The Lost Capital Asset Pricing Model

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Abstract

Just as the "lost city of Atlantis," the CAPM is empirically invisible most of the time. Yet, recent evidence shows that the CAPM relation becomes strongly statistically significant on days when important macroeconomic news is scheduled to be announced. In an economy with asymmetric information these two seemingly opposite findings coexist. Because investors hold different portfolios, the CAPM holds only for a fictitious investor whose beliefs define the market consensus. For the econometrician, who does not observe these beliefs nor the market portfolio, the unconditional CAPM relationship is "flat". On announcement days, when investors learn about macroeconomic factors to which stocks are exposed, fundamental risk better explains variations in asset returns, clearing the "shoal of mud" that stands in the way of the CAPM.

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1 Introduction

The Capital Asset Pricing Model, one of the main pillars of modern finance theory, fails in empirical tests. A common explanation for this empirical failure is the Roll (1977) critique—testing the CAPM is almost axiomatically impossible because the composition of the market portfolio is unobservable. While various attempts have been undertaken to alleviate this criticism, the CAPM relation is yet to be found in the data, largely shaping the view that it does not hold. However, recent evidence by Savor and Wilson (2014) shows that the CAPM relation becomes strongly statistically significant on days during which macroeconomic news are released, vanishing immediately right after. This fact is puzzling as it suggests that the CAPM behaves like a hidden "Atlantis" that unveils on particular occasions.

This paper is a theoretical attempt to understand why the CAPM fails most of the time, but holds on announcement days. To explain why the CAPM holds on announcements days only, a theory must be first consistent with the CAPM not holding on most days. A natural framework in which the CAPM does not hold is a rational model in which investors have asymmetric information. Information asymmetry implies that investors have different perceptions of the mean-variance frontier and thus hold different portfolios—there does not exist a unique portfolio that is mean-variance efficient to all investors. However, there exists a "market portfolio" resulting from the market-clearing condition, which requires that the supply of stocks be equal to the sum of investors' positions. A necessity for information asymmetry to be relevant in this framework is that this so-defined market portfolio be unobservable, thus sowing the seeds of the Roll (1977) critique right into the building block of the model.

Our main purpose requires that the model contain two elements. First, we assume that stock payoffs have a common, unobservable factor structure on which each asset loads differently. Second, we introduce periodic announcements through which public information is released about this common factor. Through this structure we allow macroeconomic announcements to commonly affect the cross-section of stocks. The rest of our setting is a standard stationary rational-expectations model with multiple assets (e.g., Spiegel, 1998), in which a continuum of mean-variance investors receive private signals about innovations in the common factor, which they use to infer current and past values of the factor and the supply of stocks.

Suppose now that an outside observer who holds unconditional beliefs—an econometrician—estimates a Security Market Line (SML) in this equilibrium model. Three features of the model compromise a test of the CAPM. First, the market portfolio is unobservable (Roll, 1977). The econometrician's best proxy for the market is its (unconditional) average, which biases the econometrician's estimates of a stock's true beta. Second, the information set on which investors condition their expectations is unobservable (Hansen and Richard, 1987; Dybvig and Ross, 1985). Conditioning down on a coarser information set further biases beta estimates. Third, aggregating investors' dispersed private information results in a violation of the law of iterated expectations (Allen, Morris, and Shin, 2006), which introduces another source of distortion in beta estimates. In general, without an equilibrium specification of stock excess returns, the bias resulting from these three features is merely a statistical effect that distorts the estimated security market line in an arbitrary way. In other words, "anything goes." By placing an endogenous structure on stock excess returns, our model provides economic restrictions—beyond statistical arguments—on the distortion of the SML.

In our model, the market-clearing condition requires that the market portfolio be mean-variance efficient in the eyes of the "average investor", a fictitious investor whose beliefs define the market consensus. The average investor sees the true Security Market Line, a line that crosses the origin with a slope given by the market risk premium. Since the average investor's beliefs result from aggregating all the information available in the economy, they differ from the econometrician's unconditional beliefs—they only coincide when investors are uninformed. The law of total covariance (the observed covariance must be the sum of the unexplained and explained covariances) allows us to quantify the impact of this difference in information sets on the econometrician's estimates of betas.

The main argument is that the econometrician, who holds unconditional beliefs and thus faces more uncertainty than does the average investor, perceives the comovement among assets differently than the average investor. This difference in uncertainty amplifies the dispersion in estimated betas relative to the dispersion of true betas. This effect is not merely an arbitrary statistical effect; in our model it results from equilibrium forces that place economic restrictions on this increase in the dispersion of betas. In equilibrium, assets with a high exposure to the common factor display a higher beta than the true beta, whereas assets with a low exposure to the common factor display a lower beta than the true beta. Because assets with high exposure have a beta higher than one and assets with low exposure have a beta lower than one, the econometrician's SML rotates clockwise around the market portfolio, flattening its slope and creating a positive intercept. Faced with a flat security market line, the ineluctable conclusion for the econometrician is to reject the CAPM.

The previous argument is based on the assumption that the econometrician does not have access to investors' information set. Suppose now that we allow the econometrician to estimate the CAPM conditioning on times during which investors likely receive large bulks of information. For instance, we allow the econometrician to estimate a CAPM relation conditioning on announcement days and non-announcement days, as in Savor and Wilson

(2014). Two effects arise during announcement days. First, due to uncertainty about the public announcement, investors require a high risk premium for holding the stocks right before announcement days. As a result, both market expected returns and realized returns are high during announcement days. Keeping betas of assets constant, this effect raises expected returns on all stocks, thus causing the SML to appear steeper. Second, and most importantly, because announcements provide additional information about macroeconomic fundamentals that drive the cross-section of asset payoffs, the covariance matrix of asset returns changes in a way that the market variance becomes more informative about fundamental risk on announcement days. That is, the common factor better explains variations in asset returns on announcement days. It follows that the econometrician's estimates of betas move closer to the true betas, causing the CAPM relation to look distinctly stronger.

On announcement days a stronger CAPM relation arises along with several other phenomena. In the model, the Sharpe ratio increases on the announcement day and macroeconomic uncertainty increases over a typical announcement cycle, two patterns that have been documented by Savor and Wilson (2014, 2016). We also extend the model to allow for multiple factors. We find that the econometrician's problem of estimating a security market line worsens in the multiple factor case: not only does the CAPM look flat, but the relation between average excess returns and betas is no longer linear. Finally, we allow for private information diffusion over the announcement cycle (Cieslak, Morse, and Vissing-Jorgensen, 2015), and find that the transmission of information has to take place exactly during the announcement day to be consistent with Savor and Wilson (2014).

While our results closely match the finding of Savor and Wilson (2014), we emphasize that the objective of this paper is not to provide a theoretical explanation to this particular observation. Rather, the purpose of this paper is to provide a broader view on what we should expect to happen when investors receive periodic public announcements about macroeconomic fundamentals. We argue that on most days a flat SML should be the rule rather than the exception, but that the CAPM relation should improve on announcement days. In the presence of information asymmetry, a flat SML originates from two well-known critiques. First, that the market portfolio is unobservable precludes a test of the CAPM, the so-called Roll (1977) critique. Second, that the information set on which agents condition their expectations is unobservable further complicates the task, a result sometimes referred to as the Hansen and Richard (1987) critique[1]. Finally, aggregating dispersed private information results in violating the law of total covariance, biasing betas estimates. We

[1]The issue of not being able to observe information sets and its implications for the SML has been first analyzed by Roll (1978). The "Hansen-Richard critique" designation belongs to Cochrane (2009), by analogy to the "Roll Critique." Along the same vein, Dybvig and Ross (1985) show that conditionally mean-variance returns from a managed portfolio might appear unconditionally mean-variance inefficient.

acknowledge that other possible explanations for a flat CAPM exist.[2] However, none try to jointly explain why the CAPM fails on most days, but holds on announcement days.

The remainder of the paper is organized as follows. Section 2 approaches the problem of conditioning down on a coarser information set in a simpler model and provides intuition on the distortion in beta measurement. Sections 3 and 4 present the main model and its equilibrium. Section 5 further elaborates on the distortion of the SML. Section 6 describes the effect of a periodic public announcement, Section 7 studies extensions of the model, and Section 8 concludes. All proofs are provided in the Appendix.

2 Background: The Effect of Conditioning Down

Conditioning down on a coarser information set (Dybvig and Ross, 1985; Hansen and Richard, 1987) results in mispricing and a distortion of unconditional beta estimates. However, without further economic context, this distortion is merely a statistical artifact; providing economic restrictions on this statistical effect requires an equilibrium specification of excess returns. Before building a full-fledged model of asset pricing, we provide economic intuition into this statistical distortion in a simpler context.

Consider two random variables (e.g., realized returns, or payoffs) D_1 and D_2, the values of which are driven by a common "factor" plus noise:

$$D \equiv \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \begin{pmatrix} 1+\phi \\ 1-\phi \end{pmatrix} F + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \equiv \Phi F + \epsilon, \tag{1}$$

where we assume that all random variables are independently normally distributed with zero means: $F \sim N\left(0, \tau_F^{-1}\right) \perp \epsilon \sim N\left(\mathbb{0}_{2\times 1}, \tau_\epsilon^{-1}\mathbb{I}_2\right)$.

Suppose that a continuum of economic agents know the structure of realized payoffs in Eq. (1), but do not observe the single common factor F.[3] Each agent i forms expectations about F based on both a private signal V_i and a public signal G:

$$V_i = F + \upsilon_i \tag{2}$$

$$G = F + \upsilon, \tag{3}$$

[2] Other reasons include leverage constraints (Black, 1972; Frazzini and Pedersen, 2014), inflation (Cohen, Polk, and Vuolteenaho, 2005), disagreement (Hong and Sraer, 2016), preference for volatile, skewed returns (Kumar, 2009; Bali, Cakici, and Whitelaw, 2011), market sentiment (Antoniou, Doukas, and Subrahmanyam, 2015), stochastic volatility (Campbell, Giglio, Polk, and Turley, 2012), and benchmarking of institutional investors (Baker, Bradley, and Wurgler, 2011; Buffa, Vayanos, and Woolley, 2014).

[3] In Section 7.1, we extend this static model to allow for multiple factors in assets' payoffs.

where the noise terms υ and υ_i are independently normally distributed: $\upsilon_i \sim N\left(0, \tau_\upsilon^{-1}\right), \forall i$; $\upsilon \sim N\left(0, \tau_G^{-1}\right); \quad \upsilon_i \perp \upsilon_j, \forall i \neq j$ and $\upsilon \perp \upsilon_i, \forall i$. Denoting by $\tau \equiv \tau_F + \tau_\upsilon + \tau_G$ the total precision available to agents, an agent i's expectations and posterior variances satisfy:

$$\mathbb{E}_i[D] = \frac{\tau_\upsilon}{\tau}\Phi V_i + \frac{\tau_G}{\tau}\Phi G \tag{4}$$

$$\mathbb{V}_i[D] = \frac{1}{\tau}\Phi\Phi' + \frac{1}{\tau_\epsilon}\mathbb{I}_2. \tag{5}$$

Consider now an "outside econometrician" who does not observe F nor signals about it and only observes the payoff realizations D. Because the econometrician's information set is coarser than that of economic agents in the model, her assessment of the covariance of realized payoffs is different. In particular, the law of total covariance implies

$$\mathbb{V}[D] = \mathbb{E}\left[\mathbb{V}_i[D]\right] + \mathbb{V}\left[\mathbb{E}_i[D]\right], \tag{6}$$

where the left-hand side represents the variance of the econometrician based on realizations of D, whereas the conditional expectation and conditional variance on the right-hand side are based on the information set of any agent i in the economy.

In this Gaussian framework the conditional variance in Eq. (5) is identical and constant for all agents, and represents the conditional variance of the "average agent" in the economy; we thus denote it by $\bar{\mathbb{V}}[D]$ hereafter. Aggregating over the continuum of agents further yields

$$\mathbb{V}[D] = \bar{\mathbb{V}}[D] + \int_i \mathbb{V}\left[\mathbb{E}_i[D]\right] di. \tag{7}$$

Because the expectation of the average agent and that of an agent i differ by the noise contained in an agent i's private signal,

$$\mathbb{E}_i[D] = \bar{\mathbb{E}}[D] + \Phi\frac{\tau_\upsilon}{\tau}\upsilon_i, \tag{8}$$

the law of total covariance relative to the average agent must incorporate an adjustment:

$$\mathbb{V}[D] = \bar{\mathbb{V}}[D] + \mathbb{V}\left[\bar{\mathbb{E}}[D]\right] + \frac{\tau_\upsilon}{\tau^2}\Phi\Phi'. \tag{9}$$

The adjusted law of total covariance in Eq. (9) shows that the average agent's covariance and that of the econometrician's differ for two reasons. First, and most importantly, the econometrician conditions on a coarser information set and thus obtains noisier estimates, which is reflected in the second term on the right-hand side. Second, dispersed private information introduces an adjustment that further biases econometrician's estimate, reflected in the third term on the right-hand side (an aspect on which we elaborate below).

Using the covariance matrix of the average agent and that of the econometrician, we can compute their respective beta estimates based on exogenous payoffs (we will compute beta estimates based on endogenous equilibrium returns shortly). Suppose that the "market portfolio" has equal weights in both assets; then betas satisfy

$$\beta = 2\frac{\bar{\mathbb{V}}[D]\iota'}{\iota\bar{\mathbb{V}}[D]\iota'} \text{ and } \tilde{\beta} = 2\frac{\mathbb{V}[D]\iota'}{\iota\mathbb{V}[D]\iota'}, \tag{10}$$

where ι denotes a vector of ones.

We are interested in the difference between these betas, which precisely represents the econometrician's "distortion" of betas relative to the average agent. We highlight this difference in Proposition 1.

Proposition 1. *The average agent computes the following betas*

$$\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} = \begin{bmatrix}1 + \frac{2\tau_\epsilon}{\tau+2\tau_\epsilon}\phi\\ 1 - \frac{2\tau_\epsilon}{\tau+2\tau_\epsilon}\phi\end{bmatrix}, \tag{11}$$

whereas the econometrician obtains the following betas

$$\begin{bmatrix}\tilde{\beta}_1\\ \tilde{\beta}_2\end{bmatrix} = \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} + \begin{bmatrix}1\\ -1\end{bmatrix} \underbrace{\frac{2\tau_\epsilon}{(\tau+2\tau_\epsilon)(\tau_F+2\tau_\epsilon)}\Delta_\tau\phi}_{\text{Distortion}}, \tag{12}$$

where $\Delta_\tau \equiv \tau - \tau_F > 0$ represents the "informational advantage" of the average agent relative to the econometrician.

When payoffs load on a common factor, conditioning on a coarser information set increases the dispersion in the betas measured by the econometrician. Specifically, the econometrician overestimates betas on stocks with high betas (the first asset in this example) and underestimates betas on stocks with low betas (the second asset). This effect is illustrated in panel (a) of Figure 1, in which the solid lines represent the distortion in betas, $\tilde{\beta}_i - \beta_i$, for the first asset (upward sloping line) and for the second asset (downward sloping line), respectively. Because the econometrician faces higher uncertainty ($\Delta_\tau > 0$) about the common factor relative to economic agents, she overstates the dependence of payoffs with high factor loadings and understates the dependence of those with low factor loadings.

Dispersed private information plays an additional, nontrivial role in distorting betas through its adjustment, the third term in the law of total covariance in Eq. (9). A customary result in models with dispersed information is that the law of iterated expectations fails (Allen

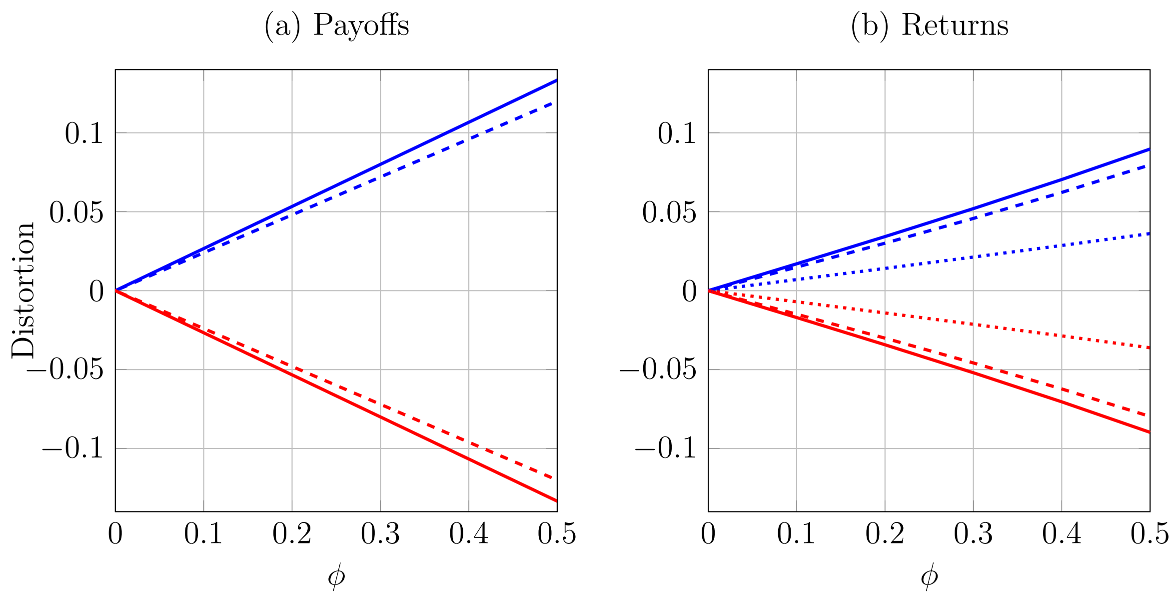

Figure 1: Beta distortion due to a coarser information set. Panel (a) shows the distortion in betas of payoffs, i.e. $\tilde{\beta}_i - \beta_i$, as described in Proposition 1. The blue solid (upward sloping) line correspond to asset 1 and the red solid (downward sloping line) corresponds to asset 2. Dashed lines are obtained by removing the higher order wedge (third term in Eq. (9)). Panel (b) shows the distortion in betas of returns with the convexity adjustment (solid lines) and without the convexity adjustment (dashed lines). The dotted lines are obtained by assuming a more precise public signal, i.e. $\tau_G = 10$ instead of $\tau_G = 1$. The calibration used for this figure is: risk aversion $\gamma = 1$ and the precision of all random variables is $\tau_F = \tau_\upsilon = \tau_G = \tau_\epsilon = \tau_X = 1$.

et al., 2006; Bacchetta and Wincoop, 2008). Importantly, in this example and throughout the paper, the law of iterated expectations fails from the viewpoint of the econometrician: the variance of individual expectations, $\mathbb{V}\left[\mathbb{E}_i[D]\right]$, differs from the variance of the first-order average expectation, $\mathbb{V}[\bar{\mathbb{E}}[D]]$. This wedge represents an additional source of covariation across payoffs in the econometrician's eyes. To illustrate its effect on betas, we simply remove the third term from Eq. (9) and plot the resulting betas (the dashed lines in panel (a) of Figure 1). Clearly, removing this higher-order effect attenuates the distortion in betas. Dispersed private information, thus, contributes to distort econometrician's beta estimates.

Our previous discussion relies on exogenous payoffs, as opposed to equilibrium returns. To place economic restrictions on the distortion in betas, we now solve for the linear equilibrium of this simple, static economy (Admati, 1985). To give an informational role to prices P in equilibrium, we make the customary assumption that the supply $X \equiv [X_1 \; X_2]'$ of assets is noisy: on average each asset still has equal weight $1/2$, but their weights are now randomly (normally) distributed with covariance matrix $\tau_X^{-1}\mathbb{I}_2$. We further assume that economic agents have utility over their terminal wealth with constant absolute risk aversion γ. For simplicity we normalize the risk-free rate to 0. We relegate all computations to Appendix A.1.

In equilibrium, the market-clearing condition creates an endogenous relation between the average agent's expectations $\bar{\mathbb{E}}[D]$ and the posterior variance $\bar{\mathbb{V}}[D]$, which we write in terms of excess returs, $Q \equiv D - P$:

$$\bar{\mathbb{E}}[Q] = \gamma \bar{\mathbb{V}}[Q] X. \tag{13}$$

Based on this relation we can further rewrite the law of total covariance in Eq. (9) in terms of excess returns as:

$$\mathbb{V}[Q] = \bar{\mathbb{V}}[Q] \left(\mathbb{I}_2 + \frac{\gamma^2}{\tau_X} \bar{\mathbb{V}}[Q] \right) + \frac{\tau_\upsilon}{\tau^2} \Phi \Phi'. \tag{14}$$

The customary assumption of noisy supply—necessary for prices to play an informational role—sows the seeds of the Roll (1977) critique right into the assumptions of the model. Specifically, in equilibrium, the market portfolio is unobservable both to economic agents and to the econometrician. Agents' expectations thus become noisy, which amplifies the posterior covariance matrix with the term in brackets in Eq. (14). Comparing again agents' betas and econometrician's betas, the solid lines in panel (b) of Figure 1 show the distortion that arises in equilibrium, due primarily to Roll's critique. As in panel (a), we drop the adjustment term associated with dispersed private information and obtain the dashed lines. The distortion in betas, thus, also appears for equilibrium returns, as this example shows.

In this example it may seem that the "econometrician" is overly unsophisticated, since her information set contains nothing but prices. As it turns out, allowing the econometrician to condition her estimates on public information does not change the result. Specifically, in Appendix A.1 we show that equilibrium returns take the following linear form:

$$Q = D - P = \Phi \frac{\tau_F}{\tau} F - \Phi \frac{\tau_G}{\tau} \upsilon + \epsilon - \begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \tag{15}$$

from which it follows that

$$\mathrm{Cov}[Q, G] = \Phi \frac{\tau_F}{\tau} \frac{1}{\tau_F} - \Phi \frac{\tau_G}{\tau} \frac{1}{\tau_G} = 0. \tag{16}$$

Allowing the econometrician to observe the public signal cannot possibly improve her estimates of the covariance matrix of returns. This is because prices already incorporate all public information available in the economy.

Although public announcements do not help the econometrician improve her beta estimates in a static model, they do in a dynamic model. Intuitively, in a dynamic model, periods with and without public announcements alternate, as opposed to a static model in which periods with and without announcement are mutually exclusive. As a result, price coefficients differ whether a scheduled announcement is imminent or whether it takes place in a distant future. In this context, the econometrician can benefit from these changes in price coefficients by estimating a CAPM relation separately on announcement and non-announcement days (Savor and Wilson, 2014). Although making this point requires a dynamic framework, panel (b) of Figure 1 offers a taste of the main result. Specifically, the dotted lines represent the distortion in betas when the public signal becomes more precise. Clearly, an increase in the precision of public information improves the econometrician's estimate of beta. We now make this broader point in a dynamic framework with scheduled public announcements.

3 Model

In this section we introduce periodic public announcements and a factor structure within a standard stationary rational-expectations equilibrium model with multiple assets (e.g. Spiegel, 1998; Watanabe, 2008).

3.1 The Economy

We consider a discrete-time economy that goes on forever. The economy is populated with a continuum of investors indexed by $i \in [0, 1]$, who have CARA utility with common risk aversion γ. Each investor i lives for two periods, entering period t with wealth W_t^i and consuming W_{t+1}^i next period. There are N risky assets (stocks) and an exogenous riskless bond with constant gross interest rate $R > 1$. At each period t the N stocks pay a vector D of dividends with a *common factor structure*:

$$D_t = \bar{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D, \tag{17}$$

where $\mathbb{1}_N$ a vector with all N components being 1, $\epsilon_t^D \sim N(\mathbf{0}_{N\times 1}, \sigma_D^2 \mathbb{I}_N)$ is an i.i.d. asset-specific innovation, $\mathbb{I}_N$ is the identity matrix of dimension N, and F denotes a factor that commonly affects all dividends on all stocks. The extent to which dividends of a given stock load on this factor is determined by the vector Φ of loadings. Without loss of generality and for ease of interpretation, we assume that the ordering of the vector of loadings is strictly ascending so that stock $n = 1$ has the smallest loading and stock $n = N$ has the highest loading. These loadings are common knowledge to all agents in the economy.[4]

The factor F_t is assumed to mean-revert over time around zero according to

$$F_t = \kappa_F F_{t-1} + \epsilon_t^F, \quad \epsilon_t^F \sim N(0, \sigma_F^2), \text{ i.i.d.,} \tag{18}$$

with $0 \leq \kappa_F \leq 1$. Furthermore, as is customary in the noisy rational expectations literature, the per-capita supply of stocks is stochastic and evolves according to:

$$X_t = (1 - \kappa_X)\bar{X} + \kappa_X X_{t-1} + \epsilon_t^X, \quad \epsilon_t^X \sim N(\mathbb{0}_N, \sigma_X^2 \mathbb{I}_N), \text{ i.i.d.,} \tag{19}$$

where $\bar{X}$ is a vector of dimension N with equal elements that sum up to one. Supply shocks prevent prices from fully revealing investors' private information.

We assume that there are overlapping generations of investors who live for two periods. Investors maximize their expected utility over their next period wealth by optimally building a portfolio with the N risky assets and the riskless bond. Specifically, investor i chooses a vector x_t^i of holdings in the N stocks at time t to maximize

$$\max_{x_t^i} \mathbb{E}_t^i \left[-\exp\left(-\gamma W_{t+1}^i \right) \right] \tag{20}$$

subject to

$$W_{t+1}^i = \left(W_t^i - (x_t^i)' P_t \right) R + (x_t^i)' \left(P_{t+1} + D_{t+1} \right), \tag{21}$$

where P_t denotes the vector of ex-dividend prices. We follow the literature and focus on linear equilibria in which the price P is a linear function of the state variables of the economy.[5] Because these state variables are Gaussian and because investors are myopic and have CARA utility, their demands take the following standard form:

$$x_t^i = \frac{1}{\gamma} \left(\mathbb{V}_t^i \left[P_{t+1} + D_{t+1} \right] \right)^{-1} \mathbb{E}_t^i \left[P_{t+1} + D_{t+1} - R P_t \right]. \tag{22}$$

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[4] We consider a single factor for simplicity of exposition. The model can accomodate a vector of multiple factors F_t.

[5] With N assets, there are 2^N linear equilibria in this model. We focus on the low-volatility equilibrium, as this is the only stable equilibrium to which a finite horizon economy would converge. Bacchetta and Wincoop (2008), Banerjee (2010), and Watanabe (2008) discuss the multiplicity of equilibria in infinite horizon models.

A linear equilibrium then results from imposing the market-clearing condition

$$\int_{i\in[0,1]} x_t^i \mathrm{d}i = X_t. \tag{23}$$

3.2 Periodic Public Announcements

We introduce periodic public announcements in the model by assuming that every T periods a new public signal centered on the fundamental F is available to all investors, according to the sequence illustrated in Figure 2.

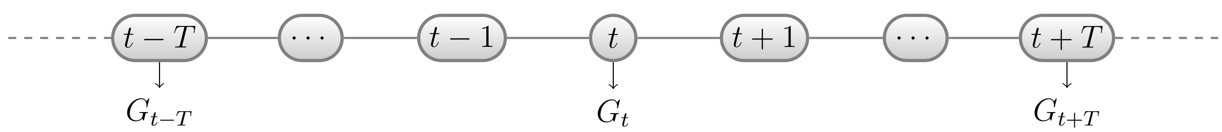

Figure 2: Timeline of periodic public announcements.

Without loss of generality, we make the convention that a public announcement is made at time t. Then for any date $t-k$, with $k \in \{-T+1, ..., T\}$ the public announcements available to investors take the form:

$$G_{t-k} = \begin{cases} F_{t-T} + \epsilon_{t-T}^G, & \forall k \in \{1, ..., T\} \\ F_t + \epsilon_t^G, & \forall k \in \{-T+1, ..., 0\}, \end{cases} \tag{24}$$

with $\epsilon_{t-T}^G, \epsilon_t^G \sim N(0, \sigma_G^2)$ and independent of each other.

In addition to public announcements, we assume that all investors commonly observe the fundamental at lag T and beyond (Townsend, 1983; Singleton, 1987). We make this assumption for tractability—without it the information structure would introduce an infinite-regress inference problem whereby investors would need to infer unobservables shocks over infinitely many periods back in time. This assumption serves to eliminate this infinite-regress problem: at any time s, the fundamental F_{s-T} becomes public information and thus investors only need to infer unobservable shocks up to lag $T-1$.

Investors observe private information at each trading date. Formally, at any date s, each investor i receives a private signal v^i about the current factor innovation:

$$v_s^i = \epsilon_s^F + \epsilon_s^i, \quad \epsilon_s^i \sim N(0, \sigma_v^2), \;\; \epsilon_s^i \perp \epsilon_s^k, \;\; \forall k \neq i. \tag{25}$$

Finally, as is customary in the literature, all investors observe current and past dividends, along with current and past prices. Specifically, we conjecture that prices at time $t-k$, with

$k \in \{0, ..., T-1\}$, take the following linear form:

$$P_{t-k} = \bar{\alpha}_k \bar{D} + \alpha_k F_{t-k-T} + \bar{\xi}_k \bar{X} + \xi_k X_{t-k-T} + d_k \tilde{D}_{t-k} + g_k G_{t-k} + a_k \tilde{\epsilon}^F_{t-k} + b_k \tilde{\epsilon}^X_{t-k}, \qquad (26)$$

where we use the following notation[6]:

- $\tilde{D}_{t-k}$ is a stacked vector of dimension NT which includes the payoffs of all N assets from time $t-k-T+1$ to time $t-k$.
- $\tilde{\epsilon}^F_{t-k}$ is a vector of dimension T containing all the fundamental shocks ϵ^F from time $t-k-T+1$ to time $t-k$.
- $\tilde{\epsilon}^X_{t-k}$ is a stacked vector of dimension NT containing the supply shocks for all assets from time $t-k-T+1$ to time $t-k$.
- The price coefficients $\bar{\alpha}_k$, α_k, $\bar{\xi}_k$, ξ_k, d_k, g_k, a_k and b_k are scalars/vectors/matrices of conformable dimension.

To understand the price structure (26), notice that any investor i observes at date $t-k$, $k \in \{0, ..., T-1\}$:

$$\left\{\{P_s\}_{s \le t-k}, \{D_s\}_{s \le t-k}, \{F_s\}_{s \le t-k-T}, \{v^i_s\}_{s \le t-k}\right\}. \qquad (27)$$

It follows that an investor i also observes $\{\epsilon^F_s\}_{s \le t-k-T}$, hence the vector of innovations $\tilde{\epsilon}^F_{t-k}$ in the common factor. Furthermore, in this dynamic setup the sequence of past dividends $\{D_{t-k-s}\}_{s=0}^{T-1}$ reveals information regarding past, unobservable factor innovations $\{\epsilon^F_{t-k-s}\}_{s=0}^{T-1}$, hence the vector $\tilde{D}_{t-k}$. Finally, to understand the structure of the vector of past supply innovations, consider the price at time $t-k-T$. This price reveals a linear combination of $\{\epsilon^F_{t-k-s}\}_{s=0}^{T-1}$ and X_{t-k-T}. Since these factor innovations are observable, so is X_{t-k-T}, hence the vector $\tilde{\epsilon}^X_{t-k}$ and the presence of F_{t-k-T} and X_{t-k-T} in the equilibrium price.

4 Equilibrium

In this section we solve for an equilibrium of the model. We start by solving investors' learning problem (Section 4.1) and then clear the market to obtain the undetermined price coefficients (Section 4.2).

[6]A detailed representation of the linear form (26) can be found in equation (78) of Appendix A.2.

4.1 Learning

When building her portfolio position, an investor i must form expectations about future excess returns next period, as apparent from equation (22). For the purpose of forecasting future excess returns, it is sufficient to form expectations about the vector $\tilde{\epsilon}^F$ of innovations in the common factor. We thus first compute an investor i' conditional expectations $\mathbb{E}_t^i[\tilde{\epsilon}_t^F]$ and conditional variance $\mathbb{V}_t^i[\tilde{\epsilon}_t^F]$ at each trading date during the periodic announcement cycle.

At time $t-k$, with $k \in \{0, ..., T-1\}$, an investor i's information set contains four sources of information regarding the vector $\tilde{\epsilon}_{t-k}^F$: past and current prices, private signals, dividends, and public announcements. Starting with past and current prices, notice from equation (26) that they reveal a combination of observable and unobservable information; accordingly, an investor i can isolate the informational part of prices P_{t-k}^a that only contains unobservables:

$$P_{t-k}^a = a_k \tilde{\epsilon}_{t-k}^F + b_k \tilde{\epsilon}_{t-k}^X, \quad k \in \{0, ..., T-1\}, \tag{28}$$

which we stack into a single vector $\tilde{P}_{t-k}^a$ of dimension NT:

$$\tilde{P}_{t-k}^a \equiv \begin{pmatrix} P_{t-k}^a & P_{t-k-1}^a & \dots & P_{t-k-T+2}^a & P_{t-k-T+1}^a \end{pmatrix}'. \tag{29}$$

Similarly, we collect all past and current private signals that investor i has gathered by time $t-k$ into a vector V_{t-k}^i of dimension T:

$$V_{t-k}^i \equiv \begin{pmatrix} v_{t-k}^i & v_{t-k-1}^i & \dots & v_{t-k-T+1}^i \end{pmatrix}'. \tag{30}$$

Furthermore, because we assume that the common factor F becomes public information after T periods, dividends and public announcements also reveal a combination of observable and unobservable information. In particular, the dividend at time $t-k$ can be written as

$$D_{t-k} = \bar{D}\mathbb{1}_N + \Phi\left(\kappa_F^T F_{t-k-T} + \sum_{\tau=0}^{T-1} \kappa_F^\tau \epsilon_{t-k-\tau}^F\right) + \epsilon_{t-k}^D \tag{31}$$

$$= \bar{D}\mathbb{1}_N + \Phi\kappa_F^T F_{t-k-T} + D_{t-k}^a, \tag{32}$$

where D_{t-k}^a represents the informational part of dividends that only contains unobservables, which we stack into a single vector $\tilde{D}_{t-k}^a$ of dimension NT:

$$\tilde{D}_{t-k}^a \equiv \begin{pmatrix} D_{t-k-T+1}^a & D_{t-k-T+2}^a & \dots & D_{t-k}^a \end{pmatrix}'. \tag{33}$$

Public announcements G_{t-k} have a similar structure:

$$G_{t-k} = \kappa_F^{k+j(k)} F_{t-k-T} + \sum_{\tau=0}^{k+j(k)-1} \kappa_F^{\tau} \epsilon_{t+j(k)-T-\tau}^{F} + \epsilon_{t+j(k)-T}^{G} \tag{34}$$

$$= \kappa_F^{k+j(k)} F_{t-k-T} + G_{t-k}^{a} \tag{35}$$

where G_{t-k}^a represents the part of public signals that only contains unobservables and the indexing function $j(k)$ equals T if $k = 0$ and 0 otherwise.

Overall, an investor i's information at time $t-k$ is fully summarized by $\tilde{P}_{t-k}^a$, V_{t-k}^i, $\tilde{D}_{t-k}^a$, and G_{t-k}^a. Since the vector grouping this information and the vector $\tilde{\epsilon}_{t-k}^F$ are jointly normally distributed, investor i forms her conditional expectations $\mathbb{E}_t^i[\tilde{\epsilon}_t^F]$ and conditional variance $\mathbb{V}_t^i[\tilde{\epsilon}_t^F]$ by projecting the former vector on the latter. We write the conditional expectation and conditional variance in Proposition 2.

Proposition 2. *At time $t-k$ an investor i's conditional expectation and variance of the fundamental innovations $\tilde{\epsilon}_{t-k}^F$ satisfy*

$$\mathbb{E}_{t-k}^i \left[\tilde{\epsilon}_{t-k}^F\right] = \underbrace{\sigma_F^2 H_k' \left(\sigma_F^2 H_k H_k' + R_k\right)^{-1}}_{\equiv M_k} \begin{pmatrix} \tilde{P}_{t-k}^a \\ V_{t-k}^i \\ \tilde{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} \tag{36}$$

and

$$\mathbb{V}_{t-k}^i \left[\tilde{\epsilon}_{t-k}^F\right] = \sigma_F^2 \left(\mathbb{I}_T - M_k H_k\right) \tag{37}$$

where the matrices H_k and R_k are defined in Appendix A.3.

Equation (36) shows how investors form expectations about the current and past innovations in the common factor F. When forecasting fundamental innovations, investors use all the public information available (including current and past prices, current and past dividends, and the periodic public signal), but also their private information gathered up to time $t-k$. The posterior variance of $\tilde{\epsilon}_{t-k}^F$ equals $\sigma_F^2 \mathbb{I}_T$ when investors have no information, but changes as soon as investors learn from private and public signals, as shown in equation (37).

Based on the conditional expectations of Proposition 2, investor i can forecast future excess returns next period, $P_{t-k+1} + D_{t-k+1}$. The conditional expectation and the conditional variance of future excess returns, $\mathbb{E}_{t-k}^i \left[P_{t-k+1} + D_{t-k+1}\right]$ and $\mathbb{V}_{t-k}^i \left[P_{t-k+1} + D_{t-k+1}\right]$, completely determine investor i's portfolio strategy, as described in equation (22). We relegate these expressions in Appendix A.3, and now aggregate investment strategies over the population of investors in order to clear the market and obtain equilibrium prices.

4.2 Equilibrium Pricing

We first rewrite individual demands in (22) at each lag k in the public announcement cycle:

$$x^i_{t-k} = \frac{1}{\gamma}\mathbb{V}^i_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right]^{-1}\left(\mathbb{E}^i_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right] - RP_{t-k}\right), \tag{38}$$

and define average market expectations at time $t-k$ as

$$\bar{\mathbb{E}}_{t-k}[P_{t-k+1} + D_{t-k+1} - RP_{t-k}] \equiv \int_{i\in[0,1]} \mathbb{E}^i_{t-k}[P_{t-k+1} + D_{t-k+1} - RP_{t-k}]\mathrm{d}i. \tag{39}$$

Furthermore, the precision of information (public and private) is the same across investors and thus we can write

$$\mathbb{V}_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right] \equiv \mathbb{V}^i_{t-k}[P_{t-k+1} + D_{t-k+1} - RP_{t-k}]. \tag{40}$$

The pair $(\bar{\mathbb{E}}, \mathbb{V})$ can be interpreted as the beliefs of a fictitious agent—the average investor—who determines the average market consensus in the economy. Using the average beliefs from (39) and (40), the market-clearing condition yields:

$$\bar{\mathbb{E}}_{t-k}\left[P_{t-k+1} + D_{t-k+1} - RP_{t-k}\right] = \gamma\mathbb{V}_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right]X_{t-k}, \tag{41}$$

which defines the conditional pricing relation that prevails in the model at each date during the public announcement cycle. Stock prices are then

$$P_{t-k} = \frac{1}{R}\bar{\mathbb{E}}_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right] - \frac{\gamma}{R}\mathbb{V}_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right]X_{t-k}. \tag{42}$$

Stock prices equal the present value of the future prices and payoffs, discounted at the risk free rate, minus a risk premium term which is proportional to the supply of assets. Sensibly, investors demand a higher risk premium to hold assets that are in large supply, and the magnitude of this risk premium term is dictated by the risk aversion coefficient γ.

We conclude the equilibrium derivation by verifying that the pricing relation in equation (41) is consistent with the price conjecture in equation (26).

Proposition 3. *In equilibrium prices take the linear form in equation* (26) *where the pricing relation in equation* (41) *implies that the coefficients* $\overline{\alpha}$, α, $\overline{\xi}$, ξ, d, g, a *and* b *solve a nonlinear* *system of equation that we provide in Appendix A.4.*

5 Distortion of the Security Market Line

In this section we estimate a security market line adopting the approach of an empiricist. In this model the true security market line is defined relative to the information set of the average agent in the economy. Our main finding is that the security market line that the empiricist estimates is flatter than the true security market line.

We start by defining excess returns as

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t, \tag{43}$$

and then apply the law of total covariance, which relates the covariance of realized excess returns (as measured by an outside observer) with the conditional variance of future excess returns (as perceived by the average agent):

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right] + \mathbb{V}\left[\bar{\mathbb{E}}_t[Q_{t+1}]\right] + \sigma_v^2 \Pi\Phi\Phi'\Pi', \tag{44}$$

where Π is a 3×3 matrix that multiplies the vector of private signals in the learning problem of each agent.

The left-hand side of Eq. (44) represents the unconditional covariance matrix, estimated from realized returns. The three terms on the right-hand side account for all sources of observed covariation in excess returns: the conditional covariance matrix of agents, the variance in expected excess returns of the average agent, and the convexity adjustment associated with the aggregation of private information. Although the econometrician observes none of these terms, the market-clearing condition imposes a relation between the conditional expectation and the conditional variance of excess returns, as shown in (41), which allows us to replace the last term in (44):

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right] + \gamma^2 \mathbb{V}\left[\mathbb{V}_t\left[Q_{t+1}\right] X_t\right] + \sigma_v^2 \Pi\Phi\Phi'\Pi'. \tag{45}$$

The relation in Eq. (45) reflects three different critiques regarding tests of equilibrium asset pricing models. First, the composition of the market portfolio X_t is unobservable, making tests of the CAPM impossible (Roll, 1977). Second, the econometrician does not observe the information on which investors condition their expectations to build their portfolios (Hansen and Richard, 1987). Third, dispersed private information introduces a positive-definite convexity adjustment term, an effect that would be absent in an economy with hierarchical private information or with a representative agent. This additional adjustment may further distort the perception of the security market line and bias the estimates of stock betas. An equilibrium analysis places an endogenous structure on excess returns, thus providing an economic interpretation to the bias resulting from each critique; it also provides economic restrictions on the direction and magnitude of this bias.

Considering that the conditional variance of future excess returns $\mathbb{V}_t\left[Q_{t+1}\right]$ moves over the public announcement cycle (it varies with each lag k), and that its fluctuations are independent of supply shocks, we can decompose Equation (45) as:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right] + \frac{\gamma^2\sigma_X^2}{1-\kappa_X^2}\mathbb{E}\left[\left(\mathbb{V}_t[Q_{t+1}]\right)^2\right] + \sigma_v^2\Pi\Phi\Phi'\Pi'. \quad (46)$$

Two effects become apparent from this relation. First, supply shocks distort the measurement of the average conditional matrix of future excess returns. While agents in the economy obtain on average $\mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right]$, the econometrician only observes the left-hand side. Without supply shocks ($\sigma_X = 0$), the econometrician's view and that of economic agents would coincide. However, because supply shocks—unobservable to both the econometrician and investors—generate fluctuations in expected returns, the unconditional covariance matrix of realized returns differs from that covariance matrix conditioned on information available to investors.

Furthermore, public announcements generate fluctuations in investors' conditional variance of future excess returns. We can isolate the effect of these fluctuations by writing

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right] + \frac{\gamma^2\sigma_X^2}{1-\kappa_X^2}\left(\mathbb{E}\left[\mathbb{V}_t[Q_{t+1}]\right]\right)^2 + \frac{\gamma^2\sigma_X^2}{1-\kappa_X^2}\mathbb{V}\left[\mathbb{V}_t[Q_{t+1}]\right] + \sigma_v^2\Pi\Phi\Phi'\Pi'. \quad (47)$$

Clearly, without public announcements, investors' conditional variance is constant and thus the third term in the decomposition above vanishes. With public announcements, however, this term further distorts empiricist's measurement of the covariance of asset returns.

We can now quantify the distortion on the security market line implied by these two effects. Since the public announcement cycle contains T different days, we can explicitly write the expectations in (46) as follows:

$$\mathbb{V}[Q_{t+1}] = \frac{1}{T}\sum_{k=0}^{T-1}\mathbb{V}_{t-k}[Q_{t-k+1}]\left(\mathbb{I}_N + \frac{\gamma\sigma_X^2}{1-\kappa_X^2}\mathbb{V}_{t-k}[Q_{t-k+1}]\right) + \sigma_v^2\Pi\Phi\Phi'\Pi'. \quad (48)$$

The unconditional covariance matrix of realized excess returns is an average of a linear-quadratic function of the covariance matrix conditioned on investors' information at each lag k of the public announcement cycle. This functional form has several implications for the unconditional CAPM relation that the econometrician observes.

Notice first that the equilibrium relation (41) implies that the market portfolio X_{t-k} is mean-variance efficient for the "average investor" with beliefs $(\bar{\mathbb{E}}_{t-k}\left[Q_{t-k+1}\right], \mathbb{V}_{t-k}\left[Q_{t-k+1}\right])$ and risk aversion γ. This observation raises two conceptual issues. First, the average investor is not part of the model. Second and most importantly, holding the market portfolio is not even feasible, since the supply is unobservable. However, holding the *average market portfolio*, $\bar{X}$, is a strategy that all investors in the model can implement. Thus, after taking the unconditional expectation of the pricing relation in equation (41) and performing a few standard manipulations, we obtain an unconditional CAPM relation that holds from the perspective of an average investor who holds the average market portfolio:

$$\mathbb{E}[Q_{t+1}] = \frac{\bar{\mathbb{V}}\left[Q_{t+1}\right]\bar{X}}{\bar{\mathbb{V}}\left[M_{t+1}\right]}\mathbb{E}[M_{t+1}], \tag{49}$$

where $M_{t+1} \equiv \bar{X}'Q_{t+1}$ is the realized return of the market portfolio and $\bar{\mathbb{V}}[\cdot]$ represents the unconditional average over all the $\mathbb{V}_{t-k}[\cdot]$ for all lags k in the announcement cycle. The ratio

$$\beta \equiv \frac{\bar{\mathbb{V}}\left[Q_{t+1}\right]\bar{X}}{\bar{\mathbb{V}}\left[M_{t+1}\right]} \tag{50}$$

represents the vector of betas as measured by the average investor in this economy. It then follows from equation (49) that the CAPM holds unconditionally for the average investor, who sees a security market line that crosses the origin with a slope given by the market risk premium.

The econometrician, instead, can only measure the unconditional variance of returns and thus obtains the following vector of betas:

$$\tilde{\beta} = \frac{\mathbb{V}\left[Q_{t+1}\right]\bar{X}}{\mathbb{V}\left[M_{t+1}\right]}, \tag{51}$$

which, in light of our discussion of the law of total variance in Eq. (44), differs from the vector of true betas in equation (50). Propositions 4 and 5 characterize this difference.

Proposition 4. *In an economy without public announcements, the econometrician overestimates betas that are higher than one and underestimates betas that are lower than one:*

$$\tilde{\beta}_i < \beta_i, \;\; \forall \tilde{\beta}_i < 1 \quad \textit{and} \quad \tilde{\beta}_i \geq \beta_i, \;\; \forall \beta_i \geq 1 \tag{52}$$

Proposition 5. *The introduction of a periodic public signals lowers the unconditional risk premium in the economy but further distorts econometrician's betas.*

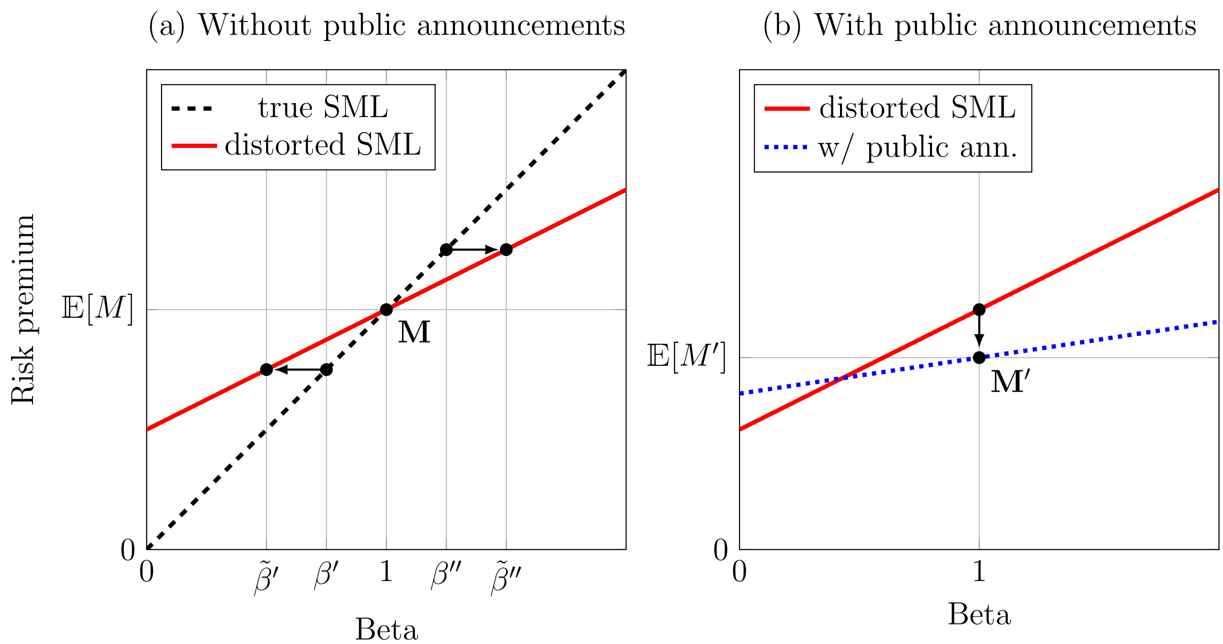

Figure 3: Distortion of the Security Market Line. The left panel illustrates the true SML (dashed black line) and the SML perceived by the econometrician (solid red line) in an economy without public announcements. The econometrician overestimates betas higher than one and underestimates betas lower than one. The right panel depicts the additional distortion of the SML induced by a periodic public announcement. While the overall unconditional market excess return is lower with public announcements, fluctuations in the conditional variance of asset returns further flattens the SML (dotted blue line).

Figure 3 illustrates the results of Propositions 4 and 5. The left panel depicts an economy without public announcements. The dashed line is the true CAPM relation (49) that holds for the average agent. A direct implication of Proposition 4 is that the SML that the econometrician perceives is flatter than the actual SML—it rotates clockwise around the market portfolio (since the true beta of the market portfolio is 1 and that both the average investor and the econometrician see the same unconditional market returns, the beta measured by the econometrician must also be equal to one). It follows that the true security market line (the dashed black line) and that perceived by the econometrician (the red solid line) must cross exactly at the market portfolio, which we denote by the point $\mathbf{M} = (1, \mathbb{E}[M])$. Since betas lower than one are underestimated and beta higher than one are overestimated, the SML rotates downwards, thus flattening its slope and creating a positive intercept.

The economic intuition for this result is the same as that in the simpler setting of Section 2; Starting from Eq. (48) and ruling out public announcements, the covariance matrix of the average investor, $\mathbb{V}_t[Q_{t+1}]$, is constant and the relationship becomes

$$\mathbb{V}[Q_{t+1}] = \mathbb{V}_t[Q_{t+1}] \left(\mathbb{I}_N + \frac{\gamma \sigma_X^2}{1-\kappa_X^2} \mathbb{V}_t[Q_{t+1}] \right) + \sigma_v^2 \Pi \Phi \Phi' \Pi'. \tag{53}$$

Another way of looking at this relation is through an eigendecomposition of the covariance matrix of the average investor. We denote this eigendecomposition as $\mathbb{V}_t[Q_{t+1}] = Q \Lambda Q^{-1}$ where Q is an orthogonal $(N \times N)$ matrix with eigenvectors as columns and Λ is a diagonal matrix with positive eigenvalues on the diagonal. Substituting this decomposition into (53) and ignoring momentarily the convexity adjustment term (which, being positive definite, further magnifies our results), we obtain another relation between the two covariances, but in terms of eigenvalues and eigenvectors:

$$\mathbb{V}\left[Q_{t+1}\right] = Q \Lambda \underbrace{\left(\mathbf{I} + \frac{\gamma^2 \sigma_X^2}{1-\kappa_X} \Lambda \right)}_{\Theta \geq \mathbb{I}_N} Q^{-1}. \tag{54}$$

This relation shows that the econometrician's and investor's covariance matrices have the same eigenvectors, but different eigenvalues. In particular, the eigenvalues of the econometrician's covariance matrix are larger. Because the econometrician holds unconditional beliefs and thus faces higher overall uncertainty than the average investor (as discussed in Section 2). This effect increases the dispersion of betas: assets with a higher exposure to the common factor display a higher beta than the true beta (which presumably is higher than one), whereas assets with a lower exposure to the common factor display a lower beta than the true beta (which presumably is lower than one), thus flattening the SML. More precisely, after a few manipulations we can write:

$$Q^{-1} \tilde{\beta} = \frac{\bar{\mathbb{V}}_t[M_{t+1}]}{\mathbb{V}[M_{t+1}]} \Theta Q^{-1} \beta, \tag{55}$$

with Θ defined in Eq. (54). That is, the model provides a relationship between betas in a "rotated" space, with the rotation matrix being Q. We can then use linear algebra arguments to show that agents' betas that are higher than one are overestimated by the econometrician, whereas agents' betas lower than one are underestimated by the econometrician.[7]

Finally, the right panel of Figure 3 shows that a second force, specific to our model with periodic public announcements, further distorts the SML. The solid line is the same

[7] In the relation (55), the fraction $\bar{\mathbb{V}}_t[M_{t+1}]/\mathbb{V}[M_{t+1}]$ represents the inverse of the generalized Rayleigh quotient pertaining to the set of eigenvalues given by Θ. Since the Rayleigh quotient is always between the maximum and minimum eigenvalue, the resulting vector will have both elements higher and lower than one, dilating the space of new betas and therefore increasing their dispersion.

in both panels and represents the SML in an economy without public announcements. In an economy with a periodic public announcement (and thus with more information), agents require lower returns to hold the assets, which lowers the *unconditional* risk premium in the economy. The market portfolio therefore moves down towards point $\mathbf{M}' = (1, \mathbb{E}[M'])$. However, periodic public announcements generate time variation in investors' conditional volatility of future excess returns. This variation materializes in the third term in Eq. (47), which further distorts the SML measured by the econometrician, as shown by the dotted line in the right panel of Figure 3.

6 CAPM on Announcement Days

The analysis so far has been focused on the unconditional CAPM relationship. We now focus on the conditional CAPM, more precisely on its behaviour during announcement cycles. From now onwards the analysis involves solving our theoretical model. We choose to show results in a model where we fix the number of assets at $N = 4$ and we assume that a public announcement arises every 4 periods.[8] This will cause the SML to move depending on whether the econometrician measures it during announcement days or during non-announcement days.

Figure 4 illustrates two effects. First, the left panel shows the unconditional CAPM during all days. The dashed line represents the true SML, whereas the solid line is distorted according to our analysis in Section 5. The SML flattens and has an intercept greater than zero—in other words, the SML is pivoting clockwise around the point $(1, \mathbb{E}[M_{t+1}])$, which leads the econometrician to conclude that the CAPM does not hold in this economy. The panel also shows the true betas (four dots on the dashed line) and econometrician's betas (four triangles on the solid line). As explained in Section 5, the econometrician underestimates betas lower than one and overestimates betas higher than one, and thus low-beta assets appear to deliver high average returns and low risk relative to high-beta stocks.

The second effect is illustrated in the right panel and shows the SML estimated by the econometrician during three types of days: all days (*all-days*, solid blue line—this is the same line as the solid line in the left panel, but scaling is different), announcement days (*a-days*, dashed line), and non-announcement days (*n-days*, dotted line). We follow the methodology in Savor and Wilson (2014) and estimate the betas over the whole sample. The three lines pivot around a common point with positive beta and positive expected returns. This point corresponds to a portfolio whose exposure to the factor F is $\phi = 0$ and thus is not affected

[8] Other parameters are: $\gamma = 2$, $\kappa_F = \kappa_X = 0.99$, $\phi_1 = 1$, $\phi_2 = 1/2$, $\phi_3 = 1/3$, $\phi_4 = 1/4$, $\sigma_F = \sigma_X = 0.06$, $\sigma_D = 0.32$, $\sigma_G = 0.001$, $\overline{X} = 1$, $\overline{D} = 0$, and $R = 1.22$.

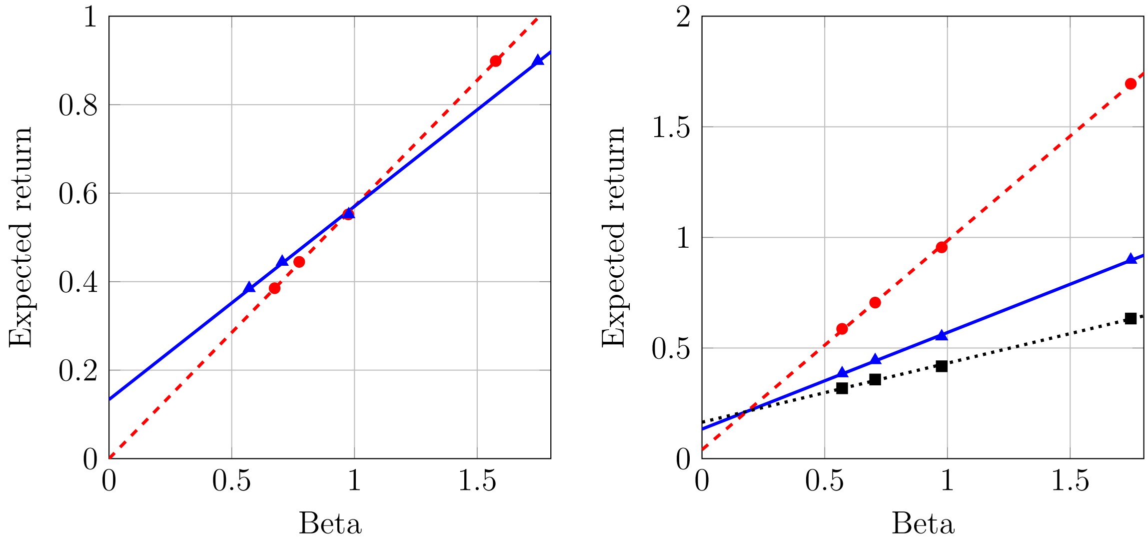

Figure 4: CAPM during announcement and non-announcement days. The left panel shows the SML distortion explained in Section 5, in line with Figure 4. The right panel shows the SML during all days (solid), announcement days (dashed), and non-announcement days (dotted).

	all-days	*a-days*	*n-days*
Average market excess return	0.57	0.98	0.43
Market volatility	0.39	0.42	0.26
Sharpe ratio	1.46	2.36	1.64

Table 1: Average market excess returns, volatility and Sharpe ratio on three different days.

about any macroeconomic information. Furthermore, on announcement days the SML slope steepens and as a result the econometrician observes a stronger CAPM relationship (Savor and Wilson, 2014).

The steepening of the CAPM during *a-days* can be understood as follows. Due to uncertainty about the public announcement, agents require a high risk premium to hold the asset at the beginning of the announcement day. Thus, both the market expected return and realized return are high during *a-days*. Holding betas of assets constant, this makes all expected returns higher and thus the overall SML appears steeper (and closer to the true SML).[9] On the contrary, on *n-days*, the agents face less uncertainty about upcoming announcements and thus all assets' expected returns are lower, which further flattens the SML (dotted line in the left panel of Figure 4) and increases its intercept.

[9] This result does not depend on holding the betas constant. In separate calculations, we estimate different betas on different types of days. Although there is variation in betas across types of days, the same steepening/flattening obtains during *a-days*/*n-days*. See Savor and Wilson (2014) for a discussion of variation of betas across types of days.

Table 1 further confirms this. The first line shows the average market excess returns during the three types of days. As expected, average excess returns are higher during *a-days*. Although the market volatility follows a similar pattern and increases during announcement days, it does so less than the average market excess returns. This results in much higher Sharpe ratios during announcement days, as documented by Savor and Wilson (2016).

Figure 5 shows the evolution of the realized return volatility over the announcement cycle. Dates on the right axis represent trading days before the announcement, which takes place at date 0. Realized volatility is highest during the announcement day, but also increases before. This increase in volatility before the actual public announcement is generated by changes in investor's expectations about the future value of stocks (He and Wang, 1995). That is, although at time $t-2$ investors do not expect a public announcement next period, they know that price functions at $t-1$ will change in order to anticipate the upcoming public announcement. Price functions at $t-2$ change through this expectation channel and thus volatility increases before the public announcement.

Figure 5 also shows the evolution of the macroeconomic uncertainty over the announcement cycle. We define macroeconomic uncertainty as the uncertainty perceived by agents when trying to forecast market returns (note that this variable is not observed by the econometrician):

$$\text{Macroeconomic uncertainty at time } t = \sqrt{\mathbb{V}_t[M_{t+1}]} \tag{56}$$

Figure 5 shows that macroeconomic uncertainty follows the same pattern with stock market volatility.

7 Additional Results

7.1 Multiple Factors

In this section we investigate how the presence of multiple factors may affect our results. The main implication is that, in contrast to the one-factor case, expected returns and econometrician's betas do not plot on a straight line—the econometrician now faces a "broken" SML, although the CAPM holds for the average agent. However, our result that high betas are inflated and low betas are deflated in the eyes of the econometrician remains valid.

To show how the presence of multiple factors alters the linearity of the econometrician's perceived SML, we consider the simplest example, which builds on the static setting of Section 2 and illustrates this point. Suppose that there are 2 factors and 3 assets and that

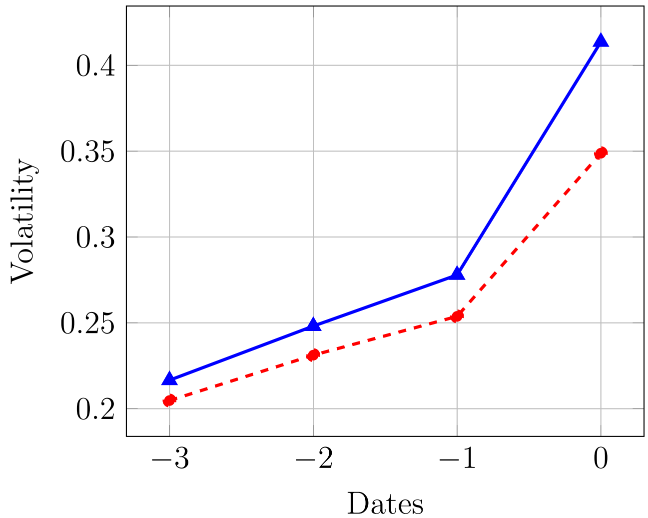

Figure 5: Market volatility during the announcement cycle. Evolution of realized volatility of excess returns (solid line) and macroeconomic uncertainty (dashed line) over public announcement cycles. Date 0 represent the date of the announcement.

factor loadings satisfy:[10]

$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} 1+\phi_1 & 0 \\ 1-\phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \underbrace{\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}}_{\mathbf{F}} + \epsilon, \tag{57}$$

with $\phi_1 > 0$ and $\phi_2 \geq 0$. With respect to the setting of Section 2, the above factor structure now accommodates a third asset, the payoff of which is driven by a separate factor, F_2. We examine the effect of this additional factor on the SML when $\phi_2 > 0$.

The above structure implies that each stock depends on one independent factor or none, and thus factors are independent under each agent i's information set:

$$\tau \equiv \mathbb{V}_i[\mathbf{F}]^{-1} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}, \tag{58}$$

where the factor precisions τ have closed-form solutions that we relegate to Appendix A.6.

We now want to construct the SML perceived by the econometrician. To do so we proceed

[10]In Appendix A.6 we solve a more general case with N assets and K factors.

as in previous sections and write the covariance matrix of the average agent as

$$\bar{\mathbb{V}}[Q] = \begin{pmatrix} \frac{(1+\phi_1^2)}{\tau_1} + \frac{1}{\tau_\epsilon} & \frac{(1-\phi_1)^2}{\tau_1} & 0 \\ \frac{(1-\phi_1)^2}{\tau_1} & \frac{(-1+\phi_1^2)}{\tau_1} + \frac{1}{\tau_\epsilon} & 0 \\ 0 & 0 & \frac{\phi_2^2}{\tau_2} + \frac{1}{\tau_\epsilon} \end{pmatrix}, \tag{59}$$

then we recover the covariance matrix of the econometrician from the law of total covariance:

$$\mathbb{V}[Q] = \bar{\mathbb{V}}[Q] + \frac{\gamma^2}{\tau_X^2}\bar{\mathbb{V}}[Q]\bar{\mathbb{V}}[Q] + \tau_v \Phi \tau^{-1}\tau^{-1}\Phi'. \tag{60}$$

From the previous sections we also know that, in the eyes of the average agent, the SML plots on a straight line that crosses the origin and slopes with the market risk premium. In particular, using the matrix $\bar{\mathbb{V}}[Q]$ we can write risk premia as

$$\mathbb{E}[Q] = \gamma \bar{\mathbb{V}}[M] \frac{\bar{\mathbb{V}}[Q]\bar{X}\iota}{\bar{\mathbb{V}}[M]} \equiv \gamma \bar{\mathbb{V}}[M]\beta \tag{61}$$

where M represents the return of an equally weighted market portfolio with weights $\bar{X} = 1/3$, and where β denotes the vector of betas of the average agent.

We now want to understand the distortion in econometrician's betas, $\tilde{\beta} = \mathbb{V}[M]^{-1}\mathbb{V}[Q]\bar{X}\iota$. To determine whether econometrician's betas will plot on a straight line, we examine whether

$$\beta_e - \iota \propto \beta_a - \iota, \tag{62}$$

where we subtract a vector of ones on both sides since the SML rotates around the point $(1, E[M])$. Clearly, unless the proportional relation in Eq. (62) is satisfied, the pair of expected excess returns and econoemtrician's betas will not plot on a straight line. In Appendix A.6, we compute the ratio of the two betas element by element

$$\frac{\beta_e - 1}{\beta_a - 1} = \frac{\bar{\mathbb{V}}[M]}{\mathbb{V}[M]} \left(\delta \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \phi_2^2 \begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} \right), \tag{63}$$

and show that $\delta > 1$ and that Δ_i, $i = 1, 2, 3$ take distinct values as soon as $\phi_2 > 0$. We conclude that the proportional relation in Eq. (62) is preserved only when $\phi_2 = 0$, i.e, when there is only one factor. Furthermore, tedious computations show that

$$\frac{\bar{\mathbb{V}}[M]}{\mathbb{V}[M]}\delta > 1 \tag{64}$$

so that we recover the result of previous sections that betas below 1 are deflated and betas above 1 are inflated. However, in general, the average agent's betas and those of the econometrician are not proportional. As a result, the econometrician's betas and excess returns will not plot a on straight line—the econometrician contemplates a broken SML. In other words, although the presence of multiple common factors breaks the linearity of the SML in the eyes of the econometrician, the broken SML still rotates clockwise around the market portfolio.

7.2 Diffusion of Private Information

A question arising at this point is whether the nature of information (public or private) matters for these results. In particular, if agents hold private information about the fundamental, it is natural to assume that this information might be diffused through social networks in-between announcements (Cieslak et al., 2015).

To address this question, we modify our setup and allow for information diffusion (Andrei and Cujean, 2016) with time-varying intensity. The details of the model are in Appendix A.5. We implement the following experiment. We assume that a public announcement is scheduled as usual every $T = 4$ periods, but that the informativeness of that signal is very small (large σ_G). However, because announcements are scheduled events anticipated by market participants, it is natural to assume that a lot of private information is exchanged around announcement days. In the context of our model, this can be done by assuming that the meeting intensity is higher during announcement days than on all other days.

Figure 6 depicts the results and shows that similar pattern arises with private information. If investors exchange a lot of information about the fundamental *during* announcement days, then the SML gets steeper and the CAPM again appears stronger. The nature of information (public or private) is therefore not particularly important for our results, although the timing of information exchange is important: the diffusion of information has to take place during announcement days, and not before. In the latter case, this will generate a steepening of the CAPM before the announcement day. This result suggests that the private information transmission during the announcement cycle is weak.

Further questions to be adressed here pertain to the trading volume around macroeconomic announcements, but also to the trading strategies of investors based on their precision of information relative to the market average precision. More specifically, in the presence of information diffusion, investors end up with different amounts of information and thus trade differently not only because they hold different signals but also because their signals are more/less precise than others' signals. Consequently, some of the investors will find it

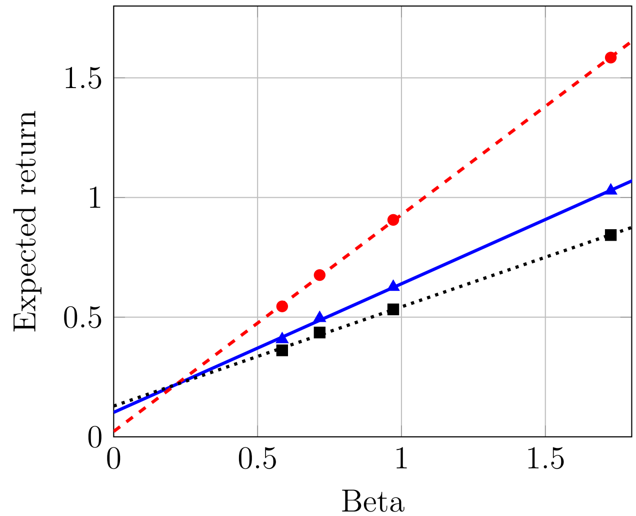

Figure 6: Information diffusion and the CAPM. CAPM during all days (solid), announcement days (dashed), and non-announcement days (dotted), when the public signal is not precise, but the information diffusion λ is intense during the announcement day only ($\lambda = 5$ on announcement days and $\lambda = 0$ otherwise).

optimal to engage in “beta-arbitrage” (buy low beta stocks ans sell high-beta stocks) whereas other investors will find it optimal to take the oposite bet.

8 Conclusion

We build an economy with periodic public announcements in which two major critiques regarding standard tests of the CAPM apply—the market portfolio and investors’ information are unobservable. In this context, one of the main result is that public announcements can alleviate these criticisms.

Several important aspects have yet to be investigated. For instance, with equally weighted stocks, the worst-case scenario for the econometrician is to find a perfectly flat security market line when the CAPM relation actually holds. However, heterogeneity in asset “size” could even cause the security market line to slope downwards, as we observe in the data. On the empirical side our model suggests that a principal component analysis of the covariance matrix of realized returns may improve tests of the CAPM.

A Appendix

A.1 Equilibrium in a static setting

As is customary in the literature, we start by conjecturing the following form for prices:

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} F + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \upsilon + \begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \tag{65}$$

where the undetermined coefficients multiplying the random variables F, υ, X_1 and X_2 will be pinned down by the market clearing condition.

Any investor i has three sources of information: (i) the private signal V_i, (ii) the public signal G, and (iii) public prices, which are informative about F. We denote the total precision of these sources of information by τ (an endogenous quantity to be determined in equilibrium):

$$\tau = \tau_F + \tau_\upsilon + \tau_G + \tau_{P1} + \tau_{P2}, \tag{66}$$

with τ_{Pj}, $j = 1, 2$ denoting the (endogenous) precision of prices as public signals. Denote by P_j^a, $j = 1, 2$, the "normalized adjusted" prices from which the public signal G has been subtracted:

$$P_1^a \equiv \frac{1}{\alpha_1 - g_1}(P_1 - G) = F + \begin{bmatrix} \frac{\xi_{11}}{\alpha_1 - g_1} & \frac{\xi_{12}}{\alpha_1 - g_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{67}$$

$$P_2^a \equiv \frac{1}{\alpha_2 - g_2}(P_2 - G) = F + \begin{bmatrix} \frac{\xi_{21}}{\alpha_2 - g_2} & \frac{\xi_{22}}{\alpha_2 - g_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{68}$$

Learning then implies:

$$\mathbb{E}_i[F] = \frac{\tau_\upsilon}{\tau} V_i + \frac{\tau_G}{\tau} G + \frac{\tau_{P1}}{\tau} P_1^a + \frac{\tau_{P2}}{\tau} P_2^a \tag{69}$$

Notice that we can use the definition of the total precision (66) and the fact that all signals are normalized to conveniently re-write Eq. (69) as follows:

$$\mathbb{E}_i[F] = \frac{\tau - \tau_F}{\tau} F + \frac{\tau_\upsilon}{\tau} \upsilon_i + \frac{\tau_G}{\tau} \upsilon + \begin{bmatrix} \frac{\tau_{P1} \frac{\xi_{11}}{\alpha_1 - g_1} + \tau_{P2} \frac{\xi_{21}}{\alpha_2 - g_2}}{\tau} & \frac{\tau_{P1} \frac{\xi_{12}}{\alpha_1 - g_1} + \tau_{P2} \frac{\xi_{22}}{\alpha_2 - g_2}}{\tau} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{70}$$

and therefore we obtain

$$\mathbb{E}_i[D] = \Phi \mathbb{E}_i[F] \tag{71}$$

$$\mathbb{V}_i[D] = \frac{1}{\tau} \Phi \Phi' + \frac{1}{\tau_\epsilon} \mathbb{I}_2. \tag{72}$$

The conditional variance is the same across investors, thus we denote it by $\bar{\mathbb{V}}[D]$. Investor i's optimal demand writes

$$\frac{1}{\gamma} \left(\bar{\mathbb{V}}[D] \right)^{-1} \left(\mathbb{E}_i[D] - P \right), \tag{73}$$

which we aggregate for market clearing:

$$\bar{\mathbb{E}}[D] - P = \gamma \bar{\mathbb{V}}[D] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \tag{74}$$

or

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \Phi \frac{\tau - \tau_F}{\tau} F + \Phi \frac{\tau_G}{\tau} \upsilon + \left(\Phi \begin{bmatrix} \frac{\tau_{P1}\frac{\xi_{11}}{\alpha_1 - g_1} + \tau_{P2}\frac{\xi_{21}}{\alpha_2 - g_2}}{\tau} & \frac{\tau_{P1}\frac{\xi_{12}}{\alpha_1 - g_1} + \tau_{P2}\frac{\xi_{22}}{\alpha_2 - g_2}}{\tau} \end{bmatrix} - \frac{\gamma}{\tau} \Phi \Phi' - \frac{\gamma}{\tau_\epsilon} \mathbb{I}_2 \right) \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{75}$$

The solution requires only solving numerically for τ_{P1} and τ_{P2}. This further determines τ and then all the coefficients in the conjecture (65) follow from Eq. (75). The equations that pin-down τ_{P1} and τ_{P2} result from the learning solution (69). After tedious calculation we obtain two equations in two unknowns:

$$\tau_{P1} = \frac{\tau_\upsilon \tau_X \tau_\epsilon^2 \left[(\tau_\upsilon + \tau_{P2})(1+\phi)^2 - \tau_{P1}(1-\phi)^2 \right]}{\gamma^2 \left[\tau + 2\tau_\epsilon (1+\phi^2) \right]^2} \tag{76}$$

$$\tau_{P2} = \frac{\tau_\upsilon \tau_X \tau_\epsilon^2 \left[(\tau_\upsilon + \tau_{P1})(1-\phi)^2 - \tau_{P2}(1+\phi)^2 \right]}{\gamma^2 \left[\tau + 2\tau_\epsilon (1+\phi^2) \right]^2}. \tag{77}$$

A.2 Detailed Price Form

$$\begin{aligned} P_{t-k} = \underbrace{\overline{\alpha}_k}_{N\times 1} \overline{D} + \underbrace{\alpha_k}_{N\times 1} F_{t-k-T} + \underbrace{\bar{\xi}_k}_{N\times 1} \overline{X} + \underbrace{\xi_k}_{N\times N} X_{t-k-T} + \underbrace{\begin{pmatrix} d_{k,-T+1} & d_{k,-T+2} & \dots & d_{k,0} \end{pmatrix}}_{N\times NT} \begin{pmatrix} D_{t-T-k+1} \\ D_{t-T-k+2} \\ \vdots \\ D_{t-k} \end{pmatrix} + g_k G_{t-} \\ + \underbrace{\begin{pmatrix} a_{k,-T+1} & a_{k,-T+2} & \dots & a_{k,0} \end{pmatrix}}_{N\times T} \begin{pmatrix} \epsilon^F_{t-k-T+1} \\ \epsilon^F_{t-k-T+2} \\ \vdots \\ \epsilon^F_{t-k} \end{pmatrix} + \underbrace{\begin{pmatrix} b_{k,-T+1} & b_{k,-T+2} & \dots & b_{k,0} \end{pmatrix}}_{N\times NT} \begin{pmatrix} \epsilon^X_{t-k-T+1} \\ \epsilon^X_{t-k-T+2} \\ \vdots \\ \epsilon^X_{t-k} \end{pmatrix} \end{aligned} \tag{78}$$

A.3 Proof of Proposition 2 (Learning)

We start by stacking all observable information in a vector. An investor i observes current and past prices, dividends, public announcements and her set of private signals. Starting with prices we first write the vector p_{t-k} in (29) as

$$p_{t-k} = \underbrace{\begin{pmatrix} a_{i(k),-T+1} & a_{i(k),-T+2} & \dots & a_{i(k),-1} & a_{i(k),0} \\ a_{i(k+1),-T+2} & a_{i(k+1),-T+3} & \dots & a_{i(k+1),0} & \mathbf{0}_{N\times 1} \\ a_{i(k+2),-T+3} & a_{i(k+2),-T+4} & \dots & \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times 1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i(k+T-1),0} & \mathbf{0}_{N\times 1} & \dots & \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times 1} \end{pmatrix}}_{A_k \ (NT\times T)} \bar{\epsilon}^F_{t-k} \tag{79}$$

$$+ \underbrace{\begin{pmatrix} b_{i(k),-T+1} & b_{i(k),-T+2} & \dots & b_{i(k),-1} & b_{i(k),0} \\ b_{i(k+1),-T+2} & b_{i(k+1),-T+3} & \dots & b_{i(k+1),0} & \mathbf{0}_{N\times N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{i(k+T-1),0} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \end{pmatrix}}_{B_k \ (NT\times NT)} \bar{\epsilon}^X_{t-k} \tag{80}$$

$$\equiv A_k \bar{\epsilon}^F_{t-k} + B_k \bar{\epsilon}^X_{t-k}. \tag{81}$$

Similarly, we express the vector of past and current private signals in (30) as

$$V^i_{t-k} = \underbrace{\begin{pmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{pmatrix}}_{\Omega \ (T\times T)} \bar{\epsilon}^F_{t-k} + \underbrace{\begin{pmatrix} \widehat{\epsilon}^i_{t-k} \\ \widehat{\epsilon}^i_{t-k-1} \\ \vdots \\ \widehat{\epsilon}^i_{t-k-T+2} \\ \widehat{\epsilon}^i_{t-k-T+1} \end{pmatrix}}_{\bar{\epsilon}^i_{t-k} \ T\times 1} \tag{82}$$

$$\equiv \Omega \bar{\epsilon}^F_{t-k} + \bar{\epsilon}^i_{t-k} \tag{83}$$

where the vector of investor-specific noise is distributed as

$$\bar{\epsilon}^i_{t-k} \sim N \left(\mathbf{0}_{T\times 1}, \underbrace{\begin{pmatrix} \sigma_v^2 & 0 & \dots & 0 \\ 0 & \sigma_v^2 (n_{t-k,1})^{-1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \sigma_v^2 (n_{t-k,T-1})^{-1} \end{pmatrix}}_{S_k \ (T\times T)} \right) \tag{84}$$

To express the informational parts of dividends in vector form, notice that the dividend at time $t-k$ can be written as

$$D_{t-k} = \overline{D}\mathbf{1}_{N\times 1} + \Phi \left(\kappa_F^T F_{t-k-T} + \sum_{\tau=0}^{T-1} \kappa_F^\tau \epsilon^F_{t-k-\tau} \right) + \epsilon^D_{t-k} \tag{85}$$

$$\equiv \Phi \kappa_F^T F_{t-k-T} + D^a_{t-k} \tag{86}$$

where D^a_{t-k} represents the part of dividends that only contains unobservables. Likewise, the dividends at time $t-k-1$ satisfies

$$D_{t-k-1} = \Phi\left(\kappa_F^{T-1}F_{t-k-T} + \sum_{\tau=0}^{T-2}\kappa_F^{\tau}\epsilon^F_{t-k-\tau-1}\right) + \epsilon^D_{t-k-1} \tag{87}$$

$$\equiv \Phi\kappa_F^{T-1}F_{t-k-T} + D^a_{t-k-1}. \tag{88}$$

Proceeding iteratively, the dividend at time $t-k-T+1$ satisfies

$$D_{t-k-T+1} = \Phi\left(\kappa_F F_{t-k-T} + \epsilon^F_{t-k-T+1}\right) + \epsilon^D_{t-k-T+1} \tag{89}$$

$$\equiv \Phi\kappa_F F_{t-k-T} + D^a_{t-k-T+1}. \tag{90}$$

Accordingly, we can express the vector of informational dividends in (33) as

$$\overline{D}^a_{t-k} = \underbrace{\begin{pmatrix} \Phi & \mathbf{0}_{N\times 1} & \ldots & \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times 1} \\ \Phi\kappa_F & \Phi & \ldots & \mathbf{0}_{N\times 1} & \mathbf{0}_{N\times 1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Phi\kappa_F^{T-2} & \Phi\kappa_F^{T-3} & \ldots & \Phi & \mathbf{0}_{N\times 1} \\ \Phi\kappa_F^{T-1} & \Phi\kappa_F^{T-2} & \ldots & \Phi\kappa_F & \Phi \end{pmatrix}}_{\Lambda\ (NT\times T)} \overline{\epsilon}^F_{t-k} + \underbrace{\begin{pmatrix} \epsilon^D_{t-k-T+1} \\ \epsilon^D_{t-k-T+2} \\ \vdots \\ \epsilon^D_{t-k-1} \\ \epsilon^D_{t-k} \end{pmatrix}}_{\overline{\epsilon}^D_{t-k}\ (NT\times 1)} \tag{91}$$

$$\equiv \Lambda\overline{\epsilon}^F_{t-k} + \overline{\epsilon}^D_{t-k}. \tag{92}$$

Finally, we can write the informational part of the public signal in (34) as

$$G^a_{t-k} = \underbrace{\begin{pmatrix} \kappa_F^{k+j(k)-1} & \kappa_F^{k+j(k)-2}\mathbf{1}_{k+j(k)-2\geq 0} & \ldots & \kappa_F^{k+j(k)-T}\mathbf{1}_{k+j(k)-T\geq 0} \end{pmatrix}}_{\Xi_k\ (1\times T)} \overline{\epsilon}^F_{t-k} + \epsilon^G_{t+j(k)-T} \tag{93}$$

$$\equiv \Xi_k\overline{\epsilon}^F_{t-k} + \epsilon^G_{t+j(k)-T}. \tag{94}$$

We now stack all observables into a single vector according to:

$$\begin{pmatrix} p_{t-k} \\ V^i_{t-k} \\ \overline{D}^a_{t-k} \\ G^a_{t-k} \end{pmatrix} = \underbrace{\begin{pmatrix} A_k \\ \Omega \\ \Lambda \\ \Xi_k \end{pmatrix}}_{H_k\ (((2N+1)T+1)\times T)} \overline{\epsilon}^F_{t-k} + \underbrace{\begin{pmatrix} B_k & \mathbf{0}_{NT\times T} & \mathbf{0}_{NT\times NT} & \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{T\times NT} & \mathbf{I}_T & \mathbf{0}_{T\times NT} & \mathbf{0}_{T\times 1} \\ \mathbf{0}_{NT\times NT} & \mathbf{0}_{NT\times T} & \mathbf{I}_{NT} & \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{1\times NT} & \mathbf{0}_{1\times T} & \mathbf{0}_{1\times NT} & 1 \end{pmatrix}}_{\Theta_k\ (((2N+1)T+1)\times((2N+1)T+1))} \begin{pmatrix} \overline{\epsilon}^X_{t-k} \\ \overline{\epsilon}^i_{t-k} \\ \overline{\epsilon}^D_{t-k} \\ \epsilon^G_{t+j(k)-T} \end{pmatrix} \tag{95}$$

where the vector of noise is distributed as

$$\begin{pmatrix} \overline{\epsilon}^X_{t-k} \\ \overline{\epsilon}^i_{t-k} \\ \overline{\epsilon}^D_{t-k} \\ \epsilon^G_{t+j(k)-T} \end{pmatrix} \sim N\left(\mathbf{0}_{((2N+1)T+1)\times 1}, \underbrace{\begin{pmatrix} \sigma_X^2\mathbf{I}_{NT} & \mathbf{0}_{NT\times T} & \mathbf{0}_{NT\times NT} & \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{T\times NT} & S_k & \mathbf{0}_{T\times NT} & \mathbf{0}_{T\times 1} \\ \mathbf{0}_{NT\times NT} & \mathbf{0}_{NT\times T} & \sigma_D^2\mathbf{I}_{NT} & \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{1\times NT} & \mathbf{0}_{1\times T} & \mathbf{0}_{1\times NT} & \sigma_G^2 \end{pmatrix}}_{\Sigma_k\ (((2N+1)T+1)\times((2N+1)T+1))}\right). \tag{96}$$

Using these matrices, define

$$R_k = \Theta_k \Sigma_k \Theta_k^\top, \tag{97}$$

which we can use to apply the projection theorem. In particular, notice that the vector:

$$\begin{pmatrix} \bar{\epsilon}^F_{t-k} \\ p_{t-k} \\ V^i_{t-k} \\ \overline{D}^a_{t-k} \\ G^a_{t-k} \end{pmatrix} \sim N\left(\mathbf{0}_{((2N+2)T+1)\times 1}, \left(\begin{array}{c|c} \sigma_F^2 \mathbf{I}_T & \sigma_F^2 H_k^\top \\ \hline \sigma_F^2 H_k & \sigma_F^2 H_k H_k^\top + R_k \end{array}\right)\right) \tag{98}$$

is jointly normally distributed. The projection theorem then implies that:

$$\mathbb{E}^i_{t-k}\left[\bar{\epsilon}^F_{t-k}\right] = \underbrace{\sigma_F^2 H_k^\top \left(\sigma_F^2 H_k H_k^\top + R_k\right)^{-1}}_{M_k \ \ (T\times((2N+1)T+1))} \begin{pmatrix} p_{t-k} \\ V^i_{t-k} \\ \overline{D}^a_{t-k} \\ G^a_{t-k} \end{pmatrix} \tag{99}$$

and

$$\underbrace{\mathbb{V}^i_{t-k}\left[\bar{\epsilon}^F_{t-k}\right]}_{(T\times T)} = \sigma_F^2 \left(\mathbf{I}_T - M_k H_k\right). \tag{100}$$

Using this result, we can now compute $\mathbb{E}^i_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right]$. We first rewrite $P_{t-k+1} + D_{t-k+1}$ using the price conjecture as

$$\begin{aligned} P_{t-k+1} + D_{t-k+1} &= D_{t-k+1} + \overline{\alpha}_{i(k+T-1)}\overline{D} + \alpha_{i(k+T-1)}F_{t-k-T+1} + \overline{\xi}_{i(k+T-1)}\overline{X} + g_{i(k+T-1)}G_{t-k+1} \\ &+ \xi_{i(k+T-1)}X_{t-k-T+1} + d_{i(k+T-1)}\overline{D}_{t-k+1} + a_{i(k+T-1)}\bar{\epsilon}^F_{t-k+1} + b_{i(k+T-1)}\bar{\epsilon}^X_{t-k+1} \\ &= \overline{\alpha}_{i(k+T-1)}\overline{D} + \alpha_{i(k+T-1)}\kappa_F F_{t-k-T} + (\overline{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N\times 1}(1-\kappa_X))\overline{X} \\ &+ \xi_{i(k+T-1)}\kappa_X X_{t-k-T} + a_{i(k+T-1)}\bar{\epsilon}^F_{t-k+1} + b_{i(k+T-1)}\bar{\epsilon}^X_{t-k+1} + g_{i(k+T-1)}G_{t-k+1} \\ &+ \alpha_{i(k+T-1)}\epsilon^F_{t-k-T+1} + \xi_{i(k+T-1)}\epsilon^X_{t-k-T+1} + D_{t-k+1} + d_{i(k+T-1)}\overline{D}_{t-k+1}. \end{aligned} \tag{101}$$

Furthermore, defining a new vector $d^\star$ as

$$d_{i(k+T-1)}\overline{D}_{t-k+1} = d_{i(k+T-1),0}D_{t-k+1} + \underbrace{\begin{pmatrix} \mathbf{0}_{N\times N} & d_{i(k+T-1),-T+1} & d_{i(k+T-1),-T+2} & \cdots & d_{i(k+T-1),-1} \end{pmatrix}}_{d^\star_{i(k+T-1)} \ \ (N\times NT)} \overline{D}_{t-k} \tag{102}$$

$$\equiv d_{i(k+T-1),0}\left(\overline{D}\mathbf{1}_{N\times 1} + \Phi\left(\kappa_F^{T+1}F_{t-k-T} + \sum_{\tau=0}^{T}\kappa_F^\tau \epsilon^F_{t-k-\tau+1}\right) + \epsilon^D_{t-k+1}\right) + d^\star_{i(k+T-1)}\overline{D}_{t-k}, \tag{103}$$

we obtain

$$P_{t-k+1}+D_{t-k+1}=\left(\overline{\alpha}_{i(k+T-1)}+(d_{i(k+T-1),0}+\mathbf{I}_N)\mathbf{1}_{N\times 1}\right)\overline{D}+(\overline{\xi}_{i(k+T-1)}+\xi_{i(k+T-1)}\mathbf{1}_{N\times 1}(1-\kappa_X))\overline{X}$$
$$+\kappa_F\left(\alpha_{i(k+T-1)}+\kappa_F^T(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi\right)F_{t-k-T}+\xi_{i(k+T-1)}\kappa_X X_{t-k-T}$$
$$+g_{i(k+T-1)}G_{t-k+1}+d^{\star}_{i(k+T-1)}\overline{D}_{t-k}+b_{i(k+T-1)}\overline{\epsilon}^X_{t-k+1}+\xi_{i(k+T-1)}\epsilon^X_{t-k-T+1}$$
$$+a_{i(k+T-1)}\overline{\epsilon}^F_{t-k+1}+\alpha_{i(k+T-1)}\epsilon^F_{t-k-T+1}+(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi\sum_{\tau=0}^{T}\kappa_F^{\tau}\epsilon^F_{t-k-\tau+1}$$
$$+(d_{i(k+T-1),0}+\mathbf{I}_N)\epsilon^D_{t-k+1}. \tag{104}$$

Now, importantly, the only time G_{t-k+1} is unobservable is at date $t-1$, right before the announcement. To see this, consider that at time $t-2$ investors attempt to forecast P_{t-1}, which is a function of G_{t-T} that they observe. At time t investors attempt to forecast P_{t+1}, which is a function of G_t that they observe. The only time this property is violated is at time $t-1$, at which investors attempt to forecast P_t, which is a function of G_t that they *do not observe yet*. In this case, we must further decompose G_t as

$$G_t=F_t+\epsilon^G_t=\kappa_F^{T+1}F_{t-1-T}+\sum_{\tau=0}^{T}\kappa_F^{\tau}\epsilon^F_{t-\tau}+\epsilon^G_t; \tag{105}$$

accordingly, we write

$$P_{t-k+1}+D_{t-k+1}=\left(\overline{\alpha}_{i(k+T-1)}+(d_{i(k+T-1),0}+\mathbf{I}_N)\mathbf{1}_{N\times 1}\right)\overline{D}+(\overline{\xi}_{i(k+T-1)}+\xi_{i(k+T-1)}\mathbf{1}_{N\times 1}(1-\kappa_X))\overline{X}$$
$$+\kappa_F\left(\alpha_{i(k+T-1)}+\kappa_F^T(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\kappa_F^{T+1}\mathbf{1}_{k=1}\right)F_{t-k-T}+\xi_{i(k+T-1)}\kappa_X X_{t-k-T}$$
$$+g_{i(k+T-1)}G_{t-k+1}\mathbf{1}_{k\neq 1}+d^{\star}_{i(k+T-1)}\overline{D}_{t-k}+b_{i(k+T-1)}\overline{\epsilon}^X_{t-k+1}+\xi_{i(k+T-1)}\epsilon^X_{t-k-T+1}$$
$$+a_{i(k+T-1)}\overline{\epsilon}^F_{t-k+1}+\alpha_{i(k+T-1)}\epsilon^F_{t-k-T+1}+\left((d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\sum_{\tau=0}^{T}\kappa_F^{\tau}\epsilon^F_{t-k-\tau+1}$$
$$+(d_{i(k+T-1),0}+\mathbf{I}_N)\epsilon^D_{t-k+1}+g_{i(T)}\epsilon^G_t\mathbf{1}_{k=1}. \tag{106}$$

Using this expression we can define a new vector $a^{\star}$ as

$$a_{i(k+T-1)}\overline{\epsilon}^F_{t-k+1}+\alpha_{i(k+T-1)}\epsilon^F_{t-k-T+1}+\left((d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\sum_{\tau=0}^{T}\kappa_F^{\tau}\epsilon^F_{t-k-\tau+1}$$
$$=\underbrace{\begin{pmatrix}\alpha_{i(k+T-1)}+\kappa_F^T\left((d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\\ a_{i(k+T-1),-T+1}+\kappa_F^{T-1}\left((d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\\ \vdots\\ a_{i(k+T-1),-1}+\kappa_F\left((d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\end{pmatrix}^{\top}}_{a^{\star}_{i(k+T-1)}\ (N\times T)}\overline{\epsilon}^F_t+\left(a_0+(d_0+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\epsilon^F_{t-k+1}$$
$$\equiv a^{\star}_{i(k+T-1)}\overline{\epsilon}^F_{t-k}+\left(a_{i(k+T-1),0}+(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\epsilon^F_{t-k+1} \tag{107}$$

and a new vector $b^\star$ as

$$
\begin{aligned}
b_{i(k+T-1)}\bar{\epsilon}^X_{t-k+1} + \xi_{i(k+T-1)}\epsilon^X_{t-k-T+1} &= \underbrace{\begin{pmatrix} \xi_{i(k+T-1)} & b_{i(k+T-1),-T+1} & b_{i(k+T-1),-T+2} & \dots & b_{i(k+T-1),-1} \end{pmatrix}}_{b^\star_{i(k+T-1)}\ (N\times NT)}\bar{\epsilon}^X_{t-k} \\
&+ b_{i(k+T-1),0}\epsilon^X_{t-k+1} \\
&= b^\star_{i(k+T-1)}\bar{\epsilon}^X_{t-k} + b_{i(k+T-1),0}\epsilon^X_{t-k+1}
\end{aligned}
\tag{108}
$$

to finally write

$$
\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + a^\star_{i(k+T-1)}\bar{\epsilon}^F_{t-k} + b^\star_{i(k+T-1)}\bar{\epsilon}^X_{t-k} \\
&+ \left(a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}\right)\epsilon^F_{t-k+1} + b_{i(k+T-1),0}\epsilon^X_{t-k+1} \\
&+ (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon^D_{t-k+1} + g_{i(T)}\epsilon^G_t\mathbf{1}_{k=1}.
\end{aligned}
\tag{109}
$$

where

$$
\begin{aligned}
&f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) := \left(\overline{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N)\mathbf{1}_{N\times 1}\right)\overline{D} \\
&+ (\overline{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N\times 1}(1-\kappa_X))\overline{X} + \xi_{i(k+T-1)}\kappa_X X_{t-k-T} + g_{i(k+T-1)}G_{t-k+1}\mathbf{1}_{k\neq 1} \\
&+ \kappa_F\left(\alpha_{i(k+T-1)} + \kappa_F^T(d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\kappa_F^{T+1}\mathbf{1}_{k=1}\right)F_{t-k-T} + d^\star_{i(k+T-1)}\overline{D}_{t-k}.
\end{aligned}
\tag{110}
$$

To compute investor i's conditional expectation and posterior variance, it is convenient to use

$$
\bar{\epsilon}^X_{t-k} = B_k^{-1}p_{t-k} - B_k^{-1}A_k\bar{\epsilon}^F_{t-k}
\tag{111}
$$

to rewrite this expression as

$$
\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + b^\star_{i(k+T-1)}B_k^{-1}p_{t-k} \\
&+ \underbrace{\left(a^\star_{i(k+T-1)} - b^\star_{i(k+T-1)}B_k^{-1}A_k\right)}_{\psi_k\ (N\times T)}\bar{\epsilon}^F_{t-k} + (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon^D_{t-k+1} + g_{i(T)}\epsilon^G_t\mathbf{1}_{k=1} \\
&+ \left(a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}\right)\epsilon^F_{t-k+1} + b_{i(k+T-1),0}\epsilon^X_{t-k+1} \\
&\equiv f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + b^\star_{i(k+T-1)}B_k^{-1}p_{t-k} + \psi_k\bar{\epsilon}^F_{t-k} \\
&+ (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon^D_{t-k+1} + g_{i(T)}\epsilon^G_t\mathbf{1}_{k=1} \\
&+ \left(a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}\right)\epsilon^F_{t-k+1} + b_{i(k+T-1),0}\epsilon^X_{t-k+1}.
\end{aligned}
\tag{112}
$$

Hence, using our previous projection results we obtain

$$
\mathbb{E}^i_{t-k}\left[P_{t-k+1} + D_{t-k+1}\right] = f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + b^\star_{i(k+T-1)}B_k^{-1}p_{t-k} + \psi_k\mathbb{E}^i_{t-k}\left[\bar{\epsilon}^F_{t-k}\right]
\tag{113}
$$

and

$$\mathbb{V}^i_{t-k}\left[P_{t-k+1}+D_{t-k+1}\right]=\sigma_F^2\psi_k\left(\mathbf{I}_T-M_kH_k\right)\psi_k^\top+(d_{i(k+T-1),0}+\mathbf{I}_N)(d_{i(k+T-1),0}+\mathbf{I}_N)^\top\sigma_D^2 \tag{114}$$

$$+\sigma_G^2 g_{i(T)}g_{i(T)}^\top\mathbf{1}_{k=1}+b_{i(k+T-1),0}b_{i(k+T-1),0}^\top\sigma_X^2 \tag{115}$$

$$+\left(a_{i(k+T-1),0}+(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)\left(a_{i(k+T-1),0}+(d_{i(k+T-1),0}+\mathbf{I}_N)\Phi+g_{i(T)}\mathbf{1}_{k=1}\right)^\top\sigma_F^2. \tag{116}$$

A.4 Proof of Proposition 3 (Equilibrium)

For the purpose of aggregating individual demands, it is important to characterize their dependence on the vector N_k of number of signals that a given investor holds at time $t-k$:

$$N_k=\begin{pmatrix}1 & n_{t-k,1} & \dots & n_{t-k,T-1}\end{pmatrix}^\top. \tag{117}$$

In particular, we can rewrite the matrix M_k as

$$M_k=\mathbb{V}^i_{t-k}\left[\bar{\epsilon}^F_{t-k}\right]H_k^\top R_k^{-1}. \tag{118}$$

Using this expression we can write the first term involved in individual demands as

$$\mathbb{V}^i_{t-k}\left[P_{t-k+1}+D_{t-k+1}\right]^{-1}\mathbb{E}^i_{t-k}\left[P_{t-k+1}+D_{t-k+1}\right]= \tag{119}$$

we denote a generallinear relation by $\mathbf{L}(\cdot)$

To setup the system of equations for equilibrium coefficients we use the law of large numbers whereby $\int_{i\in I}\bar{\epsilon}^i_{t-k}\mathrm{d}i=\mathbf{0}_{T\times 1}$, and obtain that the average expectation in (39) satisfies

$$\overline{\mathbb{E}}_{t-k}\left[P_{t-k+1}+D_{t-k+1}\right]=f(\overline{D},\overline{X},X_{t-k-T},F_{t-k-T},\overline{D}_{t-k},G_{t-k}\mathbf{1}_{k\neq 1})$$
$$+b^\star_{i(k+T-1)}B_k^{-1}p_{t-k}+\psi_kM_k\begin{pmatrix}p_{t-k}\\ \Omega\bar{\epsilon}^F_{t-k}\\ \overline{D}^a_{t-k}\\ G^a_{t-k}\end{pmatrix}. \tag{120}$$

Observing that

$$\overline{D}^a_{t-k}=\begin{pmatrix}D_{t-k-T+1}\\ D_{t-k-T+2}\\ \vdots\\ D_{t-k}\end{pmatrix}-\mathbf{1}_{N\times 1}\overline{D}-\underbrace{\begin{pmatrix}\Phi\kappa_F\\ \Phi\kappa_F^2\\ \vdots\\ \Phi\kappa_F^T\end{pmatrix}}_{L\ (NT\times 1)}F_{t-k-T} \tag{121}$$

$$\equiv\overline{D}_{t-k}-\mathbf{1}_{N\times 1}\overline{D}-LF_{t-k-T} \tag{122}$$

and that

$$G^a_{t-k}=G_{t-k}-\kappa_F^{k+j(k)}F_{t-k-T} \tag{123}$$

we can express the vector above as

$$
\begin{pmatrix} p_{t-k} \\ \Omega\bar{\epsilon}^F_{t-k} \\ \overline{D}^a_{t-k} \\ G^a_{t-k} \end{pmatrix} = \begin{pmatrix} A_k\bar{\epsilon}^F_{t-k} + B_k\overline{\eta}_{t-k} \\ \Omega\bar{\epsilon}^F_{t-k} \\ \overline{D}_{t-k} - \mathbf{1}_{N\times 1}\overline{D} - LF_{t-k-T} \\ G_{t-k} - \kappa_F^{k+j(k)} F_{t-k-T} \end{pmatrix}
$$

$$
= \underbrace{\begin{pmatrix} A_k \\ \Omega \\ \mathbf{0}_{NT\times T} \\ \mathbf{0}_{1\times T} \end{pmatrix}}_{H^\star_k \ (((2N+1)T+1)\times T)} \bar{\epsilon}^F_{t-k} + \underbrace{\begin{pmatrix} B_k \\ \mathbf{0}_{T\times NT} \\ \mathbf{0}_{NT\times NT} \\ \mathbf{0}_{1\times NT} \end{pmatrix}}_{B^\star_k \ ((2N+1)T+1\times NT)} \bar{\epsilon}^X_{t-k} + \underbrace{\begin{pmatrix} \mathbf{0}_{NT\times NT} \\ \mathbf{0}_{T\times NT} \\ \mathbf{I}_{NT} \\ \mathbf{0}_{1\times NT} \end{pmatrix}}_{C \ ((2N+1)T+1\times NT)} \overline{D}_{t-k} \tag{124}
$$

$$
- \underbrace{\begin{pmatrix} \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{T\times 1} \\ \mathbf{1}_{N\times 1} \\ 0 \end{pmatrix}}_{K \ ((2N+1)T+1\times 1)} \overline{D} + \underbrace{\begin{pmatrix} \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{T\times 1} \\ \mathbf{0}_{NT\times 1} \\ 1 \end{pmatrix}}_{E \ (((2N+1)T+1)\times 1)} G_{t-k} - \underbrace{\begin{pmatrix} \mathbf{0}_{NT\times 1} \\ \mathbf{0}_{T\times 1} \\ L \\ \kappa_F^{k+j(k)} \end{pmatrix}}_{L^\star_k \ (((2N+1)T+1)\times 1)} F_{t-k-T}
$$

$$
\equiv H^\star_k\bar{\epsilon}^F_{t-k} + B^\star_k\bar{\epsilon}^X_{t-k} + C\overline{D}_{t-k} - K\overline{D} + EG_{t-k} - L^\star F_{t-k-T}.
$$

Substituting this expression in average market expectations and using the pricing relation in (41), we can write:

$$
\begin{aligned}
RP_{t-k} &= f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + b^\star_{i(k+T-1)}\bar{\epsilon}^X_{t-k} + b^\star_{i(k+T-1)}B_k^{-1}A_k\bar{\epsilon}^F_{t-k} \\
&\quad + \psi_k M_k \left(H^\star_k\bar{\epsilon}^F_{t-k} + B^\star_k\bar{\epsilon}^X_{t-k} + C\overline{D}_{t-k} - K\overline{D} + EG_{t-k} - L^\star_k F_{t-k-T}\right) - \gamma\mathbb{V}_{t-k}[P_{t-k+1} + D_{t-k+1}]X_{t-k} \\
&= f(\overline{D}, \overline{X}, X_{t-k-T}, F_{t-k-T}, \overline{D}_{t-k}, G_{t-k}\mathbf{1}_{k\neq 1}) + \psi_k M_k \left(C\overline{D}_{t-k} - K\overline{D} + EG_{t-k} - L^\star_k F_{t-k-T}\right) \\
&\quad + \left(b^\star_{i(k+T-1)}B_k^{-1}A_k + \psi_k M_k H^\star_k\right)\bar{\epsilon}^F_{t-k} + \left(b^\star_{i(k+T-1)} + \psi_k M_k B^\star_k\right)\bar{\epsilon}^X_{t-k} \\
&\quad - \gamma\mathbb{V}_{t-k}[P_{t-k+1} + D_{t-k+1}]\left(\kappa_X^T X_{t-k-T} + \overline{X}(1-\kappa_X)\sum_{\tau=0}^{T-1}\kappa_X^\tau\mathbf{1}_{N\times 1} + \sum_{\tau=0}^{T-1}\kappa_X^\tau\epsilon^X_{t-k-\tau}\right),
\end{aligned} \tag{125}
$$

which confirms our initial price conjecture. We finally obtain the equilibrium fixed point by matching this expression with the price conjecture, which yields:

$$
\begin{pmatrix} \overline{\alpha}_k \\ \alpha_k \\ \overline{\xi}_k \\ \xi_k \\ d_k \\ g_k \\ a_k \\ b_k \end{pmatrix} = \frac{1}{R}\begin{pmatrix} \left(\overline{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N)\mathbf{1}_{N\times 1}\right) - \psi_k M_k K \\ \kappa_F\left(\alpha_{i(k+T-1)} + \kappa_F^T(d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\kappa_F^{T+1}\mathbf{1}_{k=1}\right) - \psi_k M_k L^\star_k \\ (\overline{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N\times 1}(1-\kappa_X)) - \gamma\mathbb{V}_{t-k}[P_{t-k+1} + D_{t-k+1}]\mathbf{1}_{N\times 1}(1-\kappa_X)\sum_{\tau=0}^{T-1}\kappa_X^\tau \\ \xi_{i(k+T-1)}\kappa_X - \gamma\mathbb{V}_{t-k}[P_{t-k+1} + D_{t-k+1}]\kappa_X^T \\ d^\star_{i(k+T-1)} + \psi_k M_k C \\ g_{i(k+T-1)}\mathbf{1}_{k\neq 1} + \psi_k M_k E \\ b^\star_{i(k+T-1)}B_k^{-1}A_k + \psi_k M_k H^\star_k \\ b^\star_{i(k+T-1)} + \psi_k M_k B^\star_k - \gamma\mathbb{V}_{t-k}[P_{t-k+1} + D_{t-k+1}]\begin{pmatrix} \kappa_X^{T-1}\mathbf{I}_N & \kappa_X^{T-2}\mathbf{I}_N & \dots & \kappa_X\mathbf{I}_N & \mathbf{I}_N \end{pmatrix} \end{pmatrix}. \tag{126}
$$

for all $k \in \{0, \dots, T-1\}$.

A.5 Information Percolation

We describe here the private information of investors and how it diffuses among the population of investors. At any date s each investor i receives a private signal v^i about the current factor innovation:

$$v_s^i = \epsilon_s^F + \epsilon_s^i, \quad \epsilon_s^i \sim N(0, \sigma_v^2), \ \ \epsilon_s^i \perp \epsilon_s^k, \ \ \forall k \neq i \tag{127}$$

Following Andrei and Cujean (2016) we allow investors to exchange these signals in private, bilateral meetings through the information percolation theory (Duffie, Malamud, and Manso, 2009) whereby meetings take place continuously at Poisson arrival times. Unlike Andrei and Cujean (2016), however, we allow meetings to occur with an intensity λ_s that varies over time. This meeting intensity is common to all investors. When two investors meet at time s, they exchange all the signals they have gathered about factor innovations at any relevant lag. This assumption along with the normality of individual signals imply that an agent's private information is completely summarized by her number $n_{s,k}$ of signals regarding the factor innovation at lag k for all lags k and her posterior expectation of the vector $\bar{\epsilon}_s^F$. These numbers are sufficient statistics for the information investors exchange when they meet and "talk."

We assume that the meeting intensity varies over the public signal cycles. That is, the meeting intensity between date $t-k$ and $t-k+1$ exhibits the periodic pattern highlighted in Figure 7:

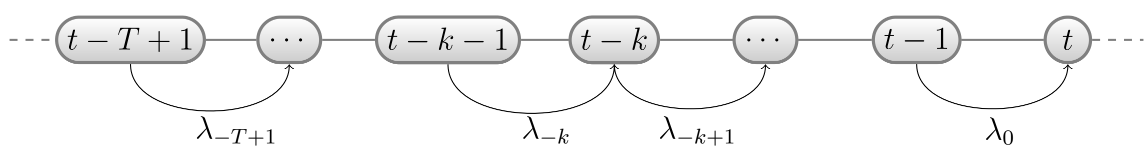

Figure 7: Information time line of private signals exchange for any $k \geq 2$.

Except from the fact that this periodic pattern repeats itself during the public announcement cycles, we do not impose any structure within the cycle. In other words, we can assume any pattern for the vector of meeting intensities (which is common knowledge to all agents in the economy):

$$\lambda_k, \forall k \in \{-T+1, ..., 0\}. \tag{128}$$

This will allow us to understand the effect of information diffusion on asset prices, especially during announcement days, where we expect investors to exchange more information than usual, for instance by chatting on Bloomberg terminals.

Following Figure 7, suppose that today $s = t-k$ is located k periods away from the announcement date. Today investor i receives one signal regarding the current factor innovation ϵ_s^F and looks for other investors with intensity λ_{-k+1}. Yesterday she was looking for investors to talk to with intensity λ_{-k} and so on and so forth until the previous announcement date at time $t-T$ at which she was looking for investors to talk to with intensity λ_{-T+1}. Right before the last announcement date she looked for other investors with intensity λ_0 and so on and so forth until the last relevant date $s-T+1$ at which she looks for someone with intensity λ_{-k+2}, consistent with the periodic pattern in λ. Note that, as of today (date s), date $s-T+1$ is the last relevant date, since all innovations prior to this date are perfectly known. Moreover, as of today, investor i has 1 signal regarding ϵ_s^F, $n_{s,1}$ signals regarding ϵ_{s-1}^F and so on.

The number of signals $n_{s,k}$ that an arbitrary investor possesses at time s regarding the factor innovation ϵ^F_{s-k} are distributed according to a certain cross-sectional distribution $\mu_{s,k}$. Adapting the computations in Duffie et al. (2009), this cross-sectional distribution can be defined recursively. In particular, the cross-sectional distribution of the number $n_{s,k}$ of signals regarding the factor innovation at lag k gathered by date s evolves according to the dynamics

$$\frac{\mathrm{d}}{\mathrm{d}u}\mu_{u,k}(n) = \lambda_{s-l}\mu_{u,k} * \mu_{u,k} - \lambda_{s-l}\mu_{u,k} \tag{129}$$

$$= \lambda_{s-l}\sum_{m=1}^{n-1}\mu_{u,k}(n-m)\mu_{u,k}(m) - \lambda_{s-l}\mu_{u,k}(n), \tag{130}$$

with $u \in [s-l, s-l+1]$, $l \in 1, ..., k$, $\mu_{s-k} = \delta_{n=1}$, and where "$*$" denotes the discrete convolution product. The summation term on the right hand side in (129) represents the rate at which new agents of a given type are created, whereas the second term in (129) captures the rate at which agents leave a given type.

A key statistic in the equilibrium construction is the first moment of this distribution. In particular, we denote the cross-sectional average number of signals regarding factor innovations at lag k gathered by time s by $\Omega_{s,k} = \sum_{n\in\mathbb{N}} n\mu_{s,k}(n)$. Using the dynamics of the cross-sectional distribution of number of signals, the cross-sectional average must satisfy

$$\frac{\mathrm{d}}{\mathrm{d}u}\Omega_{u,k} = \lambda_{s-l}\Omega_{u,k},\ \forall u \in [s-l, s-l+1],\ \ \forall l \in 1, ..., k,\ \ \Omega_{s-k} = 1. \tag{131}$$

Solving this equation recursively, the cross-sectional average number of signals regarding information at lag l gathered by date $t-k$ satisfies

$$\Omega_{t-k,l} = \exp\left(\mathbb{1}_{l\geq 1}\sum_{m=k+1}^{k+l}\lambda_{-i(m-1)}\right),\ \ \forall k \in \{0, ..., T-1\},\ \ l \in \{0, ..., T-1\}. \tag{132}$$

Finally, because information is Gaussian and investors share all their information, the collection $\{\epsilon^F_{s-k,j}\}_{j=1}^{n_{s,k}}$ of private signals about innovations at lag k gathered by date s can be summarized with a single private signal with variance $\sigma_v^2(n_{s,k})^{-1}$, which we denote by $\widehat{v}^i_{s-k}$:

$$\widehat{v}^i_{s-k} = \epsilon^F_{s-k} + \widehat{\epsilon}^i_{s-k}, \quad \text{where } \widehat{\epsilon}^i_{s-k} \equiv \left(\frac{1}{n_{s,k}}\sum_{j=1}^{n_{s,k}}\widetilde{\epsilon}^j\right) \sim N\left(0, \sigma_v^2(n_{s,k})^{-1}\right). \tag{133}$$

A.6 Multiple Factors Structure

Denote a vector of K factors by

$$\mathbf{F} = \begin{pmatrix} F_1 & F_2 & \cdots & F_K \end{pmatrix}' \sim \mathbb{N}\left(\mathbf{0}_{K\times 1}, \tau_F^{-1}\mathbf{I}_K\right) \tag{134}$$

and a vector of $N \geq K$ realized dividends by

$$\mathbf{D} = \underbrace{\begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,K} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{N,1} & \phi_{N,2} & \cdots & \phi_{N,K} \end{pmatrix}}_{\Phi\ (N\times K)} \mathbf{F} + \underbrace{\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{pmatrix}}_{\epsilon\ (N\times 1)} \equiv \Phi\mathbf{F} + \epsilon \tag{135}$$

with $\epsilon \sim \mathbb{N}\left({}_{N\times 1}, \tau_\epsilon^{-1}\mathbf{I}_N\right)$. Each asset is in supply $X \sim \mathbb{N}\left(\mathbf{0}_{N\times 1}, \tau_X^{-1}\mathbf{I}_N\right)$. Each agent i observes a vector of private signals about the K factors

$$V^i = \mathbf{F} + v^i, \;\; v^i \sim \mathbb{N}\left(\mathbf{0}_{K\times 1}, \tau_v^{-1}\mathbf{I}_K\right), \tag{136}$$

as well as a common public signal

$$G = \mathbf{F} + v, \;\; v \sim \mathbb{N}\left(\mathbf{0}_{K\times 1}, \tau_G^{-1}\mathbf{I}_K\right). \tag{137}$$

In addition they observe prices, which we conjecture satisfy

$$P = \underbrace{\begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,K} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,1} & \alpha_{N,2} & \cdots & \alpha_{N,K} \end{pmatrix}}_{\alpha\ (N\times K)} \mathbf{F} + \underbrace{\begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,K} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ g_{N,1} & g_{N,2} & \cdots & g_{N,K} \end{pmatrix}}_{g\ (N\times K)} G + \underbrace{\begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ \beta_{N,1} & \beta_{N,2} & \cdots & \beta_{N,N} \end{pmatrix}}_{\beta\ (N\times N)} X. \tag{138}$$

Since agents observe the public signal G, the effective price signal is

$$P^a \equiv P - gG = \alpha\mathbf{F} + \beta X. \tag{139}$$

Regrouping all signals in a vector we obtain

$$S^i = \begin{pmatrix} P^a \\ V^i \\ G \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha \\ \mathbf{I}_K \\ \mathbf{I}_K \end{pmatrix}}_{H\ (N+2K)\times K} \mathbf{F} + \underbrace{\begin{pmatrix} \beta & \mathbf{0}_{N\times K} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{K\times N} & \mathbf{I}_K & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times N} & \mathbf{0}_{K\times K} & \mathbf{I}_K \end{pmatrix}}_{\Theta\ (N+2K)\times(N+2K)} \begin{pmatrix} X \\ v^i \\ v \end{pmatrix} \tag{140}$$

with

$$\begin{pmatrix} X \\ v^i \\ v \end{pmatrix} \sim \mathbb{N}\left(\mathbf{0}_{(N+2K)\times 1}, \underbrace{\begin{pmatrix} \tau_X^{-1}\mathbf{I}_N & \mathbf{0}_{N\times K} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{K\times N} & \tau_v^{-1}\mathbf{I}_K & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times N} & \mathbf{0}_{K\times K} & \tau_G^{-1}\mathbf{I}_K \end{pmatrix}}_{\Sigma\ (N+2K)\times(N+2K)}\right). \tag{141}$$

Using these matrices we now define a $(N+2K)\times(N+2K)-$matrix:

$$R \equiv (\Theta\Sigma\Theta')^{-1} = \begin{pmatrix} (\beta\beta')^{-1}\tau_X & \mathbf{0}_{N\times K} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{K\times N} & \mathbf{I}_K\tau_v & \mathbf{0}_{K\times K} \\ \mathbf{0}_{K\times N} & \mathbf{0}_{K\times K} & \mathbf{I}_K\tau_G \end{pmatrix}. \tag{142}$$

The projection theorem then implies that

$$\mathbb{V}_i[\mathbf{F}] = \left(\mathbf{I}_K\tau_F + H'RH\right)^{-1} \equiv \bar{\mathbb{V}}[\mathbf{F}], \tag{143}$$

from which we define average precision τ as

$$\underbrace{\tau}_{K\times K} \equiv \bar{\mathbb{V}}[\mathbf{F}]^{-1} = \mathbf{I}_K\tau_F + H'RH. \tag{144}$$

The projection theorem also yields

$$\mathbb{E}_i[\mathbf{F}] = \tau^{-1}H'RS_i. \tag{145}$$

Aggregating over the vector of signals we can write

$$\bar{S} = \int_{i\in[0,1]} S_i \mathrm{d}i = \underbrace{\begin{pmatrix} \alpha \\ \mathbf{0}_{K\times K} \\ \mathbf{I}_K \end{pmatrix}}_{s_F\ \ (N+2K)\times K}\mathbf{F} + \underbrace{\begin{pmatrix} \mathbf{0}_{N\times K} \\ \mathbf{0}_{K\times K} \\ \mathbf{I}_K \end{pmatrix}}_{s_G\ \ (N+2K)\times K} G + \underbrace{\begin{pmatrix} \beta \\ \mathbf{0}_{K\times N} \\ \mathbf{0}_{K\times N} \end{pmatrix}}_{s_X\ \ (N+2K)\times N} X. \tag{146}$$

It follows that average expectations satisfy

$$\bar{\mathbb{E}}[\mathbf{F}] = \tau^{-1}H'R\left(s_F\mathbf{F} + s_G G + s_X X\right). \tag{147}$$

The market-clearing condition then requires that

$$\begin{aligned} \alpha\mathbf{F} + gG + \beta X &= \bar{\mathbb{E}}[D] - \gamma\bar{\mathbb{V}}[D]X && (148)\\ &= \Phi\tau^{-1}H'R\left(s_F\mathbf{F} + s_G G + s_X X\right) - \gamma\left(\Phi\tau\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right)X && (149)\\ &= \Phi\tau^{-1}H'Rs_F\mathbf{F} + \Phi\tau^{-1}H'Rs_G G + \left(\Phi\tau^{-1}H'Rs_X - \gamma\left(\Phi\tau^{-1}\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right)\right)X. && (150) \end{aligned}$$

Separating variables we obtain the following system of equations:

$$\begin{aligned} \alpha &= \Phi\tau^{-1}H'Rs_F && (151)\\ g &= \Phi\tau^{-1}H'Rs_G && (152)\\ \beta &= \Phi\tau^{-1}H'Rs_X - \gamma\left(\Phi\tau^{-1}\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right). && (153) \end{aligned}$$

We can reduce the size of this system of equations. We first define

$$\underbrace{\tau_P}_{K\times N} \equiv \alpha'\left(\beta\beta'\right)^{-1} \tag{154}$$

and write accordingly

$$\underbrace{H'R}_{K\times(N+2K)} = \begin{pmatrix} \tau_P\tau_X & \mathbf{I}_K\tau_v & \mathbf{I}_K\tau_G \end{pmatrix}. \tag{155}$$

Substituting into the system of equations we obtain g as a function of the unknown matrix τ:

$$g = \Phi\tau^{-1}\tau_G. \tag{156}$$

Similarly, rewriting total precision we can further write

$$\tau - \mathbf{I}_K\tau_F = H'RH = H'Rs_F + \mathbf{I}_K\tau_G, \tag{157}$$

from which we obtain an expression for α as a function of the unknown matrix τ:

$$\alpha = \Phi\tau^{-1}\left(\tau - \mathbf{I}_K\tau_F - \mathbf{I}_K\tau_G\right). \tag{158}$$

Finally, substituting into the equation for β we obtain an expression for β as a function of the unknown matrices τ and τ_P:

$$\beta = \gamma\left(\Phi\tau^{-1}\tau_P\tau_X - \mathbf{I}_N\right)^{-1}\left(\Phi\tau^{-1}\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right). \tag{159}$$

Hence, we need to pin down the matrices τ and τ_P. We just use the filter equation for τ:

$$\tau = \mathbf{I}_K\left(\tau_F + \tau_v + \tau_G\right) + \tau_P\tau_X\alpha \tag{160}$$

$$= \mathbf{I}_K\left(\tau_F + \tau_v + \tau_G\right) + \tau_P\tau_X\Phi\tau^{-1}\left(\tau - \mathbf{I}_K\tau_F - \mathbf{I}_K\tau_G\right), \tag{161}$$

along with the definition of the matrix τ_P:

$$\tau_P = \frac{1}{\gamma^2}\left(\Phi\tau^{-1}\left(\tau - \mathbf{I}_K\tau_F - \mathbf{I}_K\tau_G\right)\right)'\left(\Phi\tau^{-1}\tau_P\tau_X - \mathbf{I}_N\right)X\left(\Phi\tau^{-1}\tau_P\tau_X - \mathbf{I}_N\right) \tag{162}$$

where

$$X = \left(\left(\Phi\tau^{-1}\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right)\left(\Phi\tau^{-1}\Phi' + \tau_\epsilon^{-1}\mathbf{I}_N\right)'\right)^{-1}. \tag{163}$$

Regarding the simple example of Section 7.1, the above equations collapse into a system of five equations for the five unknowns τ_1, τ_2, $\tau_{P,1}$, $\tau_{P,2}$ and $\tau_{P,3}$:

$$\frac{\tau_X\left(-\tau_F - \tau_G + \tau_1\right)\left(\phi_1\tau_{P,1} - \phi_1\tau_{P,2} + \tau_{P,1} + \tau_{P,2}\right)}{\tau_1} + \tau_F + \tau_G + \tau_v = \tau_1 \tag{164}$$

$$\phi_2\tau_X\tau_{P,3} + \tau_F + \tau_G + \tau_v - \frac{\phi_2\tau_X\tau_{P,3}\left(\tau_F + \tau_G\right)}{\tau_2} = \tau_2 \tag{165}$$

$$\gamma^2\tau_1\tau_{P,1}\left(\tau_1 + 2\left(\phi_1^2 + 1\right)\tau_\epsilon\right)^2 - \tau_\epsilon^2\left(-\tau_F - \tau_G + \tau_1\right)\left(\tau_1\left(\phi_1 + 1\right) - 2\left(\phi_1^2 + 1\right)\tau_X\tau_{P,1}\right) \tag{166}$$

$$\times\left(\tau_1 - \tau_X\left(\phi_1\tau_{P,1} - \phi_1\tau_{P,2} + \tau_{P,1} + \tau_{P,2}\right)\right) = 0 \tag{167}$$

$$\tau_\epsilon^2\left(-\tau_F - \tau_G + \tau_1\right)\left(2\left(\phi_1^2 + 1\right)\tau_X\tau_{P,2} + \tau_1\left(\phi_1 - 1\right)\right) \tag{168}$$

$$\times\left(\tau_1 - \tau_X\left(\phi_1\tau_{P,1} - \phi_1\tau_{P,2} + \tau_{P,1} + \tau_{P,2}\right)\right) + \gamma^2\tau_1\tau_{P,2}\left(\tau_1 + 2\left(\phi_1^2 + 1\right)\tau_\epsilon\right)^2 = 0 \tag{169}$$

$$\phi_2\tau_\epsilon^2\left(-\tau_F - \tau_G + \tau_2\right)\left(\tau_2 - \phi_2\tau_X\tau_{P,3}\right)^2 - \gamma^2\tau_2\tau_{P,3}\left(\tau_2 + \phi_2^2\tau_\epsilon\right)^2 = 0. \tag{170}$$

This system has a unique, real solution, which is analytical in this case. In particular, defining a function

$$\varphi_i = \begin{cases} 2(1+\phi_1^2) & i=1 \\ \phi_2^2 & i=2 \end{cases} \tag{171}$$

along with

$$\bar{\tau}_i \equiv \tau_G + \tau_F + \tau_v + \tau_\epsilon \varphi_i, \quad i=1,2 \tag{172}$$

and

$$\delta_i = 4\gamma^2 \bar{\tau}_i^3 + 27\tau_v^2 \tau_X \tau_\epsilon^2 \varphi_i, \quad i=1,2. \tag{173}$$

We can then write the solution for τ_i with $i=1,2$ as:

$$\tau_i = \frac{1}{6}\left(\begin{array}{c} 2\bar{\tau}_i + \frac{2\gamma^2\bar{\tau}_i^2}{\sqrt[3]{\gamma^6\bar{\tau}_i^3 + \frac{3}{2}\left(\sqrt{3}\sqrt{\gamma^8\delta_i\varphi_i\tau_v^2\tau_X\tau_\epsilon^2} + 9\gamma^4\varphi_i\tau_v^2\tau_X\tau_\epsilon^2\right)}} \\ + \frac{2^{2/3}\sqrt[3]{2\gamma^6\bar{\tau}_i^3 + 3\sqrt{3}\sqrt{\gamma^8\delta_i\varphi_i\tau_v^2\tau_X\tau_\epsilon^2} + 27\gamma^4\varphi_i\tau_v^2\tau_X\tau_\epsilon^2}}{\gamma^2} - 6\varphi_i\tau_\epsilon \end{array} \right). \tag{174}$$

The solutions for $\tau_{P,1}$ and $\tau_{P,2}$ are analytical, but irrelevant for the example since they do not intervene in the characterization of the SML.

When comparing agent's betas versus econometrician's betas (Equation (63) in the text), we find the following values for δ and Δ_i, $i=1,2,3$:

$$\delta = \frac{2\gamma^2\left(\tau_1 + \phi_1^2\tau_\epsilon + \tau_\epsilon\right) + (\tau_1+\tau_v)\,\tau_X\tau_\epsilon}{\tau_1\tau_X\tau_\epsilon} > 1 \tag{175}$$

and

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} \frac{\left(2\tau_2\left(\phi_1^2+1\right)-\tau_1\phi_2^2\right)\gamma^2+(\tau_2-\tau_1)\tau_v\tau_X}{\tau_2\tau_X\left(\tau_2(6\phi_1+2)-\tau_1\phi_2^2\right)} \\ \frac{\left(\tau_1\phi_2^2-2\tau_2\left(\phi_1^2+1\right)\right)\gamma^2+(\tau_1-\tau_2)\tau_v\tau_X}{\tau_2\tau_X\left(\tau_1\phi_2^2+\tau_2(6\phi_1-2)\right)} \\ \frac{\left(2\tau_2\left(\phi_1^2+1\right)-\tau_1\phi_2^2\right)\gamma^2+(\tau_2-\tau_1)\tau_v\tau_X}{\tau_2\tau_X\left(2\tau_2-\tau_1\phi_2^2\right)} \end{pmatrix}. \tag{176}$$

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