathbf{xSb}}'_\Psi(\mathbf{b}'_t(\tilde{v}_{1,t} - v_t \mathbf{b}_P \pi_c) + \mathbf{c}'_t(\tilde{v}_{2,t} - v_{3,t} \mathbf{b}_P \pi_c)) \Phi \\ - v_t \Phi' \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \pi_c \\ - \mathbf{b}'_\Psi(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{array} \right) \Phi \end{array} \right) \tag{239}
\end{aligned}$$

$$\begin{aligned}
& + \Phi' \left( \begin{array}{c} \mathbf{S}^{-1} \pi_b + \\ \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t)(\mathbf{b}_P \pi_b - \mathbf{I}) \end{array} \right)' \mathbf{Sb}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \pi_c \\ - \mathbf{b}'_\Psi(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{array} \right) \Phi \\
v_{4,t-} = -(\tilde{v}_{1,t} - \pi_c v_t) \Phi' \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \pi_c \\ - \mathbf{b}'_\Psi(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{array} \right) \Phi \tag{240}
\end{aligned}$$

$$+ \tilde{v}_{3,t} - \tilde{v}_{1,t} \pi_c - \left( \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_{4,t}}{1 - \alpha_t} - \frac{v_t - \alpha_t \tilde{v}_{1,t}}{1 - \alpha_t} \pi_c \right) \bar{\mathbf{xSb}}'_\Psi \left( \begin{array}{c} \mathbf{b}'_t(\tilde{v}_{1,t} - v_t \mathbf{b}_P \pi_c) \\ + \mathbf{c}'_t(\tilde{v}_{2,t} - v_{3,t} \mathbf{b}_P \pi_c) \end{array} \right) \Phi$$

$$+ \Phi' \begin{pmatrix} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \boldsymbol{\pi}_c \\ -\mathbf{b}'_{\Psi}(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{pmatrix}' \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \begin{pmatrix} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \boldsymbol{\pi}_c \\ -\mathbf{b}'_{\Psi}(\tilde{v}_{1,t} \mathbf{b}'_t + \tilde{v}_{2,t} \mathbf{c}'_t) \end{pmatrix} \Phi.$$

where  $\boldsymbol{\pi} \equiv \Phi' \mathbf{b}_P \boldsymbol{\pi} \Phi$  and where we use that  $\mathbf{x}$  can be rewritten as:

$$\mathbf{x} = \tau_t^{-1} \Phi \begin{pmatrix} \tau_A & -\tau_M^{1/2} \tau_G^{1/2} \Phi' \boldsymbol{\xi}_t^{-1} \end{pmatrix} \equiv \Phi \bar{\mathbf{x}}, \quad (241)$$

so that  $\Phi' \mathbf{x} = \bar{\mathbf{x}}$ ; when performing the separation for the last two equations we have also used that  $\mathbf{c} \equiv \Phi \Phi' \mathbf{c}$  and then pre- and post-multiplied each equation by  $\Phi'$  and  $\Phi$ , respectively, which gives scalar equations for  $v_{3,t-}$  and  $v_{4,t-}$ . The first equation is a matrix equation, which we can transform into two equations for the scalars  $v_{t-}$  and  $v_{2,t-}$ . Specifically, isolating terms in  $\mathbf{I}$  gives the boundary condition for  $v_2$  in Eq. (61). To get the boundary condition for  $v_{t-}$  first re-arrange the first equation using the identities under step 2, which imply that:

$$(\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_{\Psi} = -\mathbf{v}_t \mathbf{b}_P + \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x}, \quad (242)$$

and which allows to rewrite the first equation as:

$$\begin{aligned} \mathbf{v}_{t-} &= (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)' (\mathbf{v}_t + (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_{\Psi} \mathbf{Q} \mathbf{b}'_{\Psi} (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)') (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \\ &\quad - (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)' (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_{\Psi} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b + \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b \\ &\quad - \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \mathbf{b}'_{\Psi} (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b), \end{aligned} \quad (243)$$

where  $\mathbf{Q}$  denotes the projection:

$$\mathbf{Q} \equiv \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} - \mathbf{S}. \quad (244)$$

Using the property that  $\mathbf{b}_P \mathbf{Q} = \mathbf{Q} \mathbf{b}'_P = \mathbf{0}$  along with Eq. (242) this equation simplifies to:

$$\begin{aligned} \mathbf{v}_{t-} &= \mathbf{v}_t + \boldsymbol{\pi}'_b \mathbf{b}'_P ((\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} - \mathbf{v}_t) \mathbf{b}_P \boldsymbol{\pi}_b + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \right)^2 (\mathbf{I} - \boldsymbol{\pi}'_b \mathbf{b}'_P) \mathbf{x} \mathbf{Q} \mathbf{x}' (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \\ &\quad - \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} ((\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)' \mathbf{x} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b + \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \mathbf{x}' (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)). \end{aligned} \quad (245)$$

We then pre- and post-multiply this equation by  $\Phi'$  and  $\Phi$ , respectively, and using the definition of  $\bar{\mathbf{x}}$  and  $\boldsymbol{\pi}_b$  gives the boundary condition in Eq. (58). This concludes step 1-3. Turning to step 4 we now want to take weighted combinations of the boundary conditions for  $v_{t-}$ ,  $v_{3,t-}$  and  $v_{4,t-}$ , and to this end the identity in Eq. (209) is crucial. Thus we first elaborate on how we obtain this identity. Exploiting the definition of  $\mathbf{S}$  and Eqs. (206) and

(207) we can write:

$$\mathbf{S}^{-1} = \mathbf{\Upsilon}^{-1} + \mathbf{b}'_{\Psi} \left( \mathbf{b}'_t \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x} - \mathbf{v}_t \mathbf{b}_P \right) + \mathbf{c}'_t \left( \frac{v_{3,t} - \alpha_t \bar{v}_{2,t}}{1 - \alpha_t} \mathbf{x} - v_{3,t} \mathbf{b}_P \right) \right). \quad (246)$$

Adding  $\mathbf{b}'_{\Psi}(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P$ , post-multiplying by  $\mathbf{y}$  and using once more the identities in step 2 we can then write:

$$(\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P) \mathbf{y} = \mathbf{\Upsilon}^{-1} \mathbf{y} - \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_{\Psi} \mathbf{x} \mathbf{y} + \frac{v_t - 2\alpha_t \bar{v}_{1,t} + \alpha_t^2 \bar{v}_t}{(1 - \alpha_t)^2} \mathbf{x}' \mathbf{x} \mathbf{y} \quad (247)$$

Furthermore, note that

$$\mathbf{x} \mathbf{y} = \tau_t^{-1} (\tau_A + \tau_{G,t}) \Phi \equiv \beta (1 - \alpha_t) \Phi, \quad (248)$$

with  $\tau_t - \tau_{t-} = \tau_A + \tau_{G,t}$  and that, using block matrix inversion we can write:

$$\mathbf{\Upsilon}^{-1} = \begin{pmatrix} \tau_t^{-1} \tau_A (\tau_{t-} + \tau_{G,t}) & \tau_t^{-1} \tau_A \tau_{G,t}^{1/2} \tau_M^{1/2} \Phi' \xi_t^{-1} \\ \tau_t^{-1} \tau_{G,t}^{1/2} \tau_M^{1/2} \tau_A \xi_t^{-1} \Phi & \tau_M \xi_t^{-1} (\mathbf{I} - \tau_{G,t} \tau_t^{-1} \Phi \Phi') \xi_t^{-1} \end{pmatrix}, \quad (249)$$

and this gives after simplifications:

$$\mathbf{\Upsilon}^{-1} \mathbf{y} = \tau_{t-} \mathbf{x}' \Phi. \quad (250)$$

Substituting back in the above gives Eq. (209). Now to take linear combinations of the  $v$ . coefficients we exploit Eq. (44):

$$\mathbf{b}_P (\ell_{t-} \boldsymbol{\pi}_c + (1 - \ell_{t-}) \boldsymbol{\pi}_b) = \mathbf{b}_P \mathbf{y}. \quad (251)$$

Combining the first and second equation in Eq. (238) (pre- and post-multiply the first equation by  $\Phi$ ), using the relation above along with the definition of  $\mathbf{Q}$  gives:

$$\begin{aligned} \bar{v}_{1,t-} &= \Phi' \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_P) \mathbf{y} - \\ \mathbf{b}'_{\Psi} \left( \begin{array}{c} (\ell_{t-} \tilde{v}_{1,t} + (1 - \ell_{t-}) v_t) \mathbf{b}'_t \\ (\ell_{t-} \tilde{v}_{2,t} + (1 - \ell_{t-}) v_{3,t}) \mathbf{c}'_t \end{array} \right) \end{array} \right) \Phi + \\ &+ \Phi' (\mathbf{I} - \boldsymbol{\pi}'_b \mathbf{b}'_P) \left( \begin{array}{c} \ell_{t-} \tilde{v}_{1,t} + (1 - \ell_{t-}) v_t - \mathbf{v}_t \mathbf{b}_P \mathbf{y} - \\ (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_{\Psi} \mathbf{Q} \left( \begin{array}{c} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_P) \mathbf{y} - \\ \mathbf{b}'_{\Psi} \left( \begin{array}{c} (\ell_{t-} \tilde{v}_{1,t} + (1 - \ell_{t-}) v_t) \mathbf{b}'_t + \\ (\ell_{t-} \tilde{v}_{2,t} + (1 - \ell_{t-}) v_{3,t}) \mathbf{c}'_t \end{array} \right) \end{array} \right) \end{array} \right) \Phi \\ &- \Phi' (\mathbf{I} - \boldsymbol{\pi}'_b \mathbf{b}'_P) (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_{\Psi} \mathbf{y} \Phi. \end{aligned} \quad (252)$$

Using the identities under step 2 further allow us to write:

$$\ell_{t-}\tilde{v}_{1,t} + (1 - \ell_{t-})v_t = \beta v_t + (1 - \beta)\bar{v}_{1,t} \quad (253)$$

and

$$\mathbf{b}'_{\Psi} \begin{pmatrix} (\ell_{t-}\tilde{v}_{1,t} + (1 - \ell_{t-})v_t)\mathbf{b}_t \\ +(\ell_{t-}\tilde{v}_{2,t} + (1 - \ell_{t-})v_{3,t})\mathbf{c}'_t \end{pmatrix} = -(\beta v_t + (1 - \beta)\bar{v}_{1,t})\mathbf{b}'_{\mathbf{P}} \quad (254)$$

$$+ \frac{\beta v_t + (1 - \beta)\bar{v}_{1,t} - \alpha_t(\beta\bar{v}_{1,t} + (1 - \beta)\bar{v}_t)}{1 - \alpha_t}\mathbf{x}'.$$

Substituting these expressions back along with Eq. (242) and the key identity in Eq. (209), and using the property that  $\mathbf{b}_{\mathbf{P}}\mathbf{Q} = 0$  and simplifying gives:

$$\bar{v}_{1,t-} = \left( (1 + \beta + \alpha_t\beta)\frac{v_t - \alpha_t\bar{v}_{1,t}}{1 - \alpha_t}\beta + \tau_{t-} \right) \Phi' \pi'_b \mathbf{b}'_{\mathbf{P}} (\mathbf{b}_{\mathbf{P}}\mathbf{S}\mathbf{b}'_{\mathbf{P}})^{-1} \mathbf{b}_{\mathbf{P}}\mathbf{S}\bar{\mathbf{x}}' \quad (255)$$

$$+ (1 + \beta + \alpha_t\beta)\bar{v}_{1,t} - (1 - \pi_b)\frac{v_t - \alpha_t\bar{v}_{1,t}}{1 - \alpha_t} \left( \tau_{t-} + (1 + \beta + \alpha_t\beta)\frac{\alpha_t\bar{v}_t - \bar{v}_{1,t}}{1 - \alpha_t} \right) \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}'.$$

Further noting that

$$\alpha_t\beta + 1 - \beta = \tau_{t-}/\tau_t, \quad (256)$$

and simplifying gives the boundary condition in Eq. (59). Combining the first and second equation in Eq. (238), using Eq. (251) and the definition of  $\mathbf{Q}$  further gives:

$$\bar{v}_{2,t-} = \Phi' \left( \begin{pmatrix} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{v}_t\mathbf{b}_t + v_{3,t}\mathbf{c}_t)\mathbf{b}_{\mathbf{P}})\mathbf{y} \\ -\mathbf{b}'_{\Psi} \begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-}\tilde{v}_{1,t})\mathbf{b}'_t \\ ((1 - \ell_{t-})v_{3,t} + \ell_{t-}\tilde{v}_{2,t})\mathbf{c}'_t \end{pmatrix} \\ -((1 - \ell_{t-})v_t + \ell_{t-}\tilde{v}_{1,t} - \mathbf{y}'\mathbf{b}_{\mathbf{P}}\mathbf{v}_t) \end{pmatrix} \mathbf{S}\mathbf{b}'_{\mathbf{P}}(\mathbf{b}_{\mathbf{P}}\mathbf{S}\mathbf{b}'_{\mathbf{P}})^{-1} \right) \mathbf{b}_{\mathbf{P}}\pi_c\Phi \quad (257)$$

$$+ \Phi' \left( \begin{pmatrix} (\mathbf{S}^{-1} + \mathbf{b}'_{\Psi}(\mathbf{v}_t\mathbf{b}_t + v_{3,t}\mathbf{c}_t)\mathbf{b}_{\mathbf{P}})\mathbf{y} \\ -\mathbf{b}'_{\Psi} \begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-}\tilde{v}_{1,t})\mathbf{b}'_t \\ ((1 - \ell_{t-})v_{3,t} + \ell_{t-}\tilde{v}_{2,t})\mathbf{c}'_t \end{pmatrix} \\ -((1 - \ell_{t-})v_t + \ell_{t-}\tilde{v}_{1,t} - \mathbf{y}'\mathbf{b}_{\mathbf{P}}\mathbf{v}_t)\mathbf{b}_{\mathbf{P}} \\ + \left( \frac{(1 - \ell_{t-})v_t + \ell_{t-}\tilde{v}_{1,t} - \alpha_t((1 - \ell_{t-})\bar{v}_{1,t} + \ell_{t-}\tilde{v}_{4,t})}{1 - \alpha_t} - \frac{v_t - \alpha_t\bar{v}_{1,t}}{1 - \alpha_t}\mathbf{y}'\mathbf{b}'_{\mathbf{P}} \right) \mathbf{x} \end{pmatrix} \mathbf{S}\mathbf{b}'_{\Psi} \begin{pmatrix} (\mathbf{b}'_t\mathbf{v}_t + v_{3,t}\mathbf{c}'_t)\mathbf{b}_{\mathbf{P}}\pi_c \\ -\tilde{v}_{1,t}\mathbf{b}'_t - \tilde{v}_{2,t}\mathbf{c}'_t \end{pmatrix} \right) \Phi$$

$$+ (1 - \ell_{t-})\tilde{v}_{1,t} + \ell_{t-}\tilde{v}_{3,t} - \tilde{v}_{1,t}\Phi'\mathbf{y}'\mathbf{b}'_{\mathbf{P}}\Phi.$$

Now observing that  $\ell_t \tilde{v}_{1,t} + (1 - \ell_{t-}) \tilde{v}_{2,t} = \tilde{v}_{4,t}$  allows us to write:

$$\frac{(1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_{1,t} - \alpha_t((1 - \ell_{t-})\bar{v}_{1,t} + \ell_{t-} \tilde{v}_{4,t})}{1 - \alpha_t} \mathbf{x} = ((1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_{1,t}) \mathbf{b}_P \quad (258)$$

$$+ \begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_{1,t}) \mathbf{b}_t \\ +((1 - \ell_{t-})v_{3,t} + \ell_{t-} \tilde{v}_{2,t}) \mathbf{c}_t \end{pmatrix} \mathbf{b}_\Psi.$$

Substituting this expression, using Eq. (242) and adding subtracting  $\mathbf{y}' \mathbf{S}^{-1}$  and simplifying the equation above becomes:

$$\bar{v}_{2,t-} = \Phi' \left( \begin{pmatrix} (\mathbf{S}^{-1} + \mathbf{b}'_\Psi (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_P) \mathbf{y} \\ -\mathbf{b}'_\Psi \begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_{1,t}) \mathbf{b}'_t \\ ((1 - \ell_{t-})v_{3,t} + \ell_{t-} \tilde{v}_{2,t}) \mathbf{c}'_t \end{pmatrix} \end{pmatrix} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \right) \mathbf{b}_P \pi_c \Phi \quad (259)$$

$$+ (1 - \ell_{t-}) \tilde{v}_{1,t} + \ell_{t-} \tilde{v}_{3,t} + \Phi' \mathbf{y}' (\mathbf{b}'_\Psi ((\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_P \pi_c - \tilde{v}_{1,t} \mathbf{b}'_t - \tilde{v}_{2,t} \mathbf{c}'_t) - \tilde{v}_{1,t} \mathbf{b}'_P) \Phi$$

$$+ \Phi' \begin{pmatrix} \mathbf{y}' (\mathbf{b}'_P (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_\Psi + \mathbf{S}^{-1}) \\ - \begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_{1,t}) \mathbf{b}_t \\ +((1 - \ell_{t-})v_{3,t} + \ell_{t-} \tilde{v}_{2,t}) \mathbf{c}_t \end{pmatrix} \mathbf{b}_\Psi \end{pmatrix} \mathbf{Q} \mathbf{b}'_\Psi \begin{pmatrix} (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_P \pi_c \\ -\tilde{v}_{1,t} \mathbf{b}'_t - \tilde{v}_{2,t} \mathbf{c}_t \end{pmatrix} \Phi.$$

We further compute the following terms:

$$(1 - \ell_{t-}) \tilde{v}_{1,t} + \ell_{t-} \tilde{v}_{3,t} = \beta \tilde{v}_{1,t} + (1 - \beta) \tilde{v}_t \quad (260)$$

$$(1 - \ell_{t-}) v_t + \ell_{t-} \tilde{v}_{1,t} = \beta v_t + (1 - \beta) \bar{v}_{1,t} \quad (261)$$

$$(1 - \ell_{t-}) v_{3,t} + \ell_{t-} \tilde{v}_{2,t} = \beta v_{3,t} + (1 - \beta) \bar{v}_{2,t} \quad (262)$$

along with (again using the identities under step 2):

$$\mathbf{b}'_\Psi \begin{pmatrix} (\mathbf{b}'_t \mathbf{v}_t + v_{3,t} \mathbf{c}'_t) \mathbf{b}_P \pi_c \\ -\tilde{v}_{1,t} \mathbf{b}'_t - \tilde{v}_{2,t} \mathbf{c}'_t \end{pmatrix} = \begin{pmatrix} v_t - \alpha_t \bar{v}_{1,t} \\ 1 - \alpha_t \end{pmatrix} \mathbf{x} - \mathbf{v}_t \mathbf{b}_P \quad \mathbf{b}_P \pi_c + \tilde{v}_{1,t} \mathbf{b}'_P - \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_t}{1 - \alpha_t} \mathbf{x}' \quad (263)$$

and

$$\begin{pmatrix} ((1 - \ell_{t-})v_t + \ell_{t-} \tilde{v}_t) \mathbf{b}_t \\ +((1 - \ell_{t-})v_{3,t} + \ell_{t-} \tilde{v}_{2,t}) \mathbf{c}_t \end{pmatrix} \mathbf{b}_\Psi = -(\beta v_t + (1 - \beta) \bar{v}_{1,t}) \mathbf{b}_P \quad (264)$$

$$+ \left( \beta \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} + (1 - \beta) \frac{\bar{v}_{1,t} - \alpha_t \bar{v}_t}{1 - \alpha_t} \right) \mathbf{x}.$$

Substituting these expressions back along with the key identity in Eq. (209) and that  $\mathbf{b}_P \mathbf{Q} = 0$  we get after simplifications:

$$\begin{aligned} \bar{v}_{2,t-} = & \beta \tilde{v}_{1,t} + (1 - \beta) \tilde{v}_t - \beta(v_t - \alpha_t \bar{v}_{1,t}) \pi_c + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \pi_c - \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_t}{1 - \alpha_t} \right) \Phi' \mathbf{y}' \bar{\mathbf{x}}' \quad (265) \\ & + \left( \frac{\alpha_t \bar{v}_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} (1 - \beta + \alpha_t \beta) + \tau_{t-} \right) \left( \begin{array}{c} \bar{\mathbf{x}} \mathbf{S} \mathbf{b}_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \pi_c \Phi \\ + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \pi_c - \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_t}{1 - \alpha_t} \right) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' \end{array} \right). \end{aligned}$$

Further substituting Eq. (248) and Eq. (256) and simplifying gives:

$$\bar{v}_{2,t-} = \frac{\tau_{t-}}{\tau_t} \tilde{v}_t + \tau_{t-} \left( \frac{\alpha_t \bar{v}_t - \bar{v}_{1,t}}{1 - \alpha_t} \tau_t^{-1} + 1 \right) \left( \begin{array}{c} \bar{\mathbf{x}} \mathbf{S} \mathbf{b}_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \pi_c \Phi \\ + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \pi_c - \frac{\tilde{v}_{1,t} - \alpha_t \tilde{v}_t}{1 - \alpha_t} \right) \bar{\mathbf{x}} \mathbf{S} \bar{\mathbf{x}}' \end{array} \right). \quad (266)$$

We now take the further combination of  $\bar{v}_{1,t-}$  and  $\bar{v}_{2,t-}$ , which gives:

$$\begin{aligned} \bar{v}_{t-} = & \frac{\tau_{t-}}{\tau_t} ((1 - \ell_{t-}) \bar{v}_{1,t} + \ell_{t-} \tilde{v}_t) \quad (267) \\ & + \tau_{t-} \left( 1 + \tau_t^{-1} \frac{\alpha_t \bar{v}_t - \bar{v}_{1,t}}{1 - \alpha_t} \right) \left( \begin{array}{c} \Phi' \mathbf{y}' \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}' \\ + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi - \frac{(1 - \ell_{t-}) v_t + \ell_{t-} \tilde{v}_t - \alpha_t ((1 - \ell_{t-}) \bar{v}_{1,t} + \ell_{t-} \tilde{v}_t)}{1 - \alpha_t} \right) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' \end{array} \right). \end{aligned}$$

Using the definition of  $\mathbf{Q}$  we can write:

$$\begin{aligned} \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' + \Phi' \mathbf{y}' \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}' = & - \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi \bar{\mathbf{x}} \mathbf{S} \bar{\mathbf{x}}' \quad (268) \\ & + \Phi' \mathbf{y}' \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{b}'_P \mathbf{x} + \mathbf{S}^{-1} \right) \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \bar{\mathbf{x}}'. \end{aligned}$$

Add and subtract  $-\mathbf{b}_P \mathbf{v}_t \mathbf{b}_P$  inside the bracket and use the identities under step 2 to get:

$$\begin{aligned} \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' + \Phi' \mathbf{y}' \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}' = & - \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi \bar{\mathbf{x}} \mathbf{S} \bar{\mathbf{x}}' \quad (269) \\ & + \Phi' \mathbf{y}' (\mathbf{b}'_P (\mathbf{b}_t \mathbf{v}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_P + \mathbf{S}^{-1} + \mathbf{b}_P \mathbf{v}_t \mathbf{b}'_P) \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \bar{\mathbf{x}}'. \end{aligned}$$

From the key identity in Eq. (209) we can write:

$$(\mathbf{b}'_P (\mathbf{b}_t \mathbf{v}_t + v_{3,t} \mathbf{c}_t) \mathbf{b}_P + \mathbf{S}^{-1}) \mathbf{y} = \left( \frac{v_t - 2\alpha_t \bar{v}_{1,t} + \alpha_t^2 \bar{v}_t}{1 - \alpha_t} \beta + \tau_{t-} \right) \bar{\mathbf{x}}' - \beta (v_t - \alpha_t \bar{v}_{1,t}) \mathbf{b}'_P \Phi. \quad (270)$$

Substituting this expression back in the above and using once more the identities under step

2 and the definition of  $\mathbf{Q}$  gives:

$$\begin{aligned} & \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \Phi' \mathbf{b}_P \mathbf{y} \Phi \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' + \Phi' \mathbf{y}' \mathbf{b}_P' (\mathbf{b}_P \mathbf{S} \mathbf{b}_P')^{-1} \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}' \\ & = \left( \frac{v_t - 2\alpha_t \bar{v}_{1,t} + \alpha_t^2 \bar{v}_t}{1 - \alpha_t} \beta + \tau_{t-} \right) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' + (1 - \alpha_t) \beta. \end{aligned} \quad (271)$$

Substituting in turn this expression back into the boundary condition for  $\bar{v}_{t-}$  along with Eq. (261) and:

$$(1 - \ell_{t-}) \bar{v}_{1,t} + \ell_{t-} \tilde{v}_t = \beta \bar{v}_{1,t} + (1 - \beta) \bar{v}_t, \quad (272)$$

gives:

$$\begin{aligned} \bar{v}_{t-} & = \tau_{t-} (\tau_t^{-1} (1 - \beta + \alpha_t \beta) \bar{v}_t + (1 - \alpha_t) \beta) \\ & + \tau_{t-} \left( 1 + \tau_t^{-1} \frac{\alpha_t \bar{v}_t - \bar{v}_{1,t}}{1 - \alpha_t} \right) \left( \tau_{t-} + \frac{\alpha_t \bar{v}_t - \bar{v}_{1,t}}{1 - \alpha_t} (1 - \beta + \alpha_t \beta) \right) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}'. \end{aligned} \quad (273)$$

Then substituting Eq. (256) gives Eq. (60), which concludes step 4. Finally, we obtain boundary conditions at the liquidation date by substituting the identity in Eq. (38) into the boundary conditions in Eqs. (34) and by comparing the result to the conjecture in Eq. (49):

$$v_{1,T-} = -\tau_\epsilon^2 (\tau_{T-} + \tau_\epsilon)^{-1} \quad (274)$$

$$v_{2,T-} = \tau_\epsilon \quad (275)$$

$$v_{3,T-} = v_{4,T-} = \tau_\epsilon \tau_{T-} (\tau_{T-} + \tau_\epsilon)^{-1}. \quad (276)$$

Adding the first two together gives Eq. (62). Similarly, the boundary conditions for the weighted sums  $\bar{v}_1$  and  $\bar{v}_2$  are:

$$\bar{v}_{1,T-} = \bar{v}_{2,T-} = \frac{\tau_\epsilon \tau_{T-}}{\tau_{T-} + \tau_\epsilon}. \quad (277)$$

We now repeat steps 1-3 on  $\mathbf{U}$  to verify the identity in Eq. (63) (step 4 is unnecessary in this case). Just like for the  $v$ . coefficients we define the sum  $u \equiv u_1 + u_2$  and will further need the weighted combination:

$$\bar{u} \equiv (1 - \ell)u + \ell u_3. \quad (278)$$

Now taking each term in Eq. (29) separately, computations similar to those on which Eq.



(217) is based show that

$$\begin{aligned} & (\mathbf{B}_\Psi \mathbf{B}'_{\mathbf{P}} (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} \mathbf{A}_{\mathbf{P}} - \mathbf{A}_\Psi)' \mathbf{U} \\ &= \left( \mathbf{u} \frac{d}{dt} (\mathbf{b} + \mathbf{c}) - (u - u_3) \frac{\tau_P}{\tau^c} \mathbf{c} + \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{u} - u) \Phi \Phi' ((\Pi_1 + \Pi_2) \mathbf{b} + \Pi_3 \mathbf{c}) \right)' \xi_0 \mathbf{M}. \end{aligned} \quad (279)$$

and from computations on which Eq. (222) is based we have:

$$\mathbf{V} \mathbf{B}_\Psi (\mathbf{I} - \mathbf{B}'_{\mathbf{P}} (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} \mathbf{B}_{\mathbf{P}}) \mathbf{B}'_{\Psi} \mathbf{U} = \tau_v \tau^{-2} \bar{u} (\bar{v}_1 \mathbf{b} + \bar{v}_2 \mathbf{c})' \Phi \Phi' \xi_0 \mathbf{M}. \quad (280)$$

Furthermore, differentiating Eq. (49) gives:

$$\dot{\mathbf{U}} = \left( \mathbf{u} \frac{d}{dt} \mathbf{b} + u_3 \frac{d}{dt} \mathbf{c} + \dot{\mathbf{u}} \mathbf{b} + u'_3 \mathbf{c} \right)' \xi_0 \mathbf{M} + (\mathbf{u} \mathbf{b} + u_3 \mathbf{c})' \dot{\xi}_0 \mathbf{M}. \quad (281)$$

Finally, Eqs. (199) and (201) imply that

$$\mathbf{b} \mathbf{B}_\Psi \mathbf{B}'_{\mathbf{P}} (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} = -\mathbf{I} - \tau_P^{1/2} ((1 - \ell) \tau^{-1} + \ell (\tau^c)^{-1}) \Phi \Phi' \Sigma_{\mathbf{P}}^{-1} \quad (282)$$

$$\mathbf{c} \mathbf{B}_\Psi \mathbf{B}'_{\mathbf{P}} (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} = \tau_P^{1/2} \ell \alpha (\tau^c)^{-1} \Phi \Phi' \Sigma_{\mathbf{P}}^{-1}, \quad (283)$$

which allows us to write:

$$(\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} \mathbf{V} - \mathbf{A}_{\mathbf{P}})' (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} \dot{\xi}_0 \mathbf{M} \quad (284)$$

$$\begin{aligned} &= \left( -(\mathbf{v} \mathbf{b} + v_3 \mathbf{c})' + \frac{\tau_P^{1/2}}{\tau^c} ((\alpha \bar{v}_1 - v) \mathbf{b} + (\alpha \bar{v}_2 - v_3) \mathbf{c})' \Phi \Phi' \Sigma_{\mathbf{P}}^{-1} - \mathbf{A}_{\mathbf{P}} \Sigma_{\mathbf{P}}^{-1} \right) \dot{\xi}_0 \mathbf{M} \\ &= -(\mathbf{v} \mathbf{b} + v_3 \mathbf{c})' \dot{\xi}_0 \mathbf{M} - \frac{\tau_P^{1/2}}{\tau^c} (\Pi_1 + \Pi_2) ((\alpha \bar{v}_1 - v) \mathbf{b} + (\alpha \bar{v}_2 - v_3) \mathbf{c})' \Phi \Phi' \xi_0 \mathbf{M} \\ &+ (\mathbf{b}' (((\Pi_1 + \Pi_2)^2 - \Pi_2^2) \Phi \Phi' + \Pi_2^2 \mathbf{I}) + \Pi_3 (\Pi_1 + \Pi_2) \mathbf{c}') \xi_0 \mathbf{M}. \end{aligned} \quad (285)$$

where the last equality uses Conjecture 1, part A and B. Substituting these expressions back into Eq. (29) and simplifying gives:

$$\begin{aligned} & (\dot{\mathbf{u}} \mathbf{a} + u'_3 \mathbf{c})' \xi_0 \mathbf{M} = \tau_v \tau^{-2} \bar{u} (\bar{v}_1 \mathbf{b} + \bar{v}_2 \mathbf{c})' \Phi \Phi' \xi_0 \mathbf{M} - ((\mathbf{v} + \mathbf{u}) \mathbf{b} + (v_3 + u_3) \mathbf{c})' \dot{\xi}_0 \mathbf{M} \\ &+ \left( (u - u_3) \left( \ell^{-1} \frac{d}{dt} \ell - \frac{\tau_P}{\tau^c} \right) \mathbf{c} + \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{u} - u) \Phi \Phi' ((\Pi_1 + \Pi_2) \mathbf{b} + \Pi_3 \mathbf{c}) \right)' \xi_0 \mathbf{M} \\ &- \frac{\tau_P^{1/2}}{\tau^c} (\Pi_1 + \Pi_2) ((\alpha \bar{v}_1 - v) \mathbf{b} + (\alpha \bar{v}_2 - v_3) \mathbf{c})' \Phi \Phi' \xi_0 \mathbf{M} \\ &+ (\mathbf{b}' (((\Pi_1 + \Pi_2)^2 - \Pi_2^2) \Phi \Phi' + \Pi_2^2 \mathbf{I}) + \Pi_3 (\Pi_1 + \Pi_2) \mathbf{c}') \xi_0 \mathbf{M}. \end{aligned} \quad (286)$$

We can now verify that the solution is as in Eq. (63) in between announcements. Suppose this is true and separate terms in  $\mathbf{b}'\Phi\Phi'\xi_0\mathbf{M}$ ,  $\mathbf{b}'\xi_0\mathbf{M}$  and  $\mathbf{c}'\xi_0\mathbf{M}$  in the above, which produces the following system of three ODEs:

$$u'_1 = \frac{\tau_P^{1/2}}{\tau^c}(\Pi_1 + \Pi_2)(\alpha\bar{u} - u - (\alpha\bar{v}_1 - v)) + \frac{\tau_v}{\tau^2}\bar{u}\bar{v}_1 + (\Pi_1 + \Pi_2)^2 - \Pi_2^2 \quad (287)$$

$$u'_2 = \Pi_2^2 + \Pi_2(v_2 + u_2) \quad (288)$$

$$u'_3 = (u - u_3) \left( \ell^{-1} \frac{d}{dt} \ell - \frac{\tau_P}{\tau^c} \right) + \frac{\tau_P^{1/2}}{\tau^c} ((\alpha\bar{u} - u)\Pi_3 - (\alpha\bar{v}_2 - v_3)(\Pi_1 + \Pi_2)) \quad (289)$$

$$+ \frac{\tau_v}{\tau^2} \bar{u}\bar{v}_2 + \Pi_3(\Pi_1 + \Pi_2),$$

with terminal conditions given by:

$$u_{1,T-} = -\frac{\tau_\epsilon^2}{\tau_\epsilon + \tau_{T-}} \equiv -v_{1,T-} \quad (290)$$

$$u_{2,T-} = -\tau_\epsilon \equiv -v_{2,T-} \quad (291)$$

$$u_{3,T-} = -\frac{\tau_\epsilon \tau_{T-}}{\tau_\epsilon + \tau_{T-}} \equiv -v_{3,T-}. \quad (292)$$

We can readily verify that the identification in Eq. (63) holds at the horizon date, and substituting Eq. (63) into Eq. (287) shows that this system is identical to that in Eqs. (223)–(225), which indeed verifies Eq. (63) holds in between announcements. We now verify that it holds at the announcement. Start by rewriting Eq. (31) as:

$$\mathbf{U}_{t-} = (\mathbf{I} + \mathbf{a}_\Psi)\mathbf{U}_t + (\mathbf{I} + \mathbf{a}_\Psi)\mathbf{V}_t\mathbf{b}_\Psi\mathbf{Q}\mathbf{b}'_\Psi\mathbf{U}_t + \mathbf{a}'_P(\mathbf{b}_P\mathbf{S}\mathbf{b}'_P)^{-1}(\xi_{0,t} - \xi_{0,t-})\mathbf{M} \quad (293)$$

$$- \mathbf{a}'_P(\mathbf{b}_P\mathbf{S}\mathbf{b}'_P)^{-1}\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\mathbf{U}_t - (\mathbf{I} + \mathbf{a}_\Psi)\mathbf{V}_t\mathbf{b}_\Psi\mathbf{S}\mathbf{b}'_P(\xi_{0,t} - \xi_{0,t-})\mathbf{M}.$$

Next we substitute the unconditional premium in Eq. (48) and  $\mathbf{U}$  in Eq. (49) (step 3), and after separating terms we obtain:

$$\mathbf{b}'_{t-}\mathbf{u}_{t-} + u_{3,t-}\mathbf{c}'_{t-} = (\mathbf{I} + \mathbf{a}_\Psi)(\mathbf{b}'_t\mathbf{u}_t + u_{3,t}\mathbf{c}'_t)(\mathbf{I} - \mathbf{b}_P\pi_b) - \mathbf{a}'_P(\mathbf{b}_P\mathbf{S}\mathbf{b}'_P)^{-1}\mathbf{b}_P\pi_b \quad (294)$$

$$+ (\mathbf{I} + \mathbf{a}_\Psi)\mathbf{V}_t\mathbf{b}_\Psi\mathbf{Q}\mathbf{b}'_\Psi(\mathbf{b}'_t\mathbf{u}_t + u_{3,t}\mathbf{c}'_t)(\mathbf{I} - \mathbf{b}_P\pi_b)$$

$$- \mathbf{a}'_P(\mathbf{b}_P\mathbf{S}\mathbf{b}'_P)^{-1}\mathbf{b}_P\mathbf{S}\mathbf{b}'_P(\mathbf{b}'_t\mathbf{u}_t + u_{3,t}\mathbf{c}'_t)(\mathbf{I} - \mathbf{b}_P\pi_b)$$

$$+ (\mathbf{I} + \mathbf{a}_\Psi)\mathbf{V}_t\mathbf{b}_\Psi\mathbf{S}\mathbf{b}'_P(\mathbf{b}_P\mathbf{S}\mathbf{b}'_P)^{-1}\mathbf{b}_P\pi_b.$$

Next we use the two observations in Eqs. (195) and (204) (step 1-2) to write:

$$(\mathbf{I} + \mathbf{a}_\Psi)(\mathbf{b}'_t \mathbf{u}_t + u_{3,t} \mathbf{c}'_t) = \mathbf{b}'_{t-}(\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)' \mathbf{u}_t + \mathbf{c}'_{t-}(\boldsymbol{\pi}'_c \mathbf{u}_t + \tilde{u}_1) \quad (295)$$

$$\mathbf{b}'_\Psi(\mathbf{b}'_t \mathbf{u}_t + u_{3,t} \mathbf{c}'_t) = -\mathbf{b}'_P \mathbf{u}_t + \frac{u_t - \alpha_t \bar{u}_t}{1 - \alpha_t} \mathbf{x}' \quad (296)$$

$$\begin{aligned} (\mathbf{I} + \mathbf{a}_\Psi) \mathbf{V}_t \mathbf{b}_\Psi &= \mathbf{b}'_{t-}(\mathbf{I} + \boldsymbol{\pi}'_b \mathbf{b}'_P) \left( \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \mathbf{x} - \mathbf{v} \mathbf{b}_P \right) \\ &+ \mathbf{c}'_{t-} \left( \left( \frac{\tilde{v}_1 - \alpha_t \tilde{v}}{1 - \alpha_t} - \frac{v_1 - \alpha_t \bar{v}_1}{1 - \alpha_t} \boldsymbol{\pi}'_c \mathbf{b}'_P \right) \mathbf{x} + (\boldsymbol{\pi}'_c \mathbf{b}'_P \mathbf{v} - \tilde{v}_1) \mathbf{b}_P \right), \end{aligned} \quad (297)$$

where the first and last lines also use Eq. (42). Substituting back and separating terms in  $\mathbf{b}$  and  $\mathbf{c}$  we get two boundary conditions for  $\mathbf{u}$  and  $u_3$ :

$$\mathbf{u}_{t-} = (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b)' \mathbf{u}_t (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) - \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b \quad (298)$$

$$\begin{aligned} &- \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \left( \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \mathbf{x}' - \mathbf{b}'_P \mathbf{u}_t \right) (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \\ &+ (\mathbf{I} - \boldsymbol{\pi}'_b \mathbf{b}'_P) \left( \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \left( \mathbf{x} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b + \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \mathbf{x} \mathbf{Q} \mathbf{x}' (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \right) \right) \end{aligned}$$

$$u_{3,t-} = \boldsymbol{\Phi}'(\mathbf{u}_t \boldsymbol{\pi}'_c + \tilde{u}_1)(\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) - \boldsymbol{\Phi}' \boldsymbol{\pi}'_c \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\Phi} \quad (299)$$

$$\begin{aligned} &- \boldsymbol{\Phi} \boldsymbol{\pi}'_c \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \left( \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \mathbf{x}' - \mathbf{b}'_P \mathbf{u}_t \right) (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \boldsymbol{\Phi} + \boldsymbol{\Phi}'(\boldsymbol{\pi}'_c \mathbf{b}'_P \mathbf{v}_t - \tilde{v}_1) \mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\Phi} \\ &+ \boldsymbol{\Phi}' \left( \frac{\tilde{v}_1 - \alpha_t \tilde{v}}{1 - \alpha_t} - \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \boldsymbol{\pi}'_c \mathbf{b}'_P \right) \left( \mathbf{x} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b + \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \mathbf{x} \mathbf{Q} \mathbf{x}' (\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \right) \boldsymbol{\Phi}. \end{aligned}$$

By isolating the terms in  $\mathbf{I}$  in the first equation and by pre- and post-multiplying this equation by  $\boldsymbol{\Phi}'$  and  $\boldsymbol{\Phi}$ , respectively, and using the notation introduced above these equations can be further rearranged as:

$$u_{t-} = u_t - \boldsymbol{\Phi}' \boldsymbol{\pi}'_b \mathbf{b}'_P ((\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} + \mathbf{u}_t) \mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\Phi} + \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} (1 - \pi_b)^2 \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' \quad (300)$$

$$+ (1 - \pi_b) \left( \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} - \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \right) \bar{\mathbf{x}} \mathbf{S} \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\Phi} - \boldsymbol{\Phi}' \boldsymbol{\pi}'_b \mathbf{b}'_P (\mathbf{v} + \mathbf{u})(\mathbf{I} - \mathbf{b}_P \boldsymbol{\pi}_b) \boldsymbol{\Phi}$$

$$u_{2,t-} = u_{2,t} - \tau_M / \xi_{2,t}^2 \pi_2^2 - \pi_2 (u_{2,t} + v_{2,t}) \quad (301)$$

$$u_{3,t-} = \tilde{u}_1 - \boldsymbol{\Phi}' \boldsymbol{\pi}'_b \mathbf{b}'_P ((\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} + \mathbf{u}_t) \mathbf{b}_P \boldsymbol{\pi}_c \boldsymbol{\Phi} - (1 - \pi_b) \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \boldsymbol{\Phi}' \boldsymbol{\pi}'_c \mathbf{b}'_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}' \quad (302)$$

$$\begin{aligned} &+ \left( \frac{\tilde{v}_1 - \alpha_t \tilde{v}}{1 - \alpha_t} - \pi_c \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \right) \left( (1 - \pi_b) \frac{u - \alpha_t \bar{u}}{1 - \alpha_t} \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' + \bar{\mathbf{x}} \mathbf{S} \mathbf{b}_P (\mathbf{b}_P \mathbf{S} \mathbf{b}'_P)^{-1} \mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\Phi} \right) \\ &- (\tilde{v}_1 + \tilde{u}_1) \pi_b + \boldsymbol{\Phi}' \boldsymbol{\pi}'_b (\mathbf{u}_t + \mathbf{v}_t) \mathbf{b}_P \boldsymbol{\pi}_c \boldsymbol{\Phi}. \end{aligned}$$

Clearly, if Eq. (63) holds then the first two equations are identical to Eqs. (58) and (61), respectively. Furthermore, using the identities under step 2 Eq. (240) can be reexpressed as:

$$v_{3,t-} = \tilde{v}_1 + \Phi' \pi'_b \mathbf{b}'_{\mathbf{P}} ((\mathbf{b}_{\mathbf{P}} \mathbf{S} \mathbf{b}'_{\mathbf{P}})^{-1} - v) \mathbf{b}_{\mathbf{P}} \pi_c \Phi - \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} (1 - \pi_b) \bar{\mathbf{x}} \mathbf{S} \mathbf{b}'_{\mathbf{P}} (\mathbf{b}_{\mathbf{P}} \mathbf{S} \mathbf{b}'_{\mathbf{P}})^{-1} \mathbf{b}_{\mathbf{P}} \pi_c \Phi \\ + \left( \frac{\tilde{v}_1 - \alpha_t \tilde{v}}{1 - \alpha_t} - \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} \pi_c \right) \left( \frac{v - \alpha_t \bar{v}_1}{1 - \alpha_t} (1 - \pi_b) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' - \bar{\mathbf{x}} \mathbf{S} \mathbf{b}'_{\mathbf{P}} (\mathbf{b}_{\mathbf{P}} \mathbf{S} \mathbf{b}'_{\mathbf{P}})^{-1} \mathbf{b}_{\mathbf{P}} \pi_b \Phi \right). \quad (303)$$

If Eq. (63) holds the third equation is identical to this equation and the claim holds.

Finally, regarding the portfolio formulation in Eq. (64) in the main text, substituting the conjecture in Eq. (28) in Eq. (174) implies that investor  $i$ 's optimal portfolio on the post-announcement subperiod take the form:

$$\mathbf{w}^i = \frac{1}{\gamma} (\mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}})^{-1} \left( \dot{\xi}_0 \mathbf{M} - \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} \mathbf{U} + (\mathbf{A}_{\mathbf{P}} - \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} \mathbf{V}) \Psi^i \right) \quad (304)$$

$$= \frac{1}{\gamma} \Sigma_{\mathbf{P}}^{-1} \left( \underbrace{\Pi_1 \Phi \Phi' + \Pi_2 \mathbf{I} + \Sigma_{\mathbf{P}} \mathbf{V} - \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{v}_1 - v) \Phi \Phi'}_{\text{traditional demand in a REE absent residual uncertainty}} \right) (\mathbf{b} \Psi^i - \xi_0 \mathbf{M}) \quad (305)$$

$$+ \frac{1}{\gamma} \Sigma_{\mathbf{P}}^{-1} \left( \underbrace{\Pi_3 \Phi \Phi' + v_3 \Sigma_{\mathbf{P}} - \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{v}_2 - v_3) \Phi \Phi'}_{\text{speculation on the wedge } \ell} \right) \mathbf{c} \Psi^i.$$

We obtain Eq. (64) by substituting the conjectured form for  $\mathbf{U}$  and  $\mathbf{V}$  in Eq. (49) and the identity in Eq. (63) in the first equality; the second equality further substitutes Eq. (282) and the conditional and unconditional risk premia of Conjecture 1, and regroups terms as follows: the first line corresponds to the traditional portfolio structure in a NRE absent residual uncertainty in final payoffs; the second term arises when  $\ell \neq 0$  and is the part of the demand through which investors speculate on this wedge.

## A.4 Proof of Proposition 4

In this appendix we aggregate individual portfolios and clear the market, which produces fixed-point equations for the price coefficients. The procedure is in four steps:

1. Derive the dynamics of state variables at the population level and aggregate to obtain equilibrium conditions.
2. Show that  $\xi$  and  $\xi_0$  take the form in Eq. (67) and obtain ODEs for the equilibrium coefficients  $\ell$  and  $\xi$ , and algebraic equations for the equilibrium speed of information revelation,  $\tau_P^{1/2}$  and  $\tau_G$ .

3. Verify Conjecture 1, which in turn gives expressions for the unknown market prices of risk,  $\Pi$ . and  $\boldsymbol{\pi}$ .
4. Substitute these coefficients and simplify to obtain the equations in the proposition.

We start with the dynamics of state variables at the population level. Specifically, an application of Ito's lemma gives:

$$d\tau_t^c \widehat{F}_t^c = \tau_{P,t} \widehat{F}_t^i dt - \tau_{P,t}^{1/2} \boldsymbol{\Phi}^\top d\widehat{\mathbf{B}}_{m,t}^i + (\tau_t \boldsymbol{\Phi}' \mathbf{x}'_t \widetilde{\mathbf{y}}_t^i + (\tau_A + \tau_{G,t}) \widehat{F}_{t-}^i) \mathbf{1}_{t=\tau}, \quad (306)$$

where the matrix  $\mathbf{x}$  is defined in Eq. (203) below. Similarly, we have:

$$d\tau_t \widehat{F}_t^i = (\tau_{P,t} + \tau_v) \widehat{F}_t^i dt - \tau_{P,t}^{1/2} \boldsymbol{\Phi}^\top d\widehat{\mathbf{B}}_{m,t}^i + \tau_v^{1/2} d\widehat{B}_t^i + \underbrace{(\tau_t \Delta \widehat{F}_t^i + \Delta \tau_t \widehat{F}_{t-}^i)}_{\equiv \tau_t \boldsymbol{\Phi}' \mathbf{x}'_t \widetilde{\mathbf{y}}_t^i + (\tau_A + \tau_{G,t}) \widehat{F}_{t-}^i} \mathbf{1}_{t=\tau}. \quad (307)$$

Subtracting one from the other we obtain:

$$d\tau_t \widehat{F}_t^i - d\tau_t^c \widehat{F}_t^c = \tau_v \widehat{F}_t^i dt + \tau_v^{1/2} d\widehat{B}_t^i. \quad (308)$$

That is, the difference between an investor's and the econometrician's views scaled by their respective precision does not jump upon announcements. From the innovation process we further have:

$$d\widehat{B}_t^i = \tau_v^{1/2} (d\widetilde{V}_t^i - \widehat{F}_t^i dt), \quad (309)$$

which, after substituting in the previous equation, gives:

$$d\tau_t \widehat{F}_t^i - d\tau_t^c \widehat{F}_t^c = \tau_v d\widetilde{V}_t^i = \tau_v \widetilde{F} dt + \tau_v^{1/2} dB_t^i. \quad (310)$$

Integrating, taking into account the initial signals in Eq. (15), yields:

$$\tau_t \widehat{F}_t^i = \tau_t^c \widehat{F}_t^c + (\tau_V + \tau_v t) \widetilde{F} + \tau_v^{1/2} (\widetilde{\epsilon}^i + B_{v,t}^i). \quad (311)$$

Denoting average market expectations at time  $t$  by  $\widehat{F}_t = \int_0^1 \widehat{F}_t^i di$  and using the law of large numbers (Duffie and Sun, 2007) whereby:

$$\int_0^1 B_t^i di = \int_0^1 \widetilde{\epsilon}^i di = 0, \quad (312)$$

we can write average market expectations of the common factor as a linear combination of

its true value,  $\tilde{F}$ , and econometrician's expectations,  $\widehat{F}^c$ :

$$\widehat{F}_t = \frac{\tau_t^c}{\tau_t} \widehat{F}_t^c + \underbrace{\frac{\tau_t - \tau_t^c}{\tau_t}}_{\equiv \alpha_t} \tilde{F}. \quad (313)$$

It follows that the average informational advantage relative to the econometrician is:

$$\int_0^1 \Delta_t^i di = \alpha_t (\tilde{F} - \widehat{F}_t^c) \equiv \alpha_t \Delta_t, \quad (314)$$

and, by observational equivalence, that:

$$\int_0^1 \widehat{\mathbf{m}}_t^i di = \tilde{\mathbf{m}}_t + (1 - \alpha_t) \lambda_t \boldsymbol{\xi}_t^{-1} \Phi \Delta_t. \quad (315)$$

Concatenating these two expressions finally delivers the desired aggregation result:

$$\int_0^1 \boldsymbol{\Psi}_t^i di = \underbrace{\begin{pmatrix} \alpha_t & \mathbf{0} \\ (1 - \alpha_t) \lambda_t \boldsymbol{\xi}_t^{-1} \Phi & \mathbf{I} \end{pmatrix}}_{\equiv \Gamma_t} \boldsymbol{\Psi}_t. \quad (316)$$

Note that this aggregation result holds at any date, including announcement dates. We now use this aggregation result to clear the market and obtain equilibrium equations at each date. We start with the equilibrium equations that hold in between announcements. Substituting optimal demands in Eq. (64) in the market-clearing equation and separating variables yields the following fixed-point equations in between announcements:

$$\left( \dot{\boldsymbol{\xi}}_0 - \mathbf{B}_P \mathbf{B}'_{\Psi} (\mathbf{u} \mathbf{b} + u_3 \mathbf{c})' \boldsymbol{\xi}_0 \right) \mathbf{M} = \gamma \mathbf{B}_P \mathbf{B}'_P \mathbf{M}, \quad (317)$$

$$(\mathbf{A}_P - \mathbf{B}_P \mathbf{B}'_{\Psi} (\mathbf{b}' \mathbf{v} \mathbf{b} + v_3 (\mathbf{b}' \mathbf{c} + \mathbf{c}' \mathbf{b}) + v_4 \mathbf{c}' \mathbf{c})) \Gamma = \gamma \mathbf{B}_P \mathbf{B}'_P \mathbf{1}^*. \quad (318)$$

We now repeat this operation immediately prior to the announcement date,  $\tau$ . For simplicity we let  $t$  be the announcement date and  $t-$  the time right before the announcement is made. Substituting the conjectured forms in Eq. (49) in an investor  $i$ 's optimal portfolio immediately prior to the announcement date, which we obtained in Eq. (182), gives:

$$\begin{aligned} \mathbf{w}_{t-}^i &= \frac{1}{\gamma} (\mathbf{b}_P \mathbf{S}_t \mathbf{b}'_P)^{-1} (\boldsymbol{\xi}_{0,t} - \boldsymbol{\xi}_{0,t-} - \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_{\Psi} (\mathbf{b}'_t \mathbf{u}_t + u_{3,t} \mathbf{c}') \boldsymbol{\xi}_{0,t}) \mathbf{M} \\ &+ \frac{1}{\gamma} (\mathbf{b}_P \mathbf{S}_t \mathbf{b}'_P)^{-1} (\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_{\Psi} ((\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_t + (v_{3,t} \mathbf{b}_t + v_{4,t} \mathbf{c}_t)' \mathbf{c}_t) (\mathbf{I} + \mathbf{a}_{\Psi})) \boldsymbol{\Psi}_{t-}. \end{aligned} \quad (319)$$

Imposing the market-clearing condition:

$$\int_0^1 \mathbf{w}_{t_k^-}^i di = \mathbf{M} + \tilde{\mathbf{m}}_{t_k^-}, \quad (320)$$

and separating variables yields the following fixed-point equations:

$$(\boldsymbol{\xi}_{0,t} - \boldsymbol{\xi}_{0,t^-} - \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_{\Psi} (\mathbf{b}'_t \mathbf{u}_t + u_{3,t} \mathbf{c}'_t) \boldsymbol{\xi}_{0,t}) \mathbf{M} = \gamma \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_P \mathbf{M}, \quad (321)$$

$$(\mathbf{a}_P - \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_{\Psi} ((\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)' \mathbf{b}_t + (v_{3,t} \mathbf{b}_t + v_{4,t} \mathbf{c}_t)' \mathbf{c}_t) (\mathbf{I} + \mathbf{a}_{\Psi})) \boldsymbol{\Gamma}_{t^-} = \gamma \mathbf{b}_P \mathbf{S}_t \mathbf{b}'_P \mathbf{1}^*. \quad (322)$$

Finally, we obtain terminal conditions for the post-announcement subperiod by aggregating portfolios at the terminal date (those in Eq. (190)) and imposing market clearing:

$$\int_0^1 \mathbf{w}_{T^-}^i di = \frac{1}{\gamma} (\tau_{T^-}^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_{\epsilon}^{-1} \mathbf{I})^{-1} (-\boldsymbol{\xi}_{0,T^-} \mathbf{M} + \mathbf{a}_{T^-} \boldsymbol{\Gamma}_{T^-} \boldsymbol{\Psi}_{T^-}) \quad (323)$$

$$= \mathbf{M} + \tilde{\mathbf{m}}_T. \quad (324)$$

Separating variables produces the desired terminal conditions:

$$\boldsymbol{\xi}_{0,T^-} = \boldsymbol{\xi}_{T^-} = -\gamma (\tau_{T^-}^{-1} \boldsymbol{\Phi} \boldsymbol{\Phi}' + \tau_{\epsilon}^{-1} \mathbf{I}), \quad (325)$$

$$\equiv -\gamma \mathbf{v}_{T^-}^{-1} \quad (326)$$

$$\lambda_{T^-} = \alpha_{T^-}, \quad (327)$$

where Eq. (327) shows that the wedge  $\ell_{T^-} = 0$  vanishes at the horizon date, as reported in Eqs. (76)–(77).

In step 2 we now obtain an equation for  $\ell$  and verify that the solution for  $\boldsymbol{\xi}_0$  and  $\boldsymbol{\xi}$  has the form in Eq. (67), which gives two ODEs for  $\xi$  and  $\xi_2$ . We then combine these equations to obtain algebraic equations for  $\tau_P^{1/2}$  and  $\tau_G^{1/2}$ . We start by separating the first column in Eq. (318) from the other  $N$  columns, which gives the following system of equations:

$$0 = (\mathbf{A}_P - \mathbf{B}_P \mathbf{B}'_{\Psi} \mathbf{V}) (\mathbf{1}^{*'} \lambda \boldsymbol{\xi}^{-1} \boldsymbol{\Phi} + \alpha \boldsymbol{\omega}) \quad (328)$$

$$\dot{\boldsymbol{\xi}} = \mathbf{B}_P (\mathbf{B}'_{\Psi} \mathbf{V} \mathbf{1}^{*'} + \gamma \mathbf{B}'_P). \quad (329)$$

The first equation determines  $\lambda$  (equivalently,  $\ell$ ) and the second equation determines  $\boldsymbol{\xi}$ . From the second equation we can verify Eq. (67) holds in between announcements. Using Eqs.

(198) and (201) we rewrite this equation as:

$$\dot{\boldsymbol{\xi}} = \boldsymbol{\Sigma}_{\mathbf{P}} \left( \boldsymbol{\Sigma}_{\mathbf{P}}(\mathbf{v}\boldsymbol{\xi} + \gamma) + \tau_P^{1/2} \frac{v - \alpha\bar{v}_1}{\tau^c} \boldsymbol{\Phi}\boldsymbol{\Phi}'\boldsymbol{\xi} \right). \quad (330)$$

Now substitute Eq. (67) and isolate terms in  $\mathbf{I}$ , which gives an ODE for  $\xi_2$

$$\xi_2' = \tau_m^{-1} \xi_2^2 (v_2 \xi_2 + \gamma), \quad (331)$$

and proceeding similarly with Eq. (325) we obtain a boundary condition for it:

$$\xi_{2,T-} = -\gamma/\tau_\epsilon \equiv -\gamma/v_{2,T-}. \quad (332)$$

The remaining terms are proportional to  $\boldsymbol{\Phi}\boldsymbol{\Phi}'$ . In particular, pre- and post-multiplying Eq. (330) by  $\boldsymbol{\Phi}'$  and  $\boldsymbol{\Phi}$ , respectively, gives an ODE for  $\xi$ :

$$\xi' = \boldsymbol{\Sigma}_{\mathbf{P}} \left( \boldsymbol{\Sigma}_{\mathbf{P}}(v\xi + \gamma) + \tau_P^{1/2} \frac{v - \alpha\bar{v}_1}{\tau^c} \xi \right) \quad (333)$$

$$= \boldsymbol{\Sigma}_{\mathbf{P}} \left( \left( \tau_m^{-1/2} \xi v - \alpha \ell \frac{\tau_P^{1/2}}{\tau^c} v_3 \right) \xi + \gamma \boldsymbol{\Sigma}_{\mathbf{P}} \right), \quad (334)$$

where the second equality uses the definition of  $\boldsymbol{\Sigma}_{\mathbf{P}}$  in Eq. (70). Repeating this operation on the boundary condition in Eq. (325) we get Eq. (76). We further show that  $\boldsymbol{\xi} \equiv \boldsymbol{\xi}_0$ . Start from the equilibrium condition in Eq. (317), which can be rewritten as:

$$\dot{\boldsymbol{\xi}}_0 = \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} (\mathbf{u}\mathbf{b} + u_3 \mathbf{c})' \boldsymbol{\xi}_0 + \gamma \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\mathbf{P}} \quad (335)$$

$$= \boldsymbol{\Sigma}_{\mathbf{P}} \left( \boldsymbol{\Sigma}_{\mathbf{P}}(-\mathbf{u}\boldsymbol{\xi}_0 + \gamma) - \tau_P^{1/2} \frac{u - \alpha\bar{u}}{\tau^c} \boldsymbol{\xi}_0 \right). \quad (336)$$

Since we have shown in Eq. (63) that  $\mathbf{u} \equiv -\mathbf{v}$  and  $u_3 = -v_3$ , if  $\boldsymbol{\xi}_0 \equiv \boldsymbol{\xi}$  this equation is identical to Eq. (330) and given the terminal condition in Eq. (325) this proves the claim, and thus proves the conjecture in Eq. (67) holds in between announcements. We now turn to Eq. (328), which represents an equation for  $\lambda$ . Note, however, that it is in fact more convenient to turn it into an algebraic equation for  $\tau_P^{1/2}$  and instead use the definition of  $\tau_P^{1/2}$  as an equation for  $\lambda$ . We first write:

$$(\mathbf{A}_{\mathbf{P}} - \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} \mathbf{V}) \mathbf{1}^{\star'} = \dot{\boldsymbol{\xi}} - \mathbf{B}_{\mathbf{P}} \mathbf{B}'_{\Psi} \mathbf{V} \mathbf{1}^{\star'} \quad (337)$$

$$= \gamma \boldsymbol{\Sigma}_{\mathbf{P}} \boldsymbol{\Sigma}_{\mathbf{P}}, \quad (338)$$



where the second line follows from Eq. (329), which we plug back into Eq. (328):

$$\gamma\lambda\Sigma_{\mathbf{P}}\Sigma_{\mathbf{P}}\xi^{-1}\Phi + \alpha(\mathbf{A}_{\mathbf{P}} - \mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\Psi}\mathbf{V})\omega = 0. \quad (339)$$

Post-multiplying the definition of  $\mathbf{A}_{\mathbf{P}}$  in Eq. (153) by  $\omega$  gives:

$$\mathbf{A}_{\mathbf{P}}\omega = -\tau_{P,t}^{1/2}\Sigma_{\mathbf{P}}\Phi, \quad (340)$$

which is a central result that underlies many properties of this kind of settings (e.g., Eq. (43)). Substituting back we obtain:

$$\gamma\lambda\Sigma_{\mathbf{P}}\Sigma_{\mathbf{P}}\xi^{-1}\Phi - \alpha(\tau_{P,t}^{1/2}\Sigma_{\mathbf{P}}\Phi + \mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\Psi}\mathbf{V}\omega) = 0. \quad (341)$$

Using Eqs. (198) and (201) we can write:

$$\mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\Psi}\mathbf{V}\omega = \mathbf{B}_{\mathbf{P}}\mathbf{B}'_{\Psi}(\bar{v}_1\mathbf{b}' + \bar{v}_2\mathbf{c}')\Phi \quad (342)$$

$$= -\bar{v}_1\Sigma_{\mathbf{P}}\Sigma_{\mathbf{P}}\Phi + \tau_{P,t}^{1/2}\frac{\alpha\bar{v} - \bar{v}_1}{\tau^c}\Sigma_{\mathbf{P}}\Phi, \quad (343)$$

which we plug back into the previous equation and after pre-multiplying by  $\Sigma_{\mathbf{P}}^{-1}$  and dividing by  $\alpha$  we get:

$$\gamma(1 - \ell)\Sigma_{\mathbf{P}}\xi^{-1}\Phi + \bar{v}_1\Sigma_{\mathbf{P}}\Phi - \tau_{P,t}^{1/2}\left(1 + \frac{\alpha\bar{v} - \bar{v}_1}{\tau^c}\right)\Phi = 0. \quad (344)$$

After pre-multiplying this equation by  $\Phi'$  we obtain an algebraic equation for  $\tau_P^{1/2}$ , which is reported in Eq. (71). Finally, we obtain an ODE for  $\lambda$  by using the definition of  $\tau_P^{1/2}$  in Eq. (45) and by plugging Eq. (330) into it and pre-multiplying by  $\Phi'$ :

$$\frac{d}{dt}\lambda = -\tau_P^{1/2}\tau_m^{-1/2}\xi + \lambda\Sigma_{\mathbf{P}}\left(\Sigma_{\mathbf{P}}\frac{v\xi + \gamma}{\xi} + \tau_{P,t}^{1/2}\frac{v - \alpha_t\bar{v}_1}{\tau^c}\right), \quad (345)$$

which concludes the second step.

We have shown that the equilibrium computation now boils down to computing  $\lambda$ ,  $\xi$ ,  $\xi_2$  and  $\tau_P$ . However, the ODEs we obtained for them in Eqs. (345), (369), (331) and (71) are not explicit as they depend on the  $v$ . coefficients, which solve the ODEs in Eqs. (223)–(226), which themselves depend on the unknown coefficients,  $\Pi$ . Hence, in a third step, we must find these coefficients by verifying Conjecture 1, and more specifically part A. We start by proving Eq. (43), which can be done without knowledge of the  $\Pi$ . coefficients. Suppose that Eq. (41) in Conjecture 1 holds true and post-multiply it by  $\omega$  and use Eq. (219), which

gives:

$$\mathbf{A}_P \boldsymbol{\omega} = ((1 - \ell)(\Pi_1 + \Pi_2) + \ell\Pi_3) \boldsymbol{\Sigma}_P \boldsymbol{\Phi}. \quad (346)$$

Comparing the result with Eq. (340) gives Eq. (43). Next, we prove that Eq. (41) in Conjecture 1 holds true. From Eq. (318) and observing that

$$\mathbf{1}^* \boldsymbol{\Gamma}^{-1} = -\boldsymbol{\xi}^{-1} \mathbf{b}, \quad (347)$$

we know that:

$$\mathbf{A}_P = \mathbf{B}_P (\mathbf{B}'_{\Psi} \mathbf{V} - \gamma \mathbf{B}'_P \boldsymbol{\xi}^{-1} \mathbf{b}). \quad (348)$$

Using the relations in Eqs. (198) and (201), we can write:

$$\mathbf{B}_P \mathbf{B}'_{\Psi} \mathbf{V} = \boldsymbol{\Sigma}_P \left( \tau_P^{1/2} \boldsymbol{\Phi} \boldsymbol{\Phi}' \left( \frac{\alpha \bar{v}_1 - v}{\tau^c} \mathbf{b} + \frac{\alpha \bar{v}_2 - v_3}{\tau^c} \mathbf{c} \right) - \boldsymbol{\Sigma}_P (\mathbf{v} \mathbf{b} + v_3 \mathbf{c}) \right), \quad (349)$$

which plugged back into the previous equation gives:

$$\mathbf{A}_P = \boldsymbol{\Sigma}_P \left( \tau_P^{1/2} \boldsymbol{\Phi} \boldsymbol{\Phi}' \left( \frac{\alpha \bar{v}_1 - v}{\tau^c} \mathbf{b} + \frac{\alpha \bar{v}_2 - v_3}{\tau^c} \mathbf{c} \right) - \boldsymbol{\Sigma}_P ((\mathbf{v} + \gamma \boldsymbol{\xi}^{-1}) \mathbf{b} + v_3 \mathbf{c}) \right). \quad (350)$$

This expression verifies the conjectured form in Eq. (41) with the identification:

$$\Pi_1 + \Pi_2 = \tau_P^{1/2} \frac{\alpha \bar{v}_1 - v}{\tau^c} - \boldsymbol{\Sigma}_P \frac{v \boldsymbol{\xi} + \gamma}{\boldsymbol{\xi}} \quad (351)$$

$$\Pi_2 = -\tau_m^{-1/2} (\boldsymbol{\xi}_2 v_2 + \gamma) \quad (352)$$

$$\Pi_3 = \tau_P^{1/2} \frac{\alpha \bar{v}_2 - v_3}{\tau^c} - \boldsymbol{\Sigma}_P v_3, \quad (353)$$

where the first equation is reported in Eq. (69). To prove part B of Conjecture 1 we start from Eq. (317) and rewrite it as:

$$\dot{\boldsymbol{\xi}}_0 \mathbf{M} = \mathbf{B}_P ((-\mathbf{B}_P + (1 - \ell) \mathbf{X}_1 - \ell \mathbf{X}_2)' \mathbf{u} + u_3 \ell (\mathbf{X}_1 + \mathbf{X}_2) + \gamma \mathbf{B}'_P \boldsymbol{\xi}_0^{-1}) \boldsymbol{\xi}_0 \mathbf{M} \quad (354)$$

$$= \boldsymbol{\Sigma}_P \underbrace{\left( -\tau_m^{-1/2} \boldsymbol{\xi} \mathbf{u} + u_3 \frac{\tau_P^{1/2}}{\tau^c} \alpha \ell \boldsymbol{\Phi} \boldsymbol{\Phi}' + \gamma \boldsymbol{\Sigma}_P \boldsymbol{\xi}_0^{-1} \right)}_{\equiv -(\Pi_1 \boldsymbol{\Phi} \boldsymbol{\Phi}' + \Pi_2 \mathbf{I})} \boldsymbol{\xi}_0 \mathbf{M} \quad (355)$$

where the first equality uses Eqs. (198) and (201), the second equality substitutes the relevant matrices and simplifies, and the underbrace uses Eq. (63) and  $\boldsymbol{\xi}_0 \equiv \boldsymbol{\xi}$ ; Eq. (47)

follows.

In step 4 we substitute the expressions in Eq. (69) into the ODEs for  $v$  in Eqs. (223)–(226) to obtain explicit equations and in those for  $\xi$  and  $\lambda$  to obtain simpler ODEs. This first reveals that  $v_2$  and  $\xi_2$  can be solved for independently of the other coefficients; in particular they solve the two coupled ODEs with constant coefficients:

$$v'_2 = -\tau_m^{-1}(\gamma + \xi_2 v_2)^2, \quad v_{2,T-} = \tau_\epsilon^{-1} \quad (356)$$

$$\xi'_2 = \tau_m^{-1} \xi_2^2 (v_2 \xi_2 + \gamma), \quad \xi_{2,T-} = -\gamma/v_{2,T-}. \quad (357)$$

Clearly, one solution to this system of equation is:

$$v_{2,t} = \tau_\epsilon^{-1} \quad (358)$$

$$\xi_{2,t} = -\gamma \tau_\epsilon, \quad (359)$$

which implies that  $\Pi_2 \equiv 0$ . Since Eq. (71) gives us  $\tau_P$ , this brings the equilibrium construction down to the computation of  $\lambda$  and  $\xi$  and the associated  $v$  coefficients; the system of ODEs for these coefficients is autonomous in  $\lambda$ ,  $\xi$ ,  $v$  and  $\bar{v}_1$ : once we know  $v$  and  $\bar{v}_1$ , we can recover  $v_3$ , and since we know  $\bar{v}$  we can recover  $v_4$ . This gives a system of four ODEs, which using the expression we obtained for  $\Pi_1 + \Pi_2 \equiv \Pi$  in Eq. (69) is written as:

$$\lambda' = -\tau_P^{1/2} \tau_m^{-1/2} \xi - \lambda \Sigma_{\mathbf{P}} \Pi \quad (360)$$

$$v' = \Pi \left( 2 \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{v}_1 - v) - \Pi \right) + \frac{\tau_v}{\tau^2} \bar{v}_1^2 \quad (361)$$

$$= \tau_P \left( \frac{\alpha \bar{v}_1 - v}{\tau^c} \right)^2 - \Sigma_{\mathbf{P}}^2 \left( \frac{v \xi + \gamma}{\xi} \right)^2 + \frac{\tau_v}{\tau^2} \bar{v}_1^2 \quad (362)$$

$$\bar{v}'_1 = \frac{\tau_P}{\tau} \bar{v}_1 + \tau_P^{1/2} \Pi \left( 1 + \frac{\alpha \bar{v} - \bar{v}_1}{\tau^c} \right) + \frac{\tau_v}{\tau^2} \bar{v} \bar{v}_1, \quad (363)$$

and Eq. (68).

We now repeat step 1-4 on the equilibrium equations in Eqs. (321) and (322) that hold immediately prior to the announcement. We separate the first column in Eq. (322) from the other  $N$  columns, which gives the following system of equations:

$$0 = (\mathbf{a}_{\mathbf{P}} - \mathbf{b}_{\mathbf{P}} \mathbf{S} \mathbf{b}'_{\Psi} \mathbf{V}_t (\mathbf{I} + \mathbf{a}_{\Psi})) (\mathbf{1}^{*'} \lambda_{t-} \boldsymbol{\xi}_{t-}^{-1} \boldsymbol{\Phi} + \alpha_{t-} \boldsymbol{\omega}_{t-}) \quad (364)$$

$$\boldsymbol{\xi}_t - \boldsymbol{\xi}_{t-} = \mathbf{b}_{\mathbf{P}} \mathbf{S} (\mathbf{b}'_{\Psi} \mathbf{V}_t (\mathbf{I} + \mathbf{a}_{\Psi}) \mathbf{1}^{*'} + \gamma \mathbf{b}'_{\mathbf{P}}). \quad (365)$$

From the second equation we can verify Eq. (67) holds upon the announcement. Using Eqs.

(204) and (207) we rewrite this equation as:

$$\boldsymbol{\xi}_t - \boldsymbol{\xi}_{t-} = \mathbf{b}_P \mathbf{S} \left( \mathbf{b}'_P (\mathbf{v}_t \boldsymbol{\xi}_t + \gamma) - \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x}' \boldsymbol{\xi}_t \right). \quad (366)$$

Now substitute Eq. (67) and isolate terms in  $\mathbf{I}$ , which gives an ODE for  $\xi_2$

$$\xi_{2,t-} = \xi_{2,t} \frac{\tau_M - \gamma \xi_{2,t}}{\tau_M + v_{2,t} \xi_{2,t}^2} \quad (367)$$

$$= \xi_{2,t-}, \quad (368)$$

where the second equality substitutes the solution post-announcement in Eq. (358). The remaining terms are proportional to  $\boldsymbol{\Phi} \boldsymbol{\Phi}'$ . In particular, pre- and post-multiplying Eq. (366) by  $\boldsymbol{\Phi}'$  and  $\boldsymbol{\Phi}$ , respectively, gives an equation for  $\xi$ :

$$\xi_t - \xi_{t-} = \sigma_P (v_t \xi_t + \gamma) + \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \xi_t \boldsymbol{\Phi}' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'. \quad (369)$$

We further show that  $\boldsymbol{\xi} \equiv \boldsymbol{\xi}_0$  on the announcement. Start from the equilibrium condition in Eq. (321), which can be rewritten as:

$$\boldsymbol{\xi}_{0,t} - \boldsymbol{\xi}_{0,t-} = \mathbf{b}_P \mathbf{S} (\mathbf{b}'_\Psi (\mathbf{b}'_t \mathbf{u}_t + u_{3,t} \mathbf{c}'_t) \boldsymbol{\xi}_{0,t} + \gamma \mathbf{b}'_P) \quad (370)$$

$$= \mathbf{b}_P \mathbf{S} \left( \mathbf{b}_P (-\mathbf{u}_t \boldsymbol{\xi}_{0,t} + \gamma) + \frac{u_t - \alpha \bar{u}_t}{1 - \alpha_t} \mathbf{x}' \boldsymbol{\xi}_{0,t} \right). \quad (371)$$

Since we have shown in Eq. (63) that  $\mathbf{u} \equiv -\mathbf{v}$  and  $u_3 = -v_3$ , if  $\boldsymbol{\xi}_0 \equiv \boldsymbol{\xi}$  this equation is identical to Eq. (366) and thus the conjecture in Eq. (67) holds at all dates. We now turn to Eq. (364), and first observe that:

$$(\mathbf{a}_P - \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi)) \mathbf{1}^{x'} = \boldsymbol{\xi}_t - \boldsymbol{\xi}_{t-} - \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi) \mathbf{1}^{x'} \quad (372)$$

$$= \gamma \mathbf{b}_P \mathbf{S} \mathbf{b}'_P, \quad (373)$$

where the second line follows from Eq. (365), which we plug back into Eq. (364):

$$\lambda_{t-} \gamma \mathbf{b}_P \mathbf{S} \mathbf{b}'_P \boldsymbol{\xi}_{t-}^{-1} \boldsymbol{\Phi} + \alpha_{t-} (\mathbf{a}_P - \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi \mathbf{V}_t (\mathbf{I} + \mathbf{a}_\Psi)) \boldsymbol{\omega}_{t-} = 0. \quad (374)$$

Post-multiplying the definition of  $\mathbf{a}_P$  in Eq. (167) by  $\boldsymbol{\omega}_{t-}$  to obtain:

$$\mathbf{a}_P \boldsymbol{\omega}_{t-} = \mathbf{b}_P \mathbf{y}, \quad (375)$$

which is the discrete counterpart to Eq. (340). Substituting back we obtain:

$$\lambda_{t-}\gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\Phi + \alpha_{t-}(\mathbf{b}_P\mathbf{y} - \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi\mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi))\omega_{t-} = 0. \quad (376)$$

Using Eqs. (204) and (207) we can write:

$$\mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi)\omega_{t-} = ((\mathbf{b}'_t\mathbf{v}_t + v_{3,t}\mathbf{c}'_t)(\mathbf{b}_{t-} - \mathbf{a}_P) + (\tilde{v}_{1,t}\mathbf{b}'_t + \tilde{v}_{2,t}\mathbf{c}'_t)\mathbf{c}_{t-})\omega_{t-} \quad (377)$$

$$= -(\mathbf{b}'_t\mathbf{v}_t + v_{3,t}\mathbf{c}'_t)\mathbf{b}_P\mathbf{y} + ((\beta v_t + (1 - \beta)\bar{v}_{1,t})\mathbf{b}'_t + (\beta v_{3,t} + (1 - \beta)\bar{v}_{2,t})\mathbf{c}'_t)\Phi, \quad (378)$$

which we plug back into the previous equation to obtain

$$\lambda_{t-}\gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\Phi + \alpha_{t-}\mathbf{b}_P\mathbf{S} \begin{pmatrix} \mathbf{S}^{-1}\mathbf{y} + \mathbf{b}'_\Psi(\mathbf{b}'_t\mathbf{v}_t + v_{3,t}\mathbf{c}'_t)\mathbf{b}_P\mathbf{y} \\ -\mathbf{b}'_\Psi((\beta v_t + (1 - \beta)\bar{v}_{1,t})\mathbf{b}'_t + (\beta v_{3,t} + (1 - \beta)\bar{v}_{2,t})\mathbf{c}'_t)\Phi \end{pmatrix} = 0. \quad (379)$$

Using Eq. (246) this equation simplifies to:

$$\lambda_{t-}\gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\Phi + \alpha_{t-}\mathbf{b}_P\mathbf{S} \begin{pmatrix} \Upsilon^{-1}\mathbf{y} + \mathbf{b}'_\Psi \left( \mathbf{b}'_t \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} + \mathbf{c}'_t \frac{v_{3,t} - \alpha_t \bar{v}_{2,t}}{1 - \alpha_t} \right) \mathbf{x}'\mathbf{y} \\ -\mathbf{b}'_\Psi((\beta v_t + (1 - \beta)\bar{v}_{1,t})\mathbf{b}'_t + (\beta v_{3,t} + (1 - \beta)\bar{v}_{2,t})\mathbf{c}'_t)\Phi \end{pmatrix} = 0. \quad (380)$$

Furthermore, substituting Eqs. (248)–(250) back the equilibrium equation becomes:

$$\lambda_{t-}\gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\Phi + \alpha_{t-}\mathbf{b}_P\mathbf{S}(\tau_{t-}\mathbf{x}' - (\alpha_t\beta + 1 - \beta)\mathbf{b}'_\Psi(\bar{v}_{1,t}\mathbf{b}'_t + \bar{v}_{2,t}\mathbf{c}'_t))\Phi = 0. \quad (381)$$

Given Eq. (256) and that

$$\mathbf{b}'_\Psi(\bar{v}_{1,t}\mathbf{b}'_t + \bar{v}_{2,t}\mathbf{c}'_t) = \frac{\bar{v}_{1,t} - \alpha_t\bar{v}_t}{1 - \alpha_t}\mathbf{x}' - \bar{v}_{1,t}\mathbf{b}'_P, \quad (382)$$

this ultimately gives after dividing by  $\alpha_{t-}$ :

$$\mathbf{b}_P\mathbf{S}\mathbf{b}'_P((1 - \ell_{t-})\gamma\xi_{t-}^{-1} + \tau_{t-}/\tau_t\bar{v}_{1,t})\Phi - \tau_{t-} \left( \frac{\bar{v}_{1,t} - \alpha_t\bar{v}_t}{\tau_t^c} - 1 \right) \mathbf{b}_P\mathbf{S}\mathbf{x}'\Phi = 0. \quad (383)$$

After pre-multiplying this equation by  $\Phi'$  we obtain an algebraic equation for  $\tau_G^{1/2}$ , which is reported in Eq. (73). Finally, we obtain an equation for  $\ell$  by using the definition of  $\tau_G^{1/2}$  in

Eq. (46) and by plugging Eq. (366) into it and pre-multiplying by  $\Phi'$ :

$$(1 - \ell_{t-})\Phi = \tau_G^{1/2}\tau_M^{-1/2}\xi_{t-}\Phi/\alpha_{t-} + (1 - \ell_t) \left( 1 - \mathbf{b}_P\mathbf{S} \left( \mathbf{b}'_P(\mathbf{v}_t + \gamma\xi_t^{-1}) - \frac{v_t - \alpha_t\bar{v}_{1,t}}{1 - \alpha_t}\mathbf{x}' \right) \right) \Phi, \quad (384)$$

which concludes the second step.

In a third step, we must verify Conjecture 1, part A. We start by proving Eq. (44), which can be done without knowledge of the  $\pi$ . coefficients. Suppose that Eq. (42) in Conjecture 1 holds true and post-multiply it by  $\omega_{t-}$  and use Eq. (219), which gives:

$$\mathbf{a}_P\omega_{t-} = \mathbf{b}_P \begin{pmatrix} (1 - \ell_{t-})\pi_1^A + \ell_{t-}\pi_2^A \\ ((1 - \ell_{t-})(\pi_1 + \pi_2) + \ell_{t-}\pi_3)\Phi \end{pmatrix}. \quad (385)$$

Comparing the result with Eq. (340) gives Eq. (44). Next, we prove that Eq. (42) in Conjecture 1 holds true. We start from the market-clearing condition in Eq. (322), in which we substitute Eq. (347):

$$\mathbf{a}_P - \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi\mathbf{V}_t(\mathbf{I} + \mathbf{a}_\Psi) = -\gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\mathbf{b}_{t-}. \quad (386)$$

Now substitute Eq. (195) in this equation, rearrange and get:

$$\begin{aligned} \mathbf{b}_t(\mathbf{I} + \mathbf{a}_\Psi) &= (\mathbf{I} + \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi(\mathbf{v}_t\mathbf{b}_t + v_{3,t}\mathbf{c}_t)')^{-1} \begin{pmatrix} \mathbf{a}_{t-} + \gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}\mathbf{b}_{t-} \\ -(\mathbf{I} + \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi(v_{3,t}\mathbf{b}_t + v_{4,t}\mathbf{c}_t)')\mathbf{c}_t(\mathbf{I} + \mathbf{a}_\Psi) \end{pmatrix} \\ &= (\mathbf{I} + \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi(\mathbf{v}_t\mathbf{b}_t + v_{3,t}\mathbf{c}_t)')^{-1} \begin{pmatrix} (\mathbf{I} - \frac{\ell_t}{\ell_{t-}}\frac{\tau_{t-}^c}{\tau_t^c}(\mathbf{I} + \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi(v_{3,t}\mathbf{b}_t + v_{4,t}\mathbf{c}_t)'))\mathbf{c}_{t-} \\ +(\mathbf{I} + \gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1})\mathbf{b}_{t-} \end{pmatrix}, \end{aligned} \quad (387)$$

$$(388)$$

where the second line uses Eq. (38),  $\mathbf{c}_{t-} = \ell_t/\ell_{t-}\mathbf{c}_t$  and  $\mathbf{c}_t(\mathbf{I} + \mathbf{a}_\Psi) = \tau_{t-}^c/\tau_t^c\mathbf{c}_t$ . Furthermore, reorganizing Eq. (366) implies:

$$(\mathbf{I} + \mathbf{b}_P\mathbf{S}\mathbf{b}'_\Psi(\mathbf{v}_t\mathbf{b}_t + v_{3,t}\mathbf{c}_t)')\xi_t\xi_{t-}^{-1} = \mathbf{I} + \gamma\mathbf{b}_P\mathbf{S}\mathbf{b}'_P\xi_{t-}^{-1}, \quad (389)$$

which substituted in the above gives:

$$\begin{aligned} \mathbf{b}_t(\mathbf{I} + \mathbf{a}_\Psi) &= \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} \mathbf{b}_{t-} \\ &+ (\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)')^{-1} \left( \mathbf{I} - \frac{\ell_t}{\ell_{t-}} \frac{\tau_{t-}^c}{\tau_t^c} (\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (v_{3,t} \mathbf{b}_t + v_{4,t} \mathbf{c}_t)') \right) \mathbf{c}_{t-}. \end{aligned} \quad (390)$$

Proceeding similarly with Eq. (364) we can reorganize it as:

$$\begin{aligned} &(\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)') (\mathbf{b}_P \mathbf{y} + (1 - \ell_{t-}) \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} \Phi - \Phi) \\ &= -\ell_{t-} \left( \mathbf{I} - \frac{\ell_t}{\ell_{t-}} \frac{\tau_{t-}^c}{\tau_t^c} (\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (v_{3,t} \mathbf{b}_t + v_{4,t} \mathbf{c}_t)') + \frac{\ell_t}{\ell_{t-}} \frac{\tau_{t-}^c}{\tau_t^c} (\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)') \right) \Phi, \end{aligned} \quad (391)$$

and thus:

$$(\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (\mathbf{v}_t \mathbf{b}_t + v_{3,t} \mathbf{c}_t)')^{-1} \left( \mathbf{I} - \frac{\ell_t}{\ell_{t-}} \frac{\tau_{t-}^c}{\tau_t^c} (\mathbf{I} + \mathbf{b}_P \mathbf{S} \mathbf{b}'_\Psi (v_{3,t} \mathbf{b}_t + v_{4,t} \mathbf{c}_t)') \right) \Phi \quad (392)$$

$$\begin{aligned} &= \frac{1}{\ell_{t-}} \left( \left( 1 - \ell_t \frac{\tau_{t-}^c}{\tau_t^c} \right) \Phi - (1 - \ell_{t-}) \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} \Phi - \mathbf{b}_P \mathbf{y} \right) \\ &= \frac{1}{\ell_{t-}} \left( (1 - \ell_t) \frac{\tau_{t-}}{\tau_t} \Phi - (1 - \ell_{t-}) \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} \Phi - \mathbf{1}^* \mathbf{y} \right), \end{aligned} \quad (393)$$

where the second equality uses Eq. (204). Since  $\mathbf{c} \equiv \Phi \Phi' \mathbf{c}$ , substituting back into Eq. (390) gives:

$$\mathbf{b}_t(\mathbf{I} + \mathbf{a}_\Psi) = \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} \mathbf{b}_{t-} + \frac{1}{\ell_{t-}} \left( (1 - \ell_t) \frac{\tau_{t-}}{\tau_t} - (1 - \ell_{t-}) \boldsymbol{\xi}_t \boldsymbol{\xi}_{t-}^{-1} + \tau_{G,t}^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_t \right) \mathbf{c}_{t-}, \quad (394)$$

which uses the definition of  $\mathbf{y}$ . Finally, using the definition of  $\tau_G$  and the fact that  $\tau_t - \tau_{t-} = \tau_t^c - \tau_{t-}^c$  shows that:

$$-\tau_M^{-1/2} \tau_{G,t}^{1/2} \Phi = \alpha_{t-} \left( (1 - \ell_t) \frac{\tau_{t-}}{\tau_t} \boldsymbol{\xi}_t^{-1} - (1 - \ell_{t-}) \boldsymbol{\xi}_{t-}^{-1} \right) \Phi, \quad (395)$$

from which it follows that:

$$\mathbf{b}_t(\mathbf{I} + \mathbf{a}_\Psi) = \boldsymbol{\xi}_t \left( \boldsymbol{\xi}_{t-}^{-1} \mathbf{b}_{t-} - \frac{1 - \alpha_{t-}}{\alpha_{t-} \ell_{t-}} \tau_{G,t}^{1/2} \tau_M^{-1/2} \mathbf{c}_{t-} \right). \quad (396)$$

Now take Eq. (322) and substitute Eq. (347) and Eq. (396) in it, and rearrange to get:

$$\begin{aligned} \mathbf{a}_P &= \mathbf{b}_P \mathbf{S} \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x}' \boldsymbol{\xi}_t - \mathbf{b}'_P (\gamma + \mathbf{v}_t \boldsymbol{\xi}_t) \right) \boldsymbol{\xi}_{t-}^{-1} \mathbf{b}_{t-} \\ &+ \frac{1}{\ell_{t-}} \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} \mathbf{x}' \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \frac{\alpha_{t-} - 1}{\alpha_{t-}} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_t + \ell_t (1 - \beta) \frac{v_{3,t} - \alpha_t \bar{v}_{2,t}}{1 - \alpha_t} \right) \\ - \mathbf{b}'_P \left( v_t \frac{\alpha_{t-} - 1}{\alpha_{t-}} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_t + \ell_t (1 - \beta) v_{3,t} \right) \end{array} \right) \mathbf{c}_{t-}. \end{aligned} \quad (397)$$

This expression verifies the conjectured form in Eq. (42) with the identification:

$$\mathbf{b}_P \boldsymbol{\pi}_b = \mathbf{b}_P \mathbf{S} \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x}' \boldsymbol{\xi}_t - \mathbf{b}'_P (\gamma + \mathbf{v}_t \boldsymbol{\xi}_t) \right) \boldsymbol{\xi}_{t-}^{-1} \quad (398)$$

$$\pi_2 = - \frac{\xi_{2,t} v_{2,t} + \gamma}{v_{2,t} + \tau_M / \xi_{2,t}} \xi_{2,t-}^{-1} \quad (399)$$

$$\mathbf{b}_P \boldsymbol{\pi}_c = \frac{1}{\ell_{t-}} \mathbf{b}_P \mathbf{S} \left( \begin{array}{c} \mathbf{x}' \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \frac{\alpha_{t-} - 1}{\alpha_{t-}} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_t + \ell_t (1 - \beta) \frac{v_{3,t} - \alpha_t \bar{v}_{2,t}}{1 - \alpha_t} \right) \\ - \mathbf{b}'_P \left( v_t \frac{\alpha_{t-} - 1}{\alpha_{t-}} \tau_G^{1/2} \tau_M^{-1/2} \boldsymbol{\xi}_t + \ell_t (1 - \beta) v_{3,t} \right) \end{array} \right) \mathbf{c}_{t-}, \quad (400)$$

where the first equation is reported in Eq. (74) (after pre- and post-multiplication by  $\Phi$ ). Finally, to prove part B of Conjecture 1 we start from Eq. (??) and rewrite it as:

$$\boldsymbol{\xi}_{0,t} - \boldsymbol{\xi}_{0,t-} = -\mathbf{b}_P \mathbf{S} \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \mathbf{x}' \boldsymbol{\xi}_t - \mathbf{b}'_P (\gamma + \mathbf{v}_t \boldsymbol{\xi}_t) \right) \quad (401)$$

$$= -\mathbf{b}_P \boldsymbol{\pi}_b \boldsymbol{\xi}_{t-}, \quad (402)$$

where the second equality follows from comparing this equation to Eqs. (397) and (42); Eq. (48) follows.

In a last step we substitute the expressions in Eq. (74) into the equations for  $v$  in Eqs. (58)–(60) to obtain explicit equations and in those for  $\xi$  and  $\lambda$  to obtain simpler equations. As in the continuous case, this first reveals that  $v_2$  and  $\xi_2$  can be solved for independently of the other coefficients; in particular substituting the solution post-announcement in Eq. (358) into Eq. (61) gives:

$$v_{2,t-} = v_{2,t}, \quad (403)$$

which given Eq. (367) shows that the solution post-announcement continues to hold pre-announcement and which implies from Eq. (399) that  $\pi_2 \equiv 0$ . Further substituting the expression we obtained for  $\mathbf{b}_P \boldsymbol{\pi}_b$  in Eq. (74) readily delivers Eq. (72).

Since Eq. (73) gives us  $\tau_G$ , this brings the equilibrium construction down to the computation of  $\lambda$  and  $\xi$  and the associated  $v$  coefficients; the system of ODEs for these coefficients



is autonomous in  $\lambda$ ,  $\xi$ ,  $v$  and  $\bar{v}_1$ : once we know  $v$  and  $\bar{v}_1$ , we can recover  $v_3$ , and since we know  $\bar{v}$  we can recover  $v_4$ . This gives a system of four ODEs, which using the expression we obtained for  $\Pi_1 + \Pi_2 \equiv \Pi$  in Eq. (69) is written as:

$$\lambda' = -\tau_P^{1/2} \tau_m^{-1/2} \xi - \lambda \Sigma_{\mathbf{P}} \Pi \quad (404)$$

$$v' = \Pi \left( 2 \frac{\tau_P^{1/2}}{\tau^c} (\alpha \bar{v}_1 - v) - \Pi \right) + \frac{\tau_v}{\tau^2} \bar{v}_1^2 \quad (405)$$

$$= \tau_P \left( \frac{\alpha \bar{v}_1 - v}{\tau^c} \right)^2 - \Sigma_{\mathbf{P}}^2 \left( \frac{v \xi + \gamma}{\xi} \right)^2 + \frac{\tau_v}{\tau^2} \bar{v}_1^2 \quad (406)$$

$$\bar{v}_1' = \frac{\tau_P}{\tau} \bar{v}_1 + \tau_P^{1/2} \Pi \left( 1 + \frac{\alpha \bar{v} - \bar{v}_1}{\tau^c} \right) + \frac{\tau_v}{\tau^2} \bar{v} \bar{v}_1, \quad (407)$$

and Eq. (68).

## A.5 Proof of Proposition 5

In this appendix we formulate the risk premia of Conjecture 1 in terms of betas for all times strictly preceding the liquidation date. Consider first all non-announcement dates  $t \in (0, \tau) \cup (\tau, T)$ . In this case, because information flows continuously investors' and empiricist's betas must agree, and equal:

$$\beta_t \equiv \hat{\beta}_t = \frac{1}{\sigma_{\mathbf{M},t}^2} d\langle \tilde{\mathbf{P}} \rangle \mathbf{M} = \frac{1}{\sigma_{\mathbf{M},t}^2} \Sigma_{\mathbf{P}} \Sigma_{\mathbf{P}} \mathbf{M} \quad (408)$$

$$= \frac{1}{\sigma_{\mathbf{M},t}^2} \left( (\sigma_{\mathbf{P}}^2 - \tau_m^{-1} \xi_{2,t}^2) \bar{\Phi} \Phi + \tau_m^{-1} \xi_{2,t}^2 / N \mathbf{1} \right), \quad (409)$$

where the instantaneous variance on market returns on non-announcement dates satisfies:

$$\sigma_{\mathbf{M},t}^2 dt = \mathbf{M}' d\langle \tilde{\mathbf{P}} \rangle \mathbf{M} = \mathbf{M}' \Sigma_{\mathbf{P}} \Sigma_{\mathbf{P}} \mathbf{M} dt \quad (410)$$

$$= \left( (\sigma_{\mathbf{P}}^2 - \tau_m^{-1} \xi_{2,t}^2) \bar{\Phi}^2 + \tau_m^{-1} \xi_{2,t}^2 / N \right) dt. \quad (411)$$

Due to hedging demands, the CAPM fails. Specifically, conditioning down the asset-pricing relation in Conjecture 1 gives:

$$\mathbb{E}[d\tilde{\mathbf{P}}_t] = \dot{\xi}_0 \mathbf{M} dt = -\Sigma_{\mathbf{P}} (\Pi_{1,t} \Phi \Phi' + \Pi_{2,t} \mathbf{I}) \xi_{0,t} \mathbf{M} dt \quad (412)$$

$$= -\bar{\Phi} \sigma_{\mathbf{P}} \Pi_t \xi_t \Phi dt, \quad (413)$$

where the second equality uses Eq. (??). Importantly, unlike the CAPM that holds at the horizon and whereby the cross section of expected returns is spanned by the two vectors  $\Phi$

and  $\mathbf{1}$ , the asset-pricing relation that holds at all other dates implies that this cross section is spanned by the single vector  $\Phi$ . As in the benchmark model of Section 2, on all other dates prior to  $T$  asset loadings  $\Phi$  entirely determine cross-sectional heterogeneity in returns. Betas, however, are spanned by both  $\Phi$  and  $\mathbf{1}$ , which implies that when substituting betas into expected returns, the relation exhibits an intercept, i.e., the CAPM fails:

$$\mathbb{E}[d\tilde{\mathbf{P}}_t] = -\frac{\sigma_{\mathbf{P}}\Pi_t\xi_t}{\sigma_{\mathbf{P}}^2 - \tau_m^{-1}\xi_{2,t}^2} (\sigma_{\mathbf{M}}^2\boldsymbol{\beta}_t - \tau_m^{-1}\xi_{2,t}^2/N \mathbf{1}) dt. \quad (414)$$

Applying this relation to the market portfolio shows that:

$$\mathbb{E}[\mathbf{M}'d\tilde{\mathbf{P}}_t] = -\sigma_{\mathbf{P}}\Pi_t\xi_t\bar{\Phi}^2 dt, \quad (415)$$

which substituted back gives:

$$\mathbb{E}[d\tilde{\mathbf{P}}_t] = \mathbb{E}[\mathbf{M}'d\tilde{\mathbf{P}}_t] \left( \frac{\sigma_{\mathbf{M}}^2}{\sigma_{\mathbf{M}}^2 - \tau_m^{-1}\gamma^2\tau_\epsilon^2/N} \boldsymbol{\beta}_t - \frac{\tau_m^{-1}\xi_{2,t}^2/N}{\sigma_{\mathbf{M}}^2 - \tau_m^{-1}\gamma^2\tau_\epsilon^2/N} \mathbf{1} \right) \quad (416)$$

$$\equiv (s_t\boldsymbol{\beta}_t + \alpha_t \mathbf{1})dt, \quad (417)$$

where the first equality uses the definition of market volatility and Eq. (??), and in the second equality  $s_t$  denotes the slope and  $\alpha_t$  the intercept of second-pass regressions at date  $t$ . In contrast to the CAPM at the horizon date, the CAPM here fails for everyone.

On announcement dates a wedge between investors' and empiricist's betas reappears. Specifically, the jump in expected returns caused by the announcement reactivate the second term in the law of total variance of Eq. (107). For investors, immediately prior to the announcement, the covariance matrix of returns satisfies:

$$\mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_{t-}^i] = \mathbf{b}_{\mathbf{P}}\Upsilon\mathbf{b}_{\mathbf{P}}' \quad (418)$$

$$= \mathbf{1}^* \Upsilon \mathbf{1}^{*'} + (1 - \lambda_t) \frac{\tau_t}{\tau_t^c} (\mathbf{x}\Upsilon \mathbf{1}^{*'} + \mathbf{1}^* \Upsilon \mathbf{x}') + (1 - \lambda_t)^2 \left( \frac{\tau_t}{\tau_t^c} \right)^2 \mathbf{x}\Upsilon \mathbf{x}'. \quad (419)$$

Using that:

$$\mathbf{x}\Upsilon \mathbf{x}' = (\tau_t^{-1} - \tau_t^{-1})\Phi\Phi' \quad (420)$$

$$\mathbf{1}^* \Upsilon \mathbf{x}' = -\tau_t^{-1}\tau_G^{1/2}\tau_M^{-1/2}\xi_t\Phi\Phi' \quad (421)$$

$$\mathbf{1}^* \Upsilon \mathbf{1}^{*'} = \tau_M^{-1}\xi_t(\mathbf{I} + \tau_t^{-1}\tau_G\Phi\Phi')\xi_t, \quad (422)$$

we can rewrite this expression as:

$$\mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_{t-}^i] = \tau_M^{-1}\boldsymbol{\xi}_t\boldsymbol{\xi}_t + \tau_{t-}^{-1} \left( \left( \tau_G^{1/2}\tau_M^{-1/2}\boldsymbol{\xi}_t - \frac{1-\lambda_t}{1-\alpha_t} \right)^2 - \left( \frac{1-\lambda_t}{1-\alpha_t} \right)^2 \frac{\tau_{t-}}{\tau_t} \right) \boldsymbol{\Phi}\boldsymbol{\Phi}'. \quad (423)$$

Turning to the econometrician, in her eyes the noise:

$$\tilde{\mathbf{y}}_t^c \equiv \mathbf{y}_t - \mathbf{h}_t\hat{\mathbf{z}}_{t-}^c \quad (424)$$

in signals and in trading caused by the announcement has covariance matrix:

$$\boldsymbol{\Upsilon}^c \equiv \begin{pmatrix} (\tau_t^c)^{-1} + \tau_A^{-1} & -(\tau_{t-}^c)^{-1}\tau_G^{1/2}\tau_M^{-1/2}\boldsymbol{\Phi}'\boldsymbol{\xi}_t \\ -(\tau_{t-}^c)^{-1}\tau_G^{1/2}\tau_M^{-1/2}\boldsymbol{\xi}_t\boldsymbol{\Phi} & \tau_M^{-1}\boldsymbol{\xi}_t(\mathbf{I} + (\tau_{t-}^c)^{-1}\tau_G\boldsymbol{\Phi}\boldsymbol{\Phi}')\boldsymbol{\xi}_t \end{pmatrix}. \quad (425)$$

Hence, the empiricist perceives the covariance matrix of returns as:

$$\mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_{t-}^c] = \tau_M^{-1}\boldsymbol{\xi}_t\boldsymbol{\xi}_t + (\tau_{t-}^c)^{-1} \left( \left( \tau_G\tau_M^{-1/2}\boldsymbol{\xi}_t - (1-\lambda_t) \right)^2 - (1-\lambda_t)^2 \frac{\tau_{t-}^c}{\tau_t^c} \right) \boldsymbol{\Phi}\boldsymbol{\Phi}'. \quad (426)$$

These two covariance matrices are related through the law of total variance as:

$$\begin{aligned} \mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_{t-}^c] &= \mathbb{V}[\Delta\tilde{\mathbf{P}}_t|\mathcal{F}_{t-}^i] \\ &+ \left( \frac{\left( \tau_M^{-1/2}\tau_G^{1/2}\boldsymbol{\xi}_t - (1-\lambda_t) \right)^2}{\tau_{t-}^c} - \frac{\left( \tau_M^{-1/2}\tau_G^{1/2}\boldsymbol{\xi}_t - \frac{1-\lambda_t}{1-\alpha_t} \right)^2}{\tau_{t-}} + \frac{(1-\lambda_t)^2\alpha_t}{(1-\alpha_t)\tau_t^c} \right) \boldsymbol{\Phi}\boldsymbol{\Phi}', \end{aligned} \quad (427)$$

or equivalently in terms of betas:

$$\hat{\boldsymbol{\beta}}_{t-} = \frac{\sigma_{\mathbf{M},t-}^2}{\hat{\sigma}_{\mathbf{M},t-}^2}\boldsymbol{\beta}_{t-} + \frac{\bar{\Phi}}{\hat{\sigma}_{\mathbf{M},t-}^2} \left( \frac{\left( \tau_M^{-1/2}\tau_G^{1/2}\boldsymbol{\xi}_t - (1-\lambda_t) \right)^2}{\tau_{t-}^c} - \frac{\left( \tau_M^{-1/2}\tau_G^{1/2}\boldsymbol{\xi}_t - \frac{1-\lambda_t}{1-\alpha_t} \right)^2}{\tau_{t-}} + \frac{(1-\lambda_t)^2\alpha_t}{(1-\alpha_t)\tau_t^c} \right) \boldsymbol{\Phi}, \quad (428)$$

where the variance of market returns as perceived by investors and the empiricist satisfy:

$$\sigma_{\mathbf{M},-}^2 = \left( \frac{1}{\tau_{t-}} \left( \tau_G^{1/2}\tau_M^{-1/2}\boldsymbol{\xi}_t - \frac{1-\lambda_t}{1-\alpha_t} \right)^2 - \left( \frac{1-\lambda_t}{1-\alpha_t} \right)^2 \frac{1}{\tau_t} + \tau_M^{-1}(\boldsymbol{\xi}_t^2 - \boldsymbol{\xi}_{2,t}^2) \right) \bar{\Phi}^2 + \frac{\boldsymbol{\xi}_{2,t}^2}{N}\tau_M^{-1} \quad (429)$$

and

$$\hat{\sigma}_{\mathbf{M},-}^2 = \left( \frac{1}{\tau_{t-}^c} \left( \tau_G^{1/2} \tau_M^{-1/2} \xi_t - (1 - \lambda_t) \right)^2 - (1 - \lambda_t)^2 \frac{1}{\tau_t^c} + \tau_M^{-1} (\xi_t^2 - \xi_{2,t}^2) \right) \bar{\Phi}^2 + \frac{\xi_{2,t}^2}{N} \tau_M^{-1}. \quad (430)$$

Now compute investors' betas based on their covariance matrix:

$$\boldsymbol{\beta}_{t-} = \frac{\xi_{2,t}^2}{\tau_M N \sigma_{\mathbf{M}}^2} \mathbf{1} + \left( \tau_M^{-1} (\xi_t^2 - \xi_{2,t}^2) + \left( \frac{1}{\tau_{t-}} \left( \tau_G^{1/2} \tau_M^{-1/2} \xi_t - \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 - \left( \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 \frac{1}{\tau_t} \right) \right) \frac{\bar{\Phi}}{\sigma_{\mathbf{M}}^2} \bar{\Phi}, \quad (431)$$

which substituted back in the relation between the two betas allows one to express their relation in terms of the vector  $\mathbf{1}$ , as opposed to  $\bar{\Phi}$ :

$$\hat{\boldsymbol{\beta}}_{t-} - \mathbf{1} = \frac{\sigma_{\mathbf{M}}^2}{\hat{\sigma}_{\mathbf{M}}^2} \left( 1 + \frac{\left( \frac{\tau_M^{-1/2} \tau_G^{1/2} \xi_t - (1 - \lambda_t)}{\tau_{t-}^c} \right)^2 - \left( \frac{\tau_M^{-1/2} \tau_G^{1/2} \xi_t - \frac{1 - \lambda_t}{1 - \alpha_t}}{\tau_{t-}} \right)^2 + \frac{(1 - \lambda_t)^2 \alpha_t}{(1 - \alpha_t) \tau_t^c}}{\frac{1}{\tau_{t-}} \left( \tau_G \tau_M^{-1/2} \xi_t - \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 - \left( \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 \frac{1}{\tau_t} + \tau_M^{-1} (\xi_t^2 - \xi_{2,t}^2)} \right) (\boldsymbol{\beta}_{t-} - \mathbf{1}). \quad (432)$$

Now referring to the form of unconditional premia in Lemma ?? and substituting in it the relevant matrix we can write:

$$\mathbb{E} \left[ \tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-} \right] = -\bar{\Phi} \xi_{t-} \left( (1 - \lambda_t) \frac{\tau_A}{\tau_t^c} \pi_1^A + \left( 1 - (1 - \lambda_t) \tau_M^{1/2} \tau_G^{1/2} / \tau_t^c \xi_t^{-1} \right) \pi \right) \bar{\Phi}. \quad (433)$$

Plugging the expression for investors' betas in it gives the beta representation from investors' perspective:

$$\mathbb{E} \left[ \tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-} \right] = \frac{-\xi_{t-} \left( (1 - \lambda_t) \frac{\tau_A}{\tau_t^c} \pi_1^A + \pi \left( 1 - (1 - \lambda_t) \tau_M^{1/2} \tau_G^{1/2} / \tau_t^c \xi_t^{-1} \right) \right)}{\tau_M^{-1} (\xi_t^2 - \xi_{2,t}^2) + \tau_{t-}^{-1} \left( \tau_G^{1/2} \tau_M^{-1/2} \xi_t - \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 - \left( \frac{1 - \lambda_t}{1 - \alpha_t} \right)^2 \frac{1}{\tau_t}} \left( \sigma_{\mathbf{M}}^2 \boldsymbol{\beta}_{t-} - \frac{\xi_{2,t}^2}{\tau_M N} \mathbf{1} \right). \quad (434)$$

Applying this relation to the market portfolio then shows that:

$$\mathbb{E} \left[ \mathbf{M}'(\tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-}) \right] = -\xi_{t-} \left( (1 - \lambda_t) \frac{\tau_A}{\tau_t^c} \pi_1^A + \pi \left( 1 - (1 - \lambda_t) \tau_M^{1/2} \tau_G^{1/2} / \tau_t^c \xi_t^{-1} \right) \right) \bar{\Phi}, \quad (435)$$

which once plugged back gives:

$$\mathbb{E} \left[ \tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-} \right] = \mathbb{E} \left[ \mathbf{M}'(\tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-}) \right] \left( \frac{\sigma_{\mathbf{M}}^2}{\sigma_{\mathbf{M}}^2 - \gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N} \boldsymbol{\beta}_{t-} - \frac{\gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N}{\sigma_{\mathbf{M}}^2 - \gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N} \mathbf{1} \right), \quad (436)$$

which is identical to Eq. (416). We conclude that for  $N$  taken to be large (a large cross section of stocks), the CAPM will be close to hold exactly. Investors and the econometrician will agree on this approximate CAPM on all non-announcement days. Yet, they will disagree about this relation on all announcement days, we now show. Substituting the vector of empiricist's betas the asset-pricing relation she perceives satisfies:

$$\mathbb{E} \left[ \tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-} \right] = \mathbb{E} \left[ \mathbf{M}'(\tilde{\mathbf{P}}_t - \tilde{\mathbf{P}}_{t-}) \right] \left( \frac{\hat{\sigma}_{\mathbf{M}}^2}{\hat{\sigma}_{\mathbf{M}}^2 - \gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N} \hat{\boldsymbol{\beta}}_{t-} - \frac{\gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N}{\hat{\sigma}_{\mathbf{M}}^2 - \gamma^2 \tau_{\epsilon}^2 \tau_M^{-1} / N} \mathbf{1} \right). \quad (437)$$

## A.6 Proof of Lemma 1

In this appendix we show that an equilibrium without a wedge  $\ell \equiv 0$  always obtains post-announcement under Assumption 2; this equilibrium is the one that obtains absent residual uncertainty. We start by noting that the solution in Eqs. (92)–(93) satisfies the terminal boundary conditions in Eqs. (76) and (77). Furthermore, suppose that this solution holds at all dates post-announcement, which imply in particular that:

$$v_t \equiv -\gamma / \xi_t. \quad (438)$$

Eq. (438) in turn implies that the two ODEs for  $v$  and  $\bar{v}_1$  (see Proposition 3) decouple from those for  $\ell$  and  $\xi$  (see Proposition 4). This corresponds to an important simplification as  $\bar{v} = \bar{v}_1 \equiv v$  (as given in Eq. (??)) and thus the coefficients in investors' value function:

$$\mathbf{U} \equiv \mathbf{a}' \mathbf{u} \xi_0 \mathbf{M} \quad (439)$$

$$\mathbf{V} \equiv \mathbf{a}' \mathbf{v} \mathbf{a}, \quad (440)$$

and equilibrium equity premia (given that  $\Pi_3 \equiv 0$ ):

$$\mathbf{A}_{\mathbf{P}} = \sigma_{\mathbf{P}} \Pi \Phi \Phi' \mathbf{a} \quad (441)$$

with  $\Pi \equiv -\tau_P^{1/2}$  (by Eq. (43)) can be described just in terms of  $\mathbf{a}$ , as opposed to  $\mathbf{b}$  and  $\mathbf{c}$  separately. Since  $v$  is known in this case, the system of ODEs in Eq. (??) simplifies to a

system of just two ODEs for  $\lambda$  and  $\xi$ :

$$\lambda' = -\tau_P^{1/2} \tau_m^{-1/2} \xi + \lambda \tau_P^{1/2} \sigma_{\mathbf{P}} \quad (442)$$

$$\xi' = \sigma_{\mathbf{P}} \tau_P^{1/2} \xi, \quad (443)$$

where  $\sigma_{\mathbf{P}} \equiv \tau_m^{-1/2} \xi - \tau_P^{1/2} / \tau$  given that, under Eq. (92),  $(\tau^c)^{-1}(1 - \lambda) \equiv \tau^{-1}$ . To verify the conjecture in Eq. (92) we observe that it implies:

$$\lambda' = -\tau^{-1}(\tau_P \lambda - (1 - \lambda)\tau_v), \quad (444)$$

which we can plug in the first ODE above to get an algebraic equation of the form:

$$\tau_P^{1/2} = -\tau_v \tau^{-1} \tau_m^{1/2} \xi^{-1}. \quad (445)$$

For the conjecture to be verified this expression must agree with that in Eq. (??), which simplifies in this case to:

$$\tau_P^{1/2} = \frac{\xi \tau (\gamma + \xi v)}{\sqrt{\tau_m} (\gamma + \xi \tau)}. \quad (446)$$

That the two must agree in turn implies an equation for  $\xi$ :

$$-\tau_v \tau^{-1} \tau_m^{1/2} \xi^{-1} = \frac{\xi \tau (\gamma + \xi v)}{\sqrt{\tau_m} (\gamma + \xi \tau)}, \quad (447)$$

which is a cubic equation. Under Assumption 2, Eq. (445) implies that:

$$\tau_P \equiv 0. \quad (448)$$

For this solution to coincide with Eq. (446),  $\xi$  must satisfy Eq. (93). Since the ODE in Eq. (443) now says  $\xi' \equiv 0$  (and  $v' = 0$ ), we conclude that  $\alpha_t \equiv \alpha_{T-}$ ,  $v_t \equiv v_{T-}$  and  $\xi_t \equiv -\gamma/v_{T-}$ , and thus the solution of Lemma 1 holds post-announcement.

## A.7 Proof of Corollary 3

To obtain the system of ODEs in Eqs. (97) we start from the ODEs in Eqs. (54), (56) and (57), impose Assumption 2, define  $k_{\tau-} \equiv \alpha_t(\alpha_t \bar{v}_t + \tau_t^c)$  for  $t \leq \tau$ , which differentiation shows

to be a constant pre-announcement, and rewrite the two first ODEs according to:

$$v'_t = \Pi_t \left( 2 \frac{\tau_P^{1/2}}{\tau_t^c} (\alpha_t \bar{v}_{1,t} - v_t) - \Pi_t \right) \quad (449)$$

$$\bar{v}'_{1,t} = \tau_P \frac{\bar{v}_{1,t}}{\tau_t} + \frac{\tau_P^{1/2}}{\alpha_t \tau_t^c} (k_{\tau-} - \alpha_t \bar{v}_{1,t}) \Pi_t. \quad (450)$$

The second ODE is further, and more conveniently written, as:

$$(\alpha_t \bar{v}_{1,t})' = \frac{\tau_P^{1/2}}{\tau_t^c} (k_{\tau-} - \alpha_t \bar{v}_{1,t}) \Pi_t. \quad (451)$$

We proceed similarly with Eq. (71):

$$\Sigma_{\mathbf{P}}(\alpha_t \bar{v}_{1,t} + \gamma Y_t) = \frac{\tau_P^{1/2}}{\tau_t^c} (k_{\tau-} - \alpha_t \bar{v}_{1,t}). \quad (452)$$

Solving this equation for  $\Sigma_{\mathbf{P}}$  and plugging back into the definition of  $\Pi$  in Eq. (69) gives:

$$\Pi_t = \frac{\tau_P^{1/2}}{\tau_t^c} (k_{\tau-} - \alpha_t \bar{v}_{1,t}) \left( \frac{\alpha_t \bar{v}_{1,t} - v_t}{k_{\tau-} - \alpha_t \bar{v}_{1,t}} - \frac{v_t + \gamma/\xi_t}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \right). \quad (453)$$

Substituting back in the ODEs for  $v$ ,  $\bar{v}_1$  and  $\xi$  (in Eq. (68)) and using the expression we just obtained for  $\Sigma_{\mathbf{P}}$  gives:

$$v'_t = \left( \frac{\tau_P^{1/2}}{\tau_t^c} \right)^2 (k_{\tau-} - \alpha_t \bar{v}_{1,t})^2 \left( \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{k_{\tau-} - \alpha_t \bar{v}_{1,t}} \right)^2 - \left( \frac{v_t + \gamma/\xi_t}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \right)^2 \right) \quad (454)$$

$$(\alpha_t \bar{v}_{1,t})' = - \left( \frac{\tau_P^{1/2}}{\tau_t^c} \right)^2 (k_{\tau-} - \alpha_t \bar{v}_{1,t})^2 \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{k_{\tau-} - \alpha_t \bar{v}_{1,t}} + \frac{v_t + \gamma/\xi_t}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \right) \quad (455)$$

$$\xi'_t = \left( \frac{\tau_P^{1/2}}{\tau_t^c} \right)^2 \xi_t \frac{(k_{\tau-} - \alpha_t \bar{v}_{1,t})^2}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{k_{\tau-} - \alpha_t \bar{v}_{1,t}} + \frac{v_t + \gamma/\xi_t}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \right). \quad (456)$$

We further obtain an ODE for the price signal-noise ratio,  $Y \equiv \lambda/\xi$ , by substituting the definition of  $\Sigma_P$  into Eq. (452):

$$\frac{\tau_P^{1/2}}{\tau_t^c} = \tau_m^{-1/2} \left( \frac{k_{\tau-} + \gamma Y_t}{\alpha_t \bar{v}_{1,t} + \gamma Y_t} \xi_t^{-1} - Y_t \right)^{-1}. \quad (457)$$

Now define the common variance,  $o_t^c \equiv (\tau_t^c)^{-1}$ , and introduce the change of dependent variable,  $t \mapsto o_t^c$ , which allows us to rewrite these ODEs as an autonomous system:

$$\frac{d}{do^c} v = (k_{\tau-} - \alpha \bar{v}_1)^2 \left( \left( \frac{v - \alpha \bar{v}_1}{k_{\tau-} - \alpha \bar{v}_1} \right)^2 - \left( \frac{v + \gamma/\xi}{\alpha \bar{v}_1 + \gamma Y} \right)^2 \right) \quad (458)$$

$$\frac{d}{do^c} \alpha \bar{v}_1 = -(k_{\tau-} - \alpha \bar{v}_1)^2 \left( \frac{v - \alpha \bar{v}_1}{k_{\tau-} - \alpha \bar{v}_1} + \frac{v + \gamma/\xi}{\alpha \bar{v}_1 + \gamma Y} \right) \quad (459)$$

$$\frac{d}{do^c} \xi = \xi \frac{(k_{\tau-} - \alpha \bar{v}_1)^2}{\alpha \bar{v}_1 + \gamma Y} \left( \frac{v - \alpha \bar{v}_1}{k_{\tau-} - \alpha \bar{v}_1} + \frac{v + \gamma/\xi}{\alpha \bar{v}_1 + \gamma Y} \right) \quad (460)$$

$$\frac{d}{do^c} Y = \frac{\tau_m^{1/2}}{o^c} \left( Y - \frac{k_{\tau-} + \gamma Y}{\alpha \bar{v}_1 + \gamma Y} \xi^{-1} \right). \quad (461)$$

Re-arranging using the ODE for  $\alpha \bar{v}_1$  then gives the system in Eqs. (97).

To obtain the boundary conditions in Eqs. (94) and (95) and in Eqs. (101) we first note that Assumption 3 allows us to simplify the boundary conditions of Proposition 4 for  $\lambda_{t-}$  and  $\xi_{t-}$  to:

$$\lambda_{t-} = 1 - \underbrace{\frac{\tau_{t-}^c}{\tau_{t-}} \frac{v_t \xi_t^2 - \xi_t \tau_{G,t}^{1/2} \tau_M^{1/2} + \tau_M}{v_t \xi_t^2 (1 + \tau_{t-}^{-1} \tau_{G,t}) + \tau_M}}_{\equiv 1 - \alpha_{t-}} \quad (462)$$

$$\xi_{t-} = \frac{-\gamma \xi_t^2 (v_t / \tau_t \tau_A + \tau_{t-} + \tau_{G,t}) + \xi_t (2\gamma + v_t \xi_t) \tau_{G,t}^{1/2} \tau_M^{1/2} + (\xi_t \tau_{t-} - \gamma (1 - \tau_{t-} / \tau_t)) \tau_M}{v_t \xi_t^2 (\tau_{t-} + \tau_{G,t}) + \tau_{t-} \tau_M}. \quad (463)$$

The first boundary condition for  $\lambda_{t-}$  directly gives Eq. (94). Further using that, under Assumption 3,  $\xi_t = -\gamma/v_t$  post-announcement we can simplify the second boundary condition for  $\xi_{t-}$  as:

$$\xi_t - \xi_{t-} = \gamma \left( \frac{v_t \xi_t^2 - \xi_t \tau_G^{1/2} \tau_M^{1/2} + \tau_M}{v_t \xi_t^2 (\tau_{t-} + \tau_G) + \tau_{t-} \tau_M} - \tau_t^{-1} \right), \quad (464)$$

which can be reorganized as in Eq. (95). Regarding the coefficients of the value function start from Eq. (59), and rewrite it as:

$$\frac{\bar{v}_{1,t-}}{\tau_{t-}} = \frac{\bar{v}_{1,t}}{\tau_t} + (\alpha_t \tau_t^c)^{-1} (k_{T-} - \alpha_t \bar{v}_{1,t}) \left( \pi_b \frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} + \frac{\alpha_t \bar{v}_{1,t} - v_t}{1 - \alpha_t} (1 - \pi_b) \bar{\mathbf{x}} \mathbf{Q} \bar{\mathbf{x}}' \right). \quad (465)$$



Under Assumption 3 direct computations and simplifications show that:

$$\bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' \equiv \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} + \frac{\tau_t \tau_M}{\xi_t^2 \tau_A} y \right)^{-1} \frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} \quad (466)$$

and from Eq. (73) we can write

$$\frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} = -\frac{\alpha_t \tau_t^c}{\tau_{t-}} (k_{T-} - \alpha_t \bar{v}_{1,t})^{-1} (\bar{v}_{1,t} \tau_{t-} / \tau_t + \gamma (1 - \ell_{t-}) \xi_{t-}^{-1}). \quad (467)$$

Substituting back and simplifying based on that under Assumption 2,  $\alpha \equiv \tau_V / \tau$ , gives:

$$\alpha_{t-} \bar{v}_{1,t-} = \alpha_\tau \bar{v}_{1,t} + (1 - \pi_b) \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{v_t - \alpha_t \bar{v}_{1,t} + \frac{\tau_V \tau_M}{\xi_t^2 \tau_A} y} - \frac{\pi_b}{1 - \pi_b} \right) (\alpha_t \bar{v}_{1,t} + \gamma Y_{t-}). \quad (468)$$

Consider now the boundary condition in Eq. (60) and rewrite it as:

$$\bar{v}_{t-} / \tau_{t-}^2 = \tau_{t-}^{-1} - \tau_t^{-1} + \bar{v}_t / \tau_t^2 + (\alpha_t \tau_t^c)^{-1} (k_{T-} - \alpha_t \bar{v}_{1,t}) \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}'. \quad (469)$$

Then substitute the expression above for  $\bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}'$  and simplify to obtain the last boundary condition in Eq. (95). Finally, consider Eq. (58) and rewrite it as:

$$v_{t-} = v_t + \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \right)^2 (1 - \pi_b)^2 \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' + \pi_b^2 (\sigma_P^{-1} - v_t) - 2 \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} (1 - \pi_b) \pi_b \frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P}. \quad (470)$$

Further note that the definition of  $\pi_b$  in Eq. (74) under Assumption 3 is:

$$\sigma_P^{-1} \pi_b = \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \frac{\mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} (1 - \pi_b), \quad (471)$$

which, substituted back in the above, gives:

$$v_{t-} = v_t (1 - \pi_b^2) - (1 - \pi_b) \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} \left( \frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} (1 - \pi_b) \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' + \pi_b \frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} \right). \quad (472)$$

From Eq. (465) we know that:

$$\frac{v_t - \alpha_t \bar{v}_{1,t}}{1 - \alpha_t} (1 - \pi_b) \bar{\mathbf{x}}\mathbf{Q}\bar{\mathbf{x}}' + \pi_b \frac{\Phi' \mathbf{b}_P \mathbf{S} \bar{\mathbf{x}}'}{\sigma_P} = \frac{1 - \alpha_t}{k_{T-} - \alpha_t \bar{v}_{1,t}} (\alpha_{t-} \bar{v}_{1,t-} - \alpha_t \bar{v}_{1,t}), \quad (473)$$

which substituted back gives:

$$v_{t-} = (1 - \pi_b) \left( v_t(1 + \pi_b) + \frac{v_t - \alpha_t \bar{v}_{1,t}}{k_{T-} - \alpha_t \bar{v}_{1,t}} (\alpha_t \bar{v}_{1,t} - \alpha_{t-} \bar{v}_{1,t-}) \right). \quad (474)$$

The remaining first two boundary conditions in Eq. (95) then follow from substituting  $\pi_b = (\xi_{t-} - \xi_t)/\xi_{t-}$  in the equations we just obtained. Furthermore note that substituting  $\xi_{t-}$  in Eq. (95) allows us to simplify  $v_{t-}$  further as:

$$v_{t-} = \underbrace{-\gamma \xi_{t-}^{-1}}_{\text{solution when } \ell \equiv 0} + \frac{\gamma v_t \xi_t^2 \tau_{G,t}^{1/2} \tau_M^{1/2}}{\xi_{t-}^2 (v_t \xi_t^2 (\tau_{t-} + \tau_{G,t}) + \tau_{t-} \tau_M)}. \quad (475)$$

## A.8 Proof of Lemma 3

In equilibrium we can reformulate hedging demands in Eq. (64) as:

$$\mathbf{1}^* \mathbf{B}'_{\Psi} ((\mathbf{v}\mathbf{b} + v_3 \mathbf{c})' (\mathbf{b}\Psi^i - \xi_0 \mathbf{M}) + (v_3 \mathbf{b} + v_4 \mathbf{c})' \mathbf{c}\Psi^i) \quad (476)$$

$$= ((\Pi + \gamma(\sigma_{\mathbf{P}}/\xi - \tau_m^{-1/2})) \Phi \Phi' + \gamma \tau_m^{-1/2} \mathbf{I}) (\mathbf{b}\Psi^i - \xi_0 \mathbf{M}) + \Pi_3 \mathbf{c}\Psi^i$$

$$= \Sigma_{\mathbf{P}}^{-1} (\mathbf{A}_{\Psi} \Psi^i + \dot{\xi}_0 \mathbf{M}) + (\gamma(\sigma_{\mathbf{P}}/\xi - \tau_m^{-1/2})) \Phi \Phi' + \gamma \tau_m^{-1/2} \mathbf{I}) (\mathbf{b}\Psi^i - \xi_0 \mathbf{M}), \quad (477)$$

which, substituted back in Eq. (64), gives the optimal portfolio policy in equilibrium:

$$\mathbf{w}^i = -\Sigma_{\mathbf{P}}^{-1} ((\sigma_{\mathbf{P}}/\xi - \tau_m^{-1/2})) \Phi \Phi' + \tau_m^{-1/2} \mathbf{I}) (\mathbf{b}\Psi^i - \xi_0 \mathbf{M}). \quad (478)$$

This expression shows that  $\Pi_3$  is set so that investors do not bet on the wedge  $\ell$ . Second, pre-multiplying this expression by  $\Phi'$  gives:

$$\Phi' \mathbf{w}^i = -\frac{1}{\xi} \Phi' \mathbf{b}\Psi^i + \Phi' \mathbf{M} \quad (479)$$

$$= -((1 - \ell)(1 - \alpha)/\xi - 1) \Phi' \Psi^i + \Phi' \mathbf{M}. \quad (480)$$

## A.9 Proof of Proposition 6

We start by expressing the dynamics of  $(\tilde{\mathbf{P}}, \Psi)$  under perfect information. Using the results of Proposition 1 we obtain (we ignore the lump at the liquidation date):

$$\begin{aligned} d \begin{pmatrix} \tilde{\mathbf{P}}_t \\ \Psi_t \end{pmatrix} &= \left( \begin{pmatrix} \mathbf{A}_P \\ \mathbf{A}_\Psi \end{pmatrix} \Psi_t + \begin{pmatrix} \dot{\xi}_t \mathbf{M} \\ \mathbf{0} \end{pmatrix} \right) dt + \begin{pmatrix} \Sigma_P \\ \Sigma_\Psi \end{pmatrix} d\mathbf{B}_{m,t} \\ &+ \mathbf{1}_{t=\tau} \left( \begin{pmatrix} \mathbf{a}_P \\ \mathbf{a}_\Psi \end{pmatrix} \Psi_{t-} + \begin{pmatrix} \mathbf{b}_P \\ \bar{\mathbf{b}}_\Psi \end{pmatrix} \tilde{\mathbf{y}} + \begin{pmatrix} (\xi_t - \xi_{t-}) \mathbf{M} \\ \mathbf{0} \end{pmatrix} \right), \end{aligned} \quad (481)$$

where

$$\Sigma_\Psi \equiv \begin{pmatrix} \tau_P^{1/2}/\tau_t^c \Phi & \tau_m^{-1/2} \mathbf{I} \end{pmatrix}' \quad (482)$$

$$\bar{\mathbf{b}}_\Psi \equiv \begin{pmatrix} -\tau_A/\tau_t^c & \tau_G^{1/2} \tau_M^{1/2}/\tau_t^c \Phi' \xi_t^{-1} \\ \mathbf{0} & \xi_t^{-1} \end{pmatrix} \quad (483)$$

$$\tilde{\mathbf{y}} \equiv \begin{pmatrix} \tilde{v} & \Delta \tilde{\mathbf{m}}'_t \xi_t \end{pmatrix}'. \quad (484)$$

Taking expectations on dollar returns conditional on the sign of the announcement yields:

$$\begin{aligned} \mathbb{E} \left[ d\tilde{\mathbf{P}}_t \mid \text{sign}(\tilde{A}) \right] &= \left( \boldsymbol{\mu}_t + \mathbf{A}_P \mathbb{E} \left[ \Psi_t \mid \text{sign}(\tilde{A}) \right] \right) dt \\ &+ \mathbf{1}_{t=\tau} \left( \Delta \boldsymbol{\mu}_t + \mathbf{a}_P \mathbb{E} \left[ \Psi_{t-} \mid \text{sign}(\tilde{A}) \right] + \mathbf{b}_P \mathbb{E} \left[ \tilde{\mathbf{y}} \mid \text{sign}(\tilde{A}) \right] \right), \end{aligned} \quad (485)$$

where  $\boldsymbol{\mu}_t \equiv \mathbb{E}[d\tilde{\mathbf{P}}_t]/dt = \dot{\xi}_t \mathbf{M}$  and  $\Delta \boldsymbol{\mu}_t = \mathbb{E}[\Delta \tilde{\mathbf{P}}_t] = (\xi_t - \xi_{t-}) \mathbf{M}$  denote unconditional expected returns in between and at announcements, respectively. Further observing that  $\mathbb{E}[\Psi_t \mid \text{sign}(\tilde{A})] = \left( \mathbb{E}[\Delta_t \mid \text{sign}(\tilde{A})] \quad \mathbf{0}' \right)'$  and  $\mathbb{E}[\tilde{\mathbf{y}} \mid \text{sign}(\tilde{A})] = \left( \mathbb{E}[\tilde{v} \mid \text{sign}(\tilde{A})] \quad \mathbf{0}' \right)'$  we can write (recall from Proposition 4 that  $\Pi_{2,t} = 0$  and  $\pi_2 = 0$  at all times):

$$\mathbf{A}_P \mathbb{E} \left[ \Psi_t \mid \text{sign}(\tilde{A}) \right] = ((1 - \ell_t)(1 - \alpha_t) \Pi_t + \ell_t \Pi_{3,t}) \mathbb{E}[\Delta_t \mid \text{sign}(\tilde{A})] \Sigma_P \Phi \quad (486)$$

$$\mathbf{a}_P \mathbb{E} \left[ \Psi_{t-} \mid \text{sign}(\tilde{A}) \right] = \mathbf{b}_P \begin{pmatrix} \pi_1^A (1 - \ell_{t-})(1 - \alpha_{t-}) + \ell_{t-} \pi_2^A \\ (\pi_1 (1 - \ell_{t-})(1 - \alpha_{t-}) + \ell_{t-} \pi_3) \Phi \end{pmatrix} \mathbb{E} \left[ \Delta_{t-} \mid \text{sign}(\tilde{A}) \right] \quad (487)$$

$$\mathbb{E} \left[ \tilde{\mathbf{y}} \mid \text{sign}(\tilde{A}) \right] = (1 - \lambda_t) \tau_A / \tau_t^c \mathbb{E}[\tilde{v} \mid \text{sign}(\tilde{A})]. \quad (488)$$

Using now the two relations in Eqs. (43) and (44) of Conjecture 1 and substituting in expected dollar returns above gives:

$$\begin{aligned} \mathbb{E} \left[ d\tilde{\mathbf{P}}_t \middle| \text{sign}(\tilde{A}) \right] &= \left( \boldsymbol{\mu}_t - \Sigma_{\mathbf{P}}(\lambda_t \Pi_t + \tau_P^{1/2}) \mathbb{E} \left[ \Delta_t \middle| \text{sign}(\tilde{A}) \right] \boldsymbol{\Phi} \right) dt \\ &+ \mathbf{1}_{t=\tau} \left( \Delta \boldsymbol{\mu}_t + (y - \lambda_t - \pi_b) \mathbb{E} \left[ \Delta_{t-} \middle| \text{sign}(\tilde{A}) \right] \boldsymbol{\Phi} + (1 - \lambda_t) \frac{\tau_A}{\tau_t^c} \mathbb{E} \left[ \tilde{v} \middle| \text{sign}(\tilde{A}) \right] \boldsymbol{\Phi} \right), \end{aligned} \quad (489)$$

where  $y$  and  $\pi_b$  are defined in Proposition 3. It remains to compute the expectations above. Extracing  $\Delta$  from  $\boldsymbol{\Psi}$  under perfect information in Eq. (481) we can write its dynamics as:

$$d\Delta_t = -\frac{\tau_P}{\tau_t^c} \Delta_t dt + \Sigma_{\boldsymbol{\Psi}} d\mathbf{B}_{m,t} + \mathbf{1}_{t=\tau} \left( -\frac{\tau_A + \tau_G}{\tau_t^c} \Delta_{t-} + (\tau_t^c)^{-1} \begin{pmatrix} -\tau_A & \tau_G^{1/2} \tau_M^{1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_t^{-1} \end{pmatrix} \tilde{\mathbf{y}} \right). \quad (490)$$

Differentiating  $\tau_t^c \Delta_t$  and integrating back then gives:

$$\Delta_t = \frac{\tau_0^c}{\tau_t^c} \Delta_0 + \int_0^t \frac{\tau_u^c}{\tau_t^c} \Sigma_{\boldsymbol{\Psi}} d\mathbf{B}_{m,u} + \mathbf{1}_{t=\tau} (\tau_t^c)^{-1} \begin{pmatrix} -\tau_A & \tau_G^{1/2} \tau_M^{1/2} \boldsymbol{\Phi}' \boldsymbol{\xi}_t^{-1} \end{pmatrix} \tilde{\mathbf{y}}. \quad (491)$$

Now taking expectations on both sides we get:

$$\mathbb{E}[\Delta_t | \text{sign}(\tilde{A})] = \frac{\tau_0^c}{\tau_t^c} \Delta_0 \mathbb{E}[\Delta_0 | \text{sign}(\tilde{A})] - \mathbf{1}_{t=\tau} \frac{\tau_A}{\tau_t^c} \mathbb{E}[\tilde{v} | \text{sign}(\tilde{A})]. \quad (492)$$

At time 0, the empiricist's applies Bayes' rule to the initial price signal, which gives:

$$\Delta_0 = \frac{\tau_F}{\tau_0^c} \tilde{F} - \frac{\xi_0 \lambda_0 \tau_F^{-1}}{\tau_0^c} \boldsymbol{\Phi}' \tilde{\mathbf{m}}, \quad (493)$$

so that its conditional expectation satisfies:

$$\mathbb{E} \left[ \Delta_0 \middle| \text{sign}(\tilde{A}) \right] = \frac{\tau_F}{\tau_t^c} \mathbb{E} \left[ \tilde{F} \middle| \text{sign}(\tilde{A}) \right] - \mathbf{1}_{t=\tau} \frac{\tau_A}{\tau_t^c} \mathbb{E} \left[ \tilde{v} \middle| \text{sign}(\tilde{A}) \right]. \quad (494)$$

Finally, applying Bayes' rule we can write:

$$\mathbb{E} \left[ \tilde{F} \middle| \text{sign}(\tilde{A}) \right] = \frac{\int_{\mathbb{R}_+} \int_{\mathbb{R}} x \frac{\tau_A^{1/2}}{\sqrt{2\pi}} \exp(-\tau_A/2(a-x)^2) \frac{\tau_F^{1/2}}{\sqrt{2\pi}} \exp(-\tau_F/2x^2) dx da}{\mathbb{P}(\tilde{A} \geq 0)} \quad (495)$$

$$= \sqrt{\frac{2}{\pi} \frac{\tau_A/\tau_F}{\tau_A + \tau_F}} \quad (496)$$

and similarly

$$\mathbb{E} \left[ \tilde{v} | \text{sign}(\tilde{A}) \right] = \sqrt{\frac{2}{\pi} \frac{\tau_F / \tau_A}{\tau_A + \tau_F}}. \quad (497)$$

Substituting back into the conditional expectation for  $\Delta$  gives:

$$\mathbb{E}[\Delta_t | \text{sign}(\tilde{A})] = \frac{1}{\tau_t^c} \sqrt{\frac{2}{\pi} \frac{\tau_A \tau_F}{\tau_A + \tau_F}} (1 - \mathbf{1}_{t=\tau}), \quad (498)$$

and substituting this expression in turn into expected return we get:

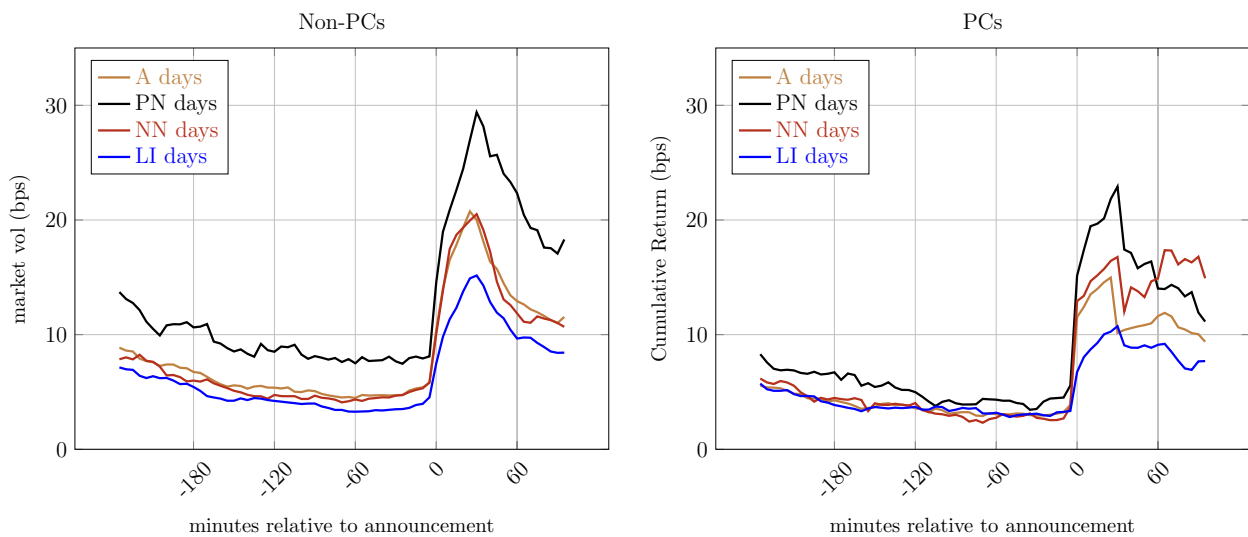
$$\begin{aligned} \mathbb{E} \left[ d\tilde{\mathbf{P}}_t \middle| \text{sign}(\tilde{A}) \right] &= \left( \boldsymbol{\mu}_t - \Sigma_{\mathbf{P}}(\lambda_t \Pi_t + \tau_P^{1/2}) \text{sign}(\tilde{A}) \frac{1}{\tau_t^c} \sqrt{\frac{2}{\pi} \frac{\tau_A \tau_F}{\tau_A + \tau_F}} (1 - \mathbf{1}_{t=\tau}) \boldsymbol{\Phi} \right) dt \quad (499) \\ &+ \mathbf{1}_{t=\tau} \left( \Delta \boldsymbol{\mu}_t + \text{sign}(\tilde{A}) \frac{1}{\tau_t^c} \sqrt{\frac{2}{\pi} \frac{\tau_A \tau_F}{\tau_A + \tau_F}} (y - \lambda_t - \pi_b + 1 - \lambda_t) \right) \boldsymbol{\Phi} \end{aligned}$$

Pre-multiplying by  $\mathbf{M}'$  we obtain the desired expression.

## B Data appendix

Although not necessarily the case theoretically, perhaps intuitively we may also expect higher trading volume and market volatility in the run-up to the announcement. Figure 11 plots intraday market volatility over 30-minute rolling windows, and reveals two patterns. First, market volatility is higher prior to announcements on PN days in sample A. In particular, the difference between market volatility on PN days and NN days before the announcement in sample A is statistically significant, and only becomes insignificant closely around the announcement (see appendix). In sample B, however, pre-announcement volatility does not differ among the different types of announcements, in line with the symmetric and weak pre-announcement drifts over this period. Second, there are spikes in volatility at the announcement, suggesting that the market reacts (strongly) to published information. These spikes have on average been smaller in sample B, and thus suggests that the reaction to the announcement has decreased since the introduction of PCs. This findings is consistent with the intuition that improved forward guidance through PCs reduce uncertainty.

[to be completed]



**Figure 11: Market volatility on announcement days.** This figure shows the average time-series volatility of a market proxy consisting of S&P500 stocks which are traded on the NYSE. The time period of sample A runs from January 2001 through March 2011. The time period of sample B runs from April 2011 (when press conferences after FOMC announcements were introduced) through December 2020. Positive news (PN) days are announcement days with a daily return that falls in the upper 75% quantile of the daily return distribution of the respective sample, and negative news (NN) days returns in the lower 25% quantile. Low information days neither belong to PN nor NN days.