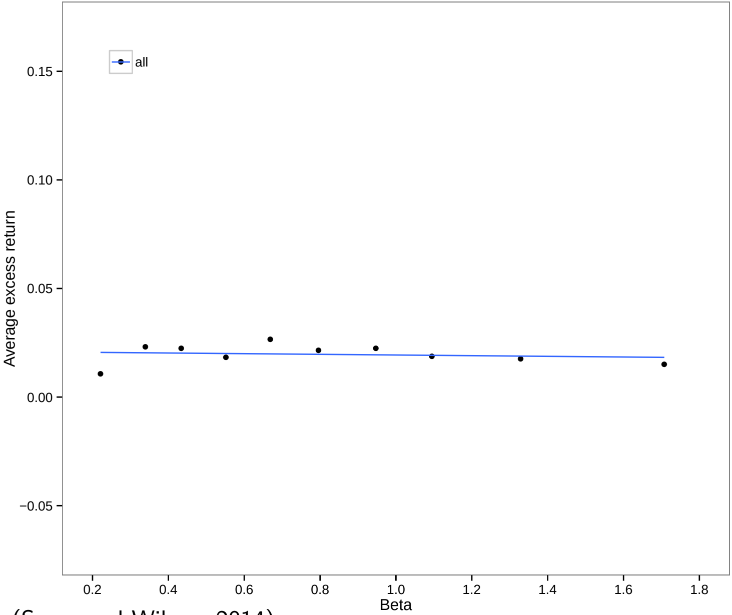
The Lost Capital Asset Pricing Model

Daniel Andrei (UCLA)
Julien Cujean (U of Maryland)
Mungo Wilson (U of Oxford)

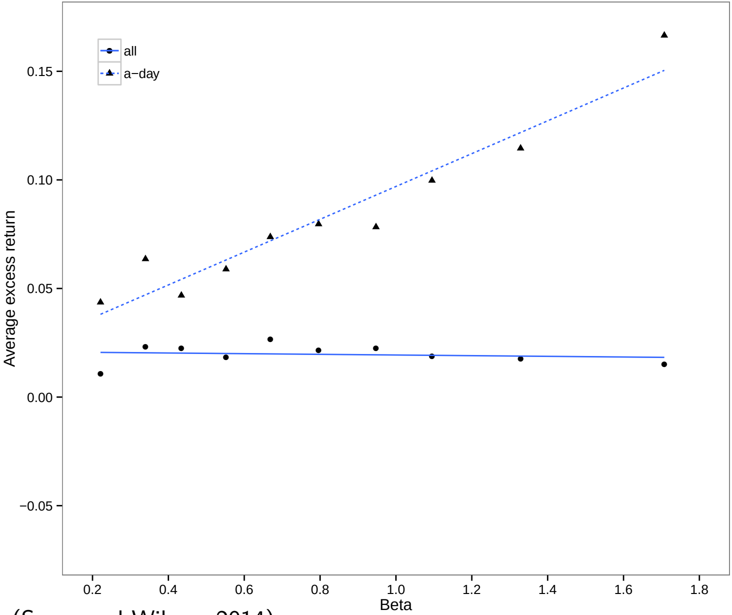
UCLA, November 2016

The lost city of Atlantis



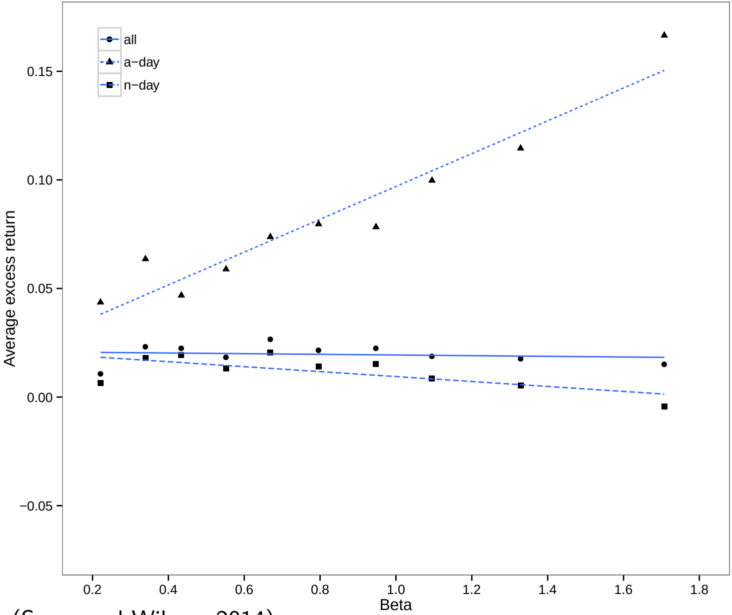
Source: (Savor and Wilson, 2014)

The lost city of Atlantis



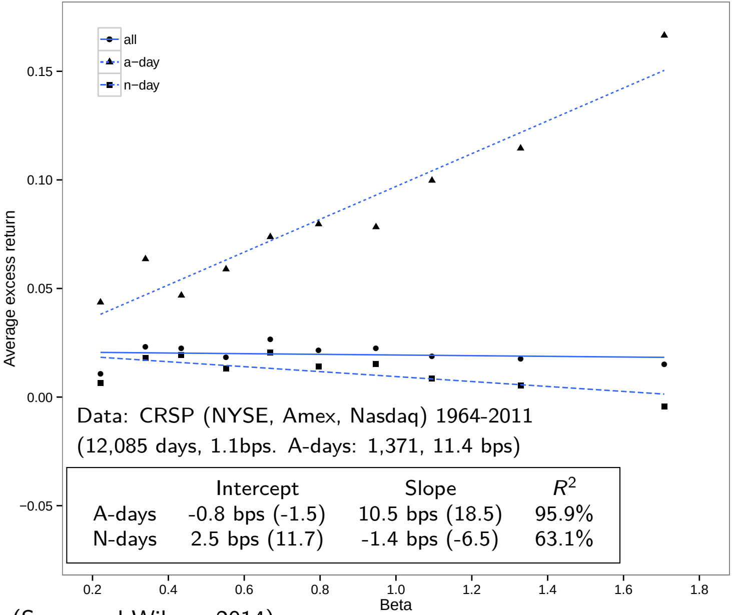
Source: (Savor and Wilson, 2014)

The lost city of Atlantis



Source: (Savor and Wilson, 2014)

The lost city of Atlantis



Data: CRSP (NYSE, Amex, Nasdaq) 1964-2011

(12,085 days, 1.1bps. A-days: 1,371, 11.4 bps)

	Intercept	Slope	R^2
A-days	-0.8 bps (-1.5)	10.5 bps (18.5)	95.9%
N-days	2.5 bps (11.7)	-1.4 bps (-6.5)	63.1%

Source: (Savor and Wilson, 2014)

Possible explanations for a flat CAPM

- Leverage constraints: Black (1972), Frazzini and Pedersen (2014)
- Inflation: Cohen, Polk, and Vuolteenaho (2005)
- Disagreement: Hong and Sraer (2016)
- Preference for volatile, skewed returns: Kumar (2009), Bali, Cakici, and Whitelaw (2011)
- Market sentiment: Antoniou, Doukas, and Subrahmanyam (2015)
- Stochastic volatility: Campbell, Giglio, Polk, and Turley (2016)
- Benchmarking of institutional investors: Baker, Bradley, and Wurgler (2011), Buffa, Vayanos, and Woolley (2014)

Our explanation (reconciling both why CAPM fails most of the time, but seems to hold on announcement days)

Our explanation (reconciling both why CAPM fails most of the time, but seems to hold on announcement days)

- Roll (1977) critique and Hansen and Richard (1987) critique generate the first fact in a noisy rational expectations equilibrium

Our explanation (reconciling both why CAPM fails most of the time, but seems to hold on announcement days)

- Roll (1977) critique and Hansen and Richard (1987) critique generate the first fact in a noisy rational expectations equilibrium
- Public announcements affects all risky assets, alleviate both critiques, and generate the second fact

Conditioning information

- Consider a market with N assets and excess returns:

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$$

Conditioning information

- Consider a market with N assets and excess returns:

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$$

- Law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\mathbb{E}_t[Q_{t+1}])$$

Conditioning information

- Consider a market with N assets and excess returns:

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$$

- Law of total covariance:

$$\underset{\substack{\uparrow \\ \text{Measured by} \\ \text{the empiricist}}}{\boxed{\mathbb{V}[Q_{t+1}]}} = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\mathbb{E}_t[Q_{t+1}])$$

Conditioning information

- Consider a market with N assets and excess returns:

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$$

- Law of total covariance:

$$\boxed{\mathbb{V}[Q_{t+1}]} = \mathbb{E}\left(\boxed{\mathbb{V}_t[Q_{t+1}]}\right) + \mathbb{V}\left(\boxed{\mathbb{E}_t[Q_{t+1}]}\right)$$

Measured by the empiricist

Equilibrium model

Conditioning information

- Consider a market with N assets and excess returns:

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t$$

- Law of total covariance:

$$\boxed{\mathbb{V}[Q_{t+1}]} = \mathbb{E}\left(\boxed{\mathbb{V}_t[Q_{t+1}]}\right) + \mathbb{V}\left(\boxed{\mathbb{E}_t[Q_{t+1}]}\right)$$

Measured by the empiricist

Equilibrium model

- The law of total covariance <u>**and**</u> the equilibrium model allow us to quantify the effect of conditioning down on a coarser information set.

Model, based on Spiegel (1998)

- N stocks pay

$$D_t = \overline{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D$$

Model, based on Spiegel (1998)

- N stocks pay

$$D_t = \overline{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D$$

- F is a common factor (unobservable):

$$F_t = \kappa_F F_{t-1} + \epsilon_t^F$$

Model, based on Spiegel (1998)

- N stocks pay

$$D_t = \overline{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D$$

- F is a common factor (unobservable):

$$F_t = \kappa_F F_{t-1} + \epsilon_t^F$$

- Supply (unobservable — Roll's critique):

$$X_t = (1 - \kappa_X)\overline{X}\mathbb{1}_N + \kappa_X X_{t-1} + \epsilon_t^X$$

Model, based on Spiegel (1998)

- N stocks pay

$$D_t = \overline{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D$$

- F is a common factor (unobservable):

$$F_t = \kappa_F F_{t-1} + \epsilon_t^F$$

- Supply (unobservable — Roll's critique):

$$X_t = (1 - \kappa_X)\overline{X}\mathbb{1}_N + \kappa_X X_{t-1} + \epsilon_t^X$$

- Innovations:

$$\epsilon_t^D \sim N(\mathbb{0}_N, \sigma_D^2 \mathbb{I}_N),\ \epsilon_t^F \sim N(0, \sigma_F^2),\ \epsilon_t^X \sim N(\mathbb{0}_N, \sigma_X^2 \mathbb{I}_N)$$

Investors

- Continuum, indexed by $i \in [0,1]$

$$\max_{x_t^i} \mathbb{E}_t^i \left[-\exp(-\gamma W_{t+1}^i) \right]$$

Investors

- Continuum, indexed by $i \in [0,1]$

$$\max_{x_t^i} \mathbb{E}_t^i \left[-\exp(-\gamma W_{t+1}^i) \right]$$

- Optimal demands:

$$x_t^i = \frac{1}{\gamma} (\mathbb{V}_t^i [Q_{t+1}])^{-1} \mathbb{E}_t^i [Q_{t+1}]$$

Investors

- Continuum, indexed by $i \in [0, 1]$

$$\max_{x_t^i} \mathbb{E}_t^i \left[- \exp(-\gamma W_{t+1}^i) \right]$$

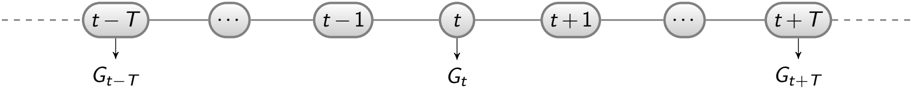
- Optimal demands:

$$x_t^i = \frac{1}{\gamma} (\mathbb{V}_t^i [Q_{t+1}])^{-1} \mathbb{E}_t^i [Q_{t+1}]$$

- Market clearing:

$$\int_i x_t^i di = X_t$$

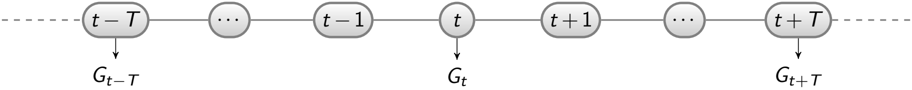
Information



$t-T$ $\cdots$ $t-1$ t $t+1$ $\cdots$ $t+T$

G_{t-T} G_t G_{t+T}

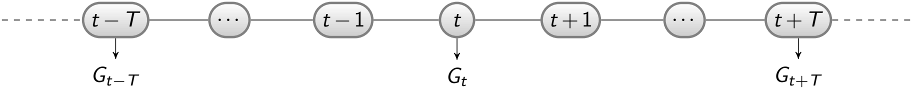
Information



- Public announcement every T periods:

$$G_t = F_t + \epsilon_t^G$$

Information



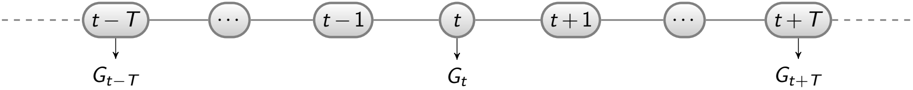
- Public announcement every T periods:

$$G_t = F_t + \epsilon_t^G$$

- Private information (at every date):

$$v_t^i = \epsilon_t^F + \epsilon_t^i$$

Information



- Public announcement every T periods:

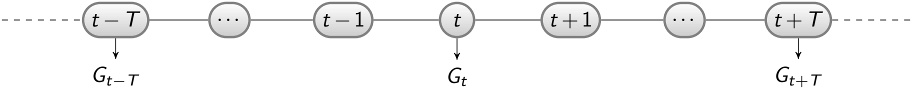
$$G_t = F_t + \epsilon_t^G$$

- Private information (at every date):

$$v_t^i = \epsilon_t^F + \epsilon_t^i$$

- F_{t-T} is revealed at time t (Townsend, 1983; Singleton, 1987)

Information



$t-T$ $\cdots$ $t-1$ t $t+1$ $\cdots$ $t+T$

G_{t-T} G_t G_{t+T}

- Public announcement every T periods:

$$G_t = F_t + \epsilon_t^G$$

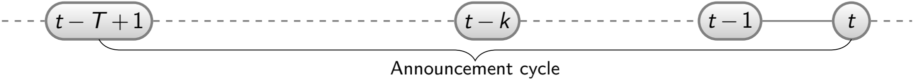
- Private information (at every date):

$$v_t^i = \epsilon_t^F + \epsilon_t^i$$

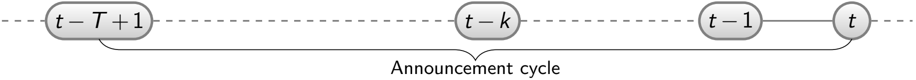
- F_{t-T} is revealed at time t (Townsend, 1983; Singleton, 1987)
- Equilibrium prices:

$$P_t = L(\overline{D}, F, \overline{X}, X, D, G, \epsilon^F, \epsilon^X)$$

Equilibrium: time-varying coefficients in a stationary model



Equilibrium: time-varying coefficients in a stationary model



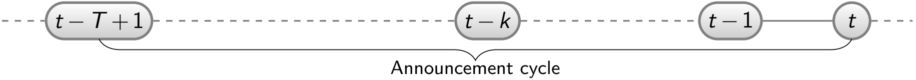
$t-T+1$ $t-k$ $t-1$ t

Announcement cycle

- Prices over the "announcement cycle":

$$P_{t-k} = \overline{\alpha}_k \overline{D} + \alpha_k F_{t-k-T} + \overline{\xi}_k \overline{X} + \xi_k X_{t-k-T} + d_k \mathcal{D}_{t-k} + g_k G_{t-k} + a_k \overline{\epsilon}^F_{t-k} + b_k \overline{\epsilon}^X_{t-k}$$

Equilibrium: time-varying coefficients in a stationary model

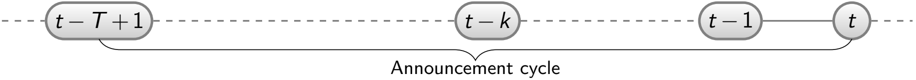


- Prices over the “announcement cycle”:

$F_{t-T}+\epsilon^G_{t-T},\ k\geq 1$
$F_t+\epsilon^G_t,\ k=0$

$$P_{t-k}=\overline{\alpha}_k\overline{D}+\alpha_kF_{t-k-T}+\overline{\xi}_k\overline{X}+\xi_kX_{t-k-T}+d_k\mathcal{D}_{t-k}+g_k\boxed{G_{t-k}}+a_k\overline{\epsilon}^F_{t-k}+b_k\overline{\epsilon}^X_{t-k}$$

Equilibrium: time-varying coefficients in a stationary model



- Prices over the "announcement cycle":

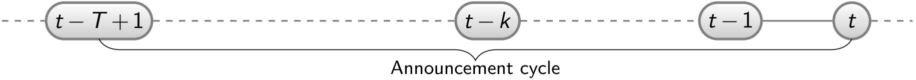
$$F_{t-T} + \epsilon^G_{t-T},\ k \geq 1$$
$$F_t + \epsilon^G_t,\ k = 0$$

$$P_{t-k} = \overline{\alpha}_k \overline{D} + \alpha_k F_{t-k-T} + \overline{\xi}_k \overline{X} + \xi_k X_{t-k-T} + d_k \mathcal{D}_{t-k} + g_k \boxed{G_{t-k}} + a_k \overline{\epsilon}^F_{t-k} + b_k \overline{\epsilon}^X_{t-k}$$

- Investors form beliefs about future payoffs:

$$\mathbb{E}^i_{t-k}[P_{t-k+1} + D_{t-k+1}]$$
$$\mathbb{V}^i_{t-k}[P_{t-k+1} + D_{t-k+1}]$$

Equilibrium: time-varying coefficients in a stationary model



- Prices over the "announcement cycle":

$$F_{t-T}+\epsilon^G_{t-T},\ k \geq 1$$
$$F_t+\epsilon^G_t,\ k=0$$

$$P_{t-k} = \overline{\alpha}_k \overline{D} + \alpha_k F_{t-k-T} + \overline{\xi}_k \overline{X} + \xi_k X_{t-k-T} + d_k \mathcal{D}_{t-k} + g_k \boxed{G_{t-k}} + a_k \overline{\epsilon}^F_{t-k} + b_k \overline{\epsilon}^X_{t-k}$$

- Investors form beliefs about future payoffs:

$$\mathbb{E}^i_{t-k}[P_{t-k+1}+D_{t-k+1}]$$
$$\mathbb{V}^i_{t-k}[P_{t-k+1}+D_{t-k+1}]$$

- At $t-1$ $(k=1)$, investors anticipate the next period public signal

Market clearing & CAPM

Market clearing & CAPM

- Market clearing:

$$\overline{\mathbb{E}}_{t-k}\underbrace{\left[P_{t-k+1}+D_{t-k+1}-RP_{t-k}\right]}_{Q_{t-k+1}}=\gamma\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]X_{t-k}$$

Market clearing & CAPM

- Market clearing:

$$\overline{\mathbb{E}}_{t-k}\underbrace{\left[P_{t-k+1}+D_{t-k+1}-RP_{t-k}\right]}_{Q_{t-k+1}}=\gamma\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]X_{t-k}$$

- The pair $(\overline{\mathbb{E}},\overline{\mathbb{V}})$ can be interpreted as the beliefs of a fictitious agent (the "average investor")

Market clearing & CAPM

- Market clearing:

$$\overline{\mathbb{E}}_{t-k}\underbrace{\left[P_{t-k+1}+D_{t-k+1}-RP_{t-k}\right]}_{Q_{t-k+1}}=\gamma\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]X_{t-k}$$

- The pair $(\overline{\mathbb{E}},\overline{\mathbb{V}})$ can be interpreted as the beliefs of a fictitious agent (the "average investor")
- The market portfolio is mean-variance efficient from the perspective of the average investor

Market clearing & CAPM

- Market clearing:

$$\overline{\mathbb{E}}_{t-k}\underbrace{\left[P_{t-k+1}+D_{t-k+1}-RP_{t-k}\right]}_{Q_{t-k+1}}=\gamma\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]X_{t-k}$$

- The pair $(\overline{\mathbb{E}},\overline{\mathbb{V}})$ can be interpreted as the beliefs of a fictitious agent (the "average investor")
- The market portfolio is mean-variance efficient from the perspective of the average investor
- Take unconditional expectation:

$$\mathbb{E}[Q_{t+1}]=\gamma\underbrace{\mathbb{E}[\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]]}_{\equiv\mathbb{E}(\mathbb{V}_t[Q_{t+1}])}\overline{X}$$

Market clearing & CAPM

- Market clearing:

$$\overline{\mathbb{E}}_{t-k}\underbrace{\left[P_{t-k+1}+D_{t-k+1}-RP_{t-k}\right]}_{Q_{t-k+1}}=\gamma\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]X_{t-k}$$

- The pair $(\overline{\mathbb{E}},\overline{\mathbb{V}})$ can be interpreted as the beliefs of a fictitious agent (the "average investor")
- The market portfolio is mean-variance efficient from the perspective of the average investor
- Take unconditional expectation:

$$\mathbb{E}[Q_{t+1}]=\gamma\underbrace{\mathbb{E}[\overline{\mathbb{V}}_{t-k}\left[Q_{t-k+1}\right]]}_{\equiv\mathbb{E}(\mathbb{V}_t[Q_{t+1}])}\overline{X}\quad\Rightarrow\quad\mathbb{E}[Q_{t+1}]=\underbrace{\frac{\mathbb{E}(\mathbb{V}_t[Q_{t+1}])\overline{X}}{\mathbb{E}(\mathbb{V}_t[M_{t+1}])}}_{\beta}\mathbb{E}[M_{t+1}]$$

Empiricist's view

- Recall the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\mathbb{E}_t[Q_{t+1}])$$

Empiricist's view

- Recall the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\underset{\substack{\uparrow \\ \mathbb{E}_t[Q_{t+1}] = \gamma \mathbb{V}_t[Q_{t+1}] X_t}}{\boxed{\mathbb{E}_t[Q_{t+1}]}})$$

Empiricist's view

- Recall the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}\Big(\underset{\substack{\uparrow \\ \mathbb{E}_t[Q_{t+1}] = \gamma \mathbb{V}_t[Q_{t+1}] X_t}}{\boxed{\mathbb{E}_t[Q_{t+1}]}}\Big)$$

- Two effects distort empiricist's view of the economy:
  1. Unobservable market portfolio
  2. $\mathbb{E}_t[Q_{t+1}]$ and $\mathbb{V}_t[Q_{t+1}]$ move together

Empiricist's view

- Recall the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\underset{\substack{\uparrow \\ \mathbb{E}_t[Q_{t+1}] = \gamma \mathbb{V}_t[Q_{t+1}] X_t}}{\boxed{\mathbb{E}_t[Q_{t+1}]}})$$

- Two effects distort empiricist's view of the economy:
  1. Unobservable market portfolio
  2. $\mathbb{E}_t[Q_{t+1}]$ and $\mathbb{V}_t[Q_{t+1}]$ move together
- Consider only the first effect (for exposition):

$$\mathbb{V}[Q_{t+1}] = \mathbb{V}_t[Q_{t+1}]\left(\mathbb{I} + \gamma^2 \frac{\sigma_X^2}{1-\kappa_X^2} \mathbb{V}_t[Q_{t+1}]\right)$$

Empiricist's view

- Recall the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\underset{\substack{\uparrow \\ \mathbb{E}_t[Q_{t+1}] = \gamma \mathbb{V}_t[Q_{t+1}] X_t}}{\boxed{\mathbb{E}_t[Q_{t+1}]}})$$

- Two effects distort empiricist's view of the economy:
  1. Unobservable market portfolio
  2. $\mathbb{E}_t[Q_{t+1}]$ and $\mathbb{V}_t[Q_{t+1}]$ move together
- Consider only the first effect (for exposition):

$$\mathbb{V}[Q_{t+1}] = \mathbb{V}_t[Q_{t+1}] \left(\mathbb{I} + \gamma^2 \frac{\sigma_X^2}{1 - \kappa_X^2} \mathbb{V}_t[Q_{t+1}] \right)$$

Main result

The empiricist overestimates betas when $\beta > 1$ and underesimates betas when $\beta < 1$

Intuition

$$\begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1+\varphi \\ 1-\varphi \end{bmatrix} F_t + \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix}$$

Intuition

$$\begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1+\varphi \\ 1-\varphi \end{bmatrix} F_t + \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix}$$

Agents:

$$\begin{bmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon^2 \\ \rho\sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{bmatrix}$$

$$\beta_1 = 1$$

$$\beta_2 = 1$$

Intuition

$$\begin{bmatrix} X_{1,t+1} \\ X_{2,t+1} \end{bmatrix} = \begin{bmatrix} 1+\varphi \\ 1-\varphi \end{bmatrix} F_t + \begin{bmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \end{bmatrix}$$

Agents:

$$\begin{bmatrix} \sigma_\varepsilon^2 & \rho\sigma_\varepsilon^2 \\ \rho\sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{bmatrix}$$

$$\beta_1 = 1$$

$$\beta_2 = 1$$

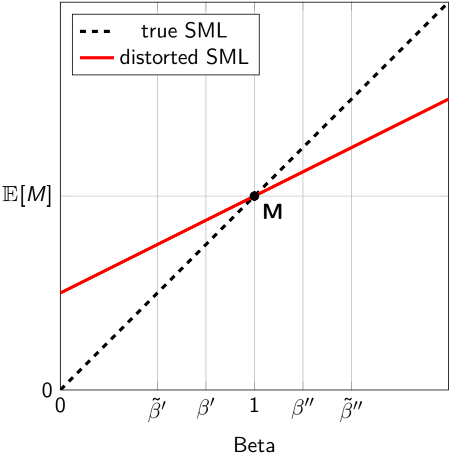
Econometrician:

$$\begin{bmatrix} (1+\varphi)^2\sigma_F^2 + \sigma_\varepsilon^2 & (1+\varphi)(1-\varphi)\sigma_F^2 + \rho\sigma_\varepsilon^2 \\ (1+\varphi)(1-\varphi)\sigma_F^2 + \rho\sigma_\varepsilon^2 & (1-\varphi)^2\sigma_F^2 + \sigma_\varepsilon^2 \end{bmatrix}$$

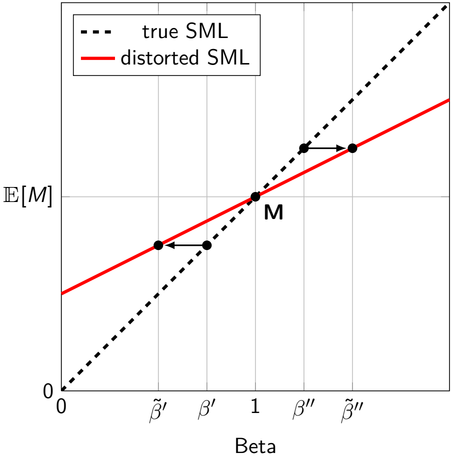
$$\tilde{\beta}_1 = 1 + \frac{2\varphi\sigma_F^2}{2\sigma_F^2 + (1+\rho)\sigma_\varepsilon^2} > 1$$

$$\tilde{\beta}_2 = 1 - \frac{2\varphi\sigma_F^2}{2\sigma_F^2 + (1+\rho)\sigma_\varepsilon^2} < 1$$

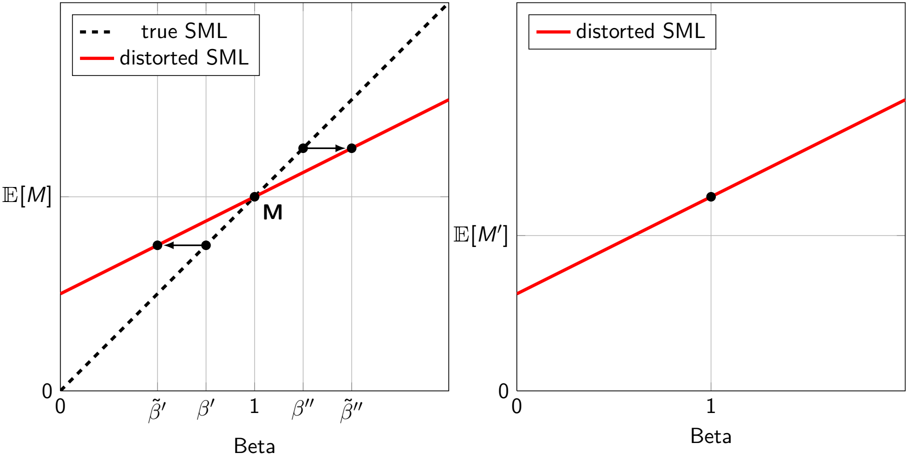
Unconditional CAPM: Distortion



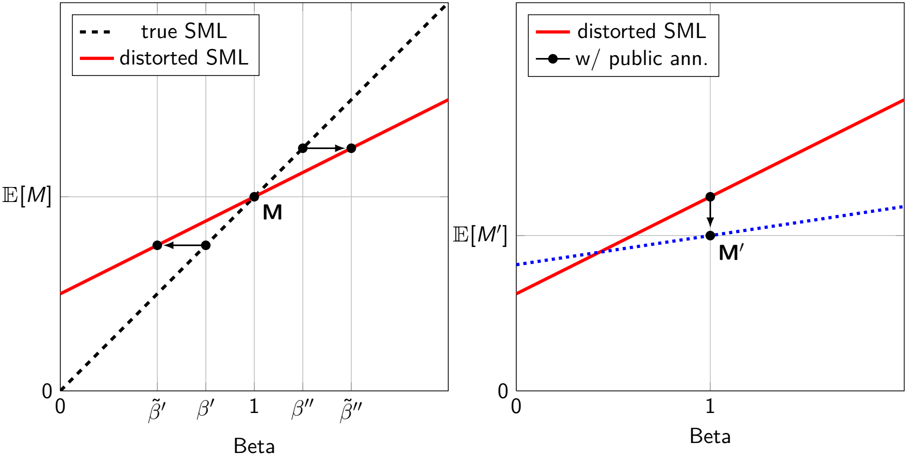
Unconditional CAPM: Distortion



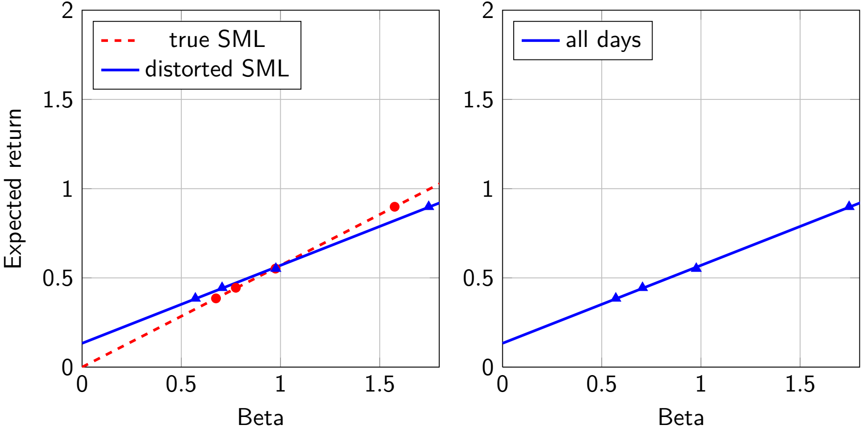
Unconditional CAPM: Distortion



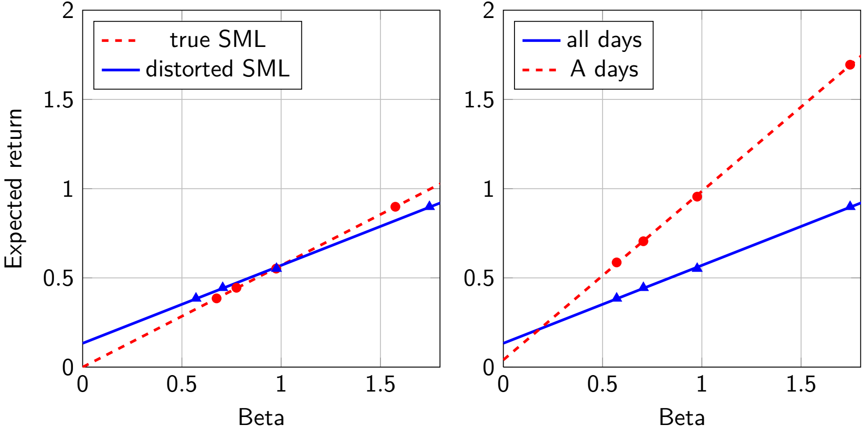
Unconditional CAPM: Distortion



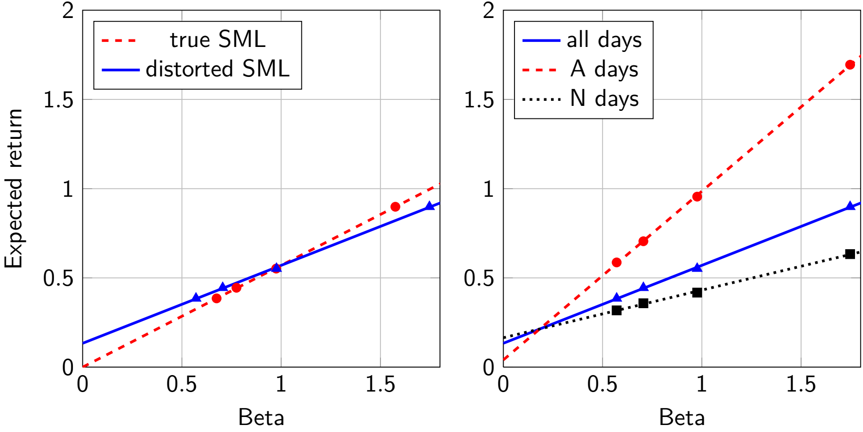
CAPM on different days



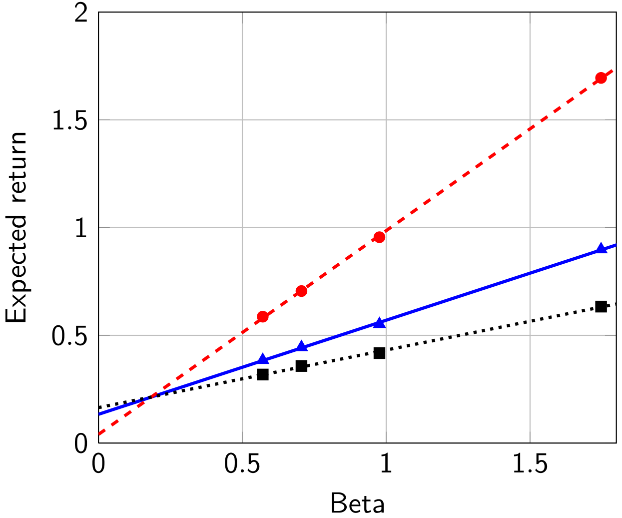
CAPM on different days



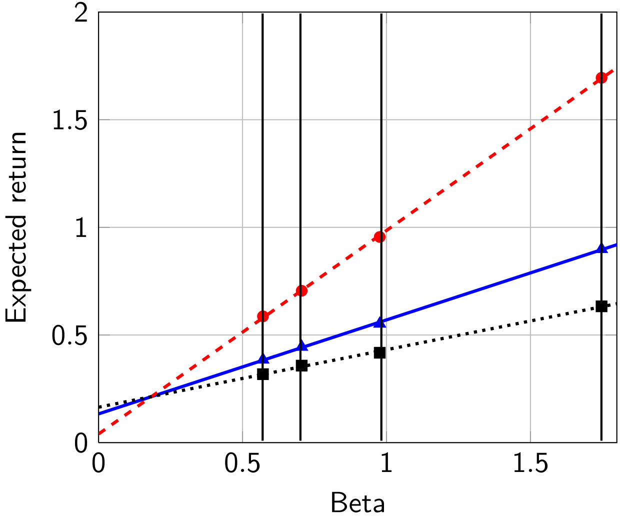
CAPM on different days



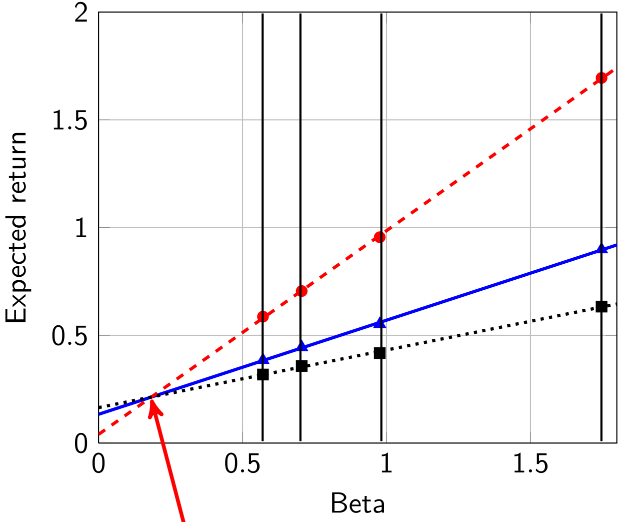
Intuition



Intuition

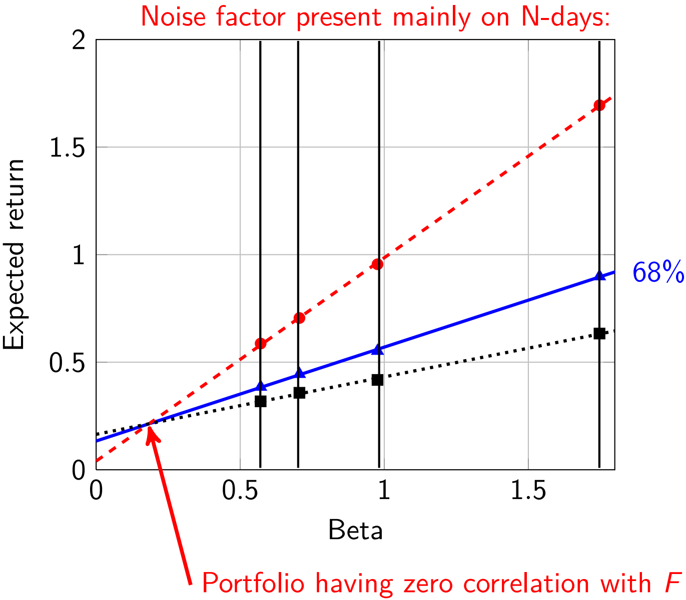


Intuition



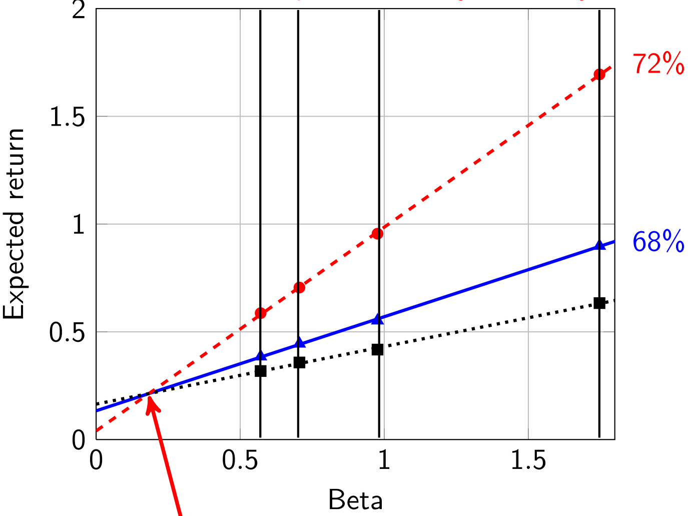
Portfolio having zero correlation with F

Intuition


Noise factor present mainly on N-days:
2
1.5
1
0.5
0
0
0.5
1
1.5
Expected return
Beta
68%
Portfolio having zero correlation with F

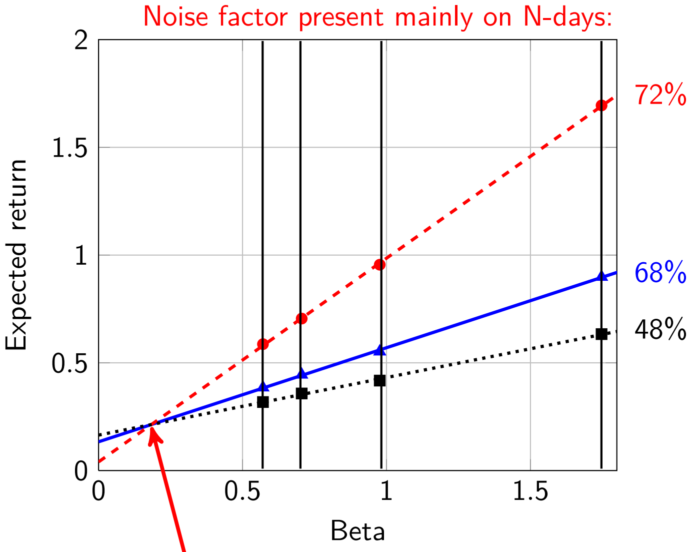
Intuition

Noise factor present mainly on N-days:



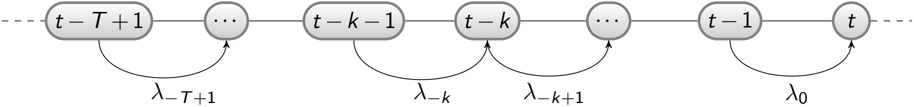
Portfolio having zero correlation with F

Intuition

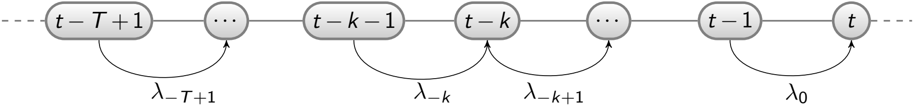


Portfolio having zero correlation with F

Diffusion of private information

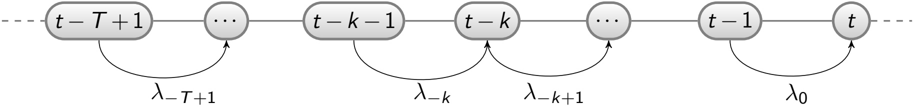


Diffusion of private information



- It matters when investors “talk” about the announcement:
  - If they talk the day before (in anticipation of the news), then the CAPM is steeper before the announcement.
  - If they talk during the day (to figure out the consequences of the new information), then CAPM gets even steeper.

Diffusion of private information



- It matters when investors “talk” about the announcement:
  - If they talk the day before (in anticipation of the news), then the CAPM is steeper before the announcement.
  - If they talk during the day (to figure out the consequences of the new information), then CAPM gets even steeper.
- Two additional dimensions are interesting here:
  1. Disagreement
  2. Trading volume

Further questions

- Size effects (two assets with same exposure to F but different sizes might have different betas)

Further questions

- Size effects (two assets with same exposure to F but different sizes might have different betas)
- Multiple factors (stocks do not necessarily align)

Further questions

- Size effects (two assets with same exposure to F but different sizes might have different betas)
- Multiple factors (stocks do not necessarily align)
- Empirical work:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left(\mathbb{V}_t[Q_{t+1}]\right) + \mathbb{V}(\mathbb{E}_t[Q_{t+1}])$$

References I

Hansen, L. P. and S. F. Richard (1987). The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models. *Econometrica: Journal of the Econometric Society*, 587–613.

Roll, R. (1977). A critique of the asset pricing theory's tests part i: On past and potential testability of the theory. *Journal of financial economics 4*(2), 129–176.

Savor, P. and M. Wilson (2014). Asset pricing: A tale of two days. *Journal of Financial Economics 113*(2), 171–201.

Singleton, K. J. (1987). Asset prices in a time series model with disparately informed, competitive traders. *New Approaches to Monetary Economics*.

Townsend, R. M. (1983). Forecasting the forecasts of others. *The Journal of Political Economy*, 546–588.