Information Percolation, Momentum and Reversal

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Abstract: We propose a joint theory of time-series momentum and reversal based on a rational-expectations model. We show that a necessary condition for momentum to arise in this framework is that information flows at an increasing rate. We focus on word-of-mouth communication as a mechanism that enforces this condition and generates short-term momentum and long-term reversal. Investors with heterogeneous trading strategies—contrarian and momentum traders—coexist in the marketplace. Although a significant proportion of investors are momentum traders, momentum is not completely eliminated. Word-of-mouth communication spreads rumors and generates price run-ups and reversals. Our theoretical predictions are in line with empirical findings.

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1 Introduction

One of the most pervasive facts in finance is price momentum. It is documented everywhere, both across and within countries and asset classes. It appears in the cross-section of returns, where it refers to securities' *relative performance*, but also in the time-series of returns, where it refers to a security's *own performance*.[1] Both forms of momentum, "cross-sectional momentum" and "time-series momentum," are followed by a phase of reversal over longer horizons. Time-series momentum and reversal are the focus of this paper.

Reconciling the existence of short-term momentum and long-term reversal with a rational explanation is challenging. Rational investors can easily detect predictable patterns and trade on them, thereby eliminating them. Leading theories of momentum and reversal are therefore mostly behavioral.[2] But the weak link between momentum and various measures of investor sentiment (Moskowitz et al., 2012) indicates that behavioral models have yet to identify the main source driving momentum and reversal. This paper provides a joint explanation for time-series momentum and reversal in which momentum arises and persists in the absence of behavioral biases.

We use a rational-expectations framework (Grossman and Stiglitz, 1980) to derive a condition on the "shape" of information arrival that is necessary for momentum to exist. Our building block is an economy in which a large population of risk-averse agents trade a risky asset over several rounds. Investors observe a flow of private information and a flow of public information conveyed by equilibrium prices. We introduce the concept of *precision elasticity*, which measures how average market precision responds to a change in the average precision of private information. Momentum only obtains when precision elasticity is greater than one,

[1] See Rouwenhorst (1998), Asness, Moskowitz, and Pedersen (2013), Jegadeesh and Titman (1993), and Moskowitz, Ooi, and Pedersen (2012). Cross-sectional and time-series momentum are related, but distinct empirical anomalies. Importantly, Moskowitz et al. (2012) show that time-series momentum is not fully explained by cross-sectional momentum.

[2] These theories propose various behavioral biases to generate momentum and reversal: "momentum traders," "conservative investors," or "attribution bias" generate momentum, whereas "newswatchers," "representativeness heuristic," or "overconfidence" generate reversals. See Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999).

so that average market precision increases faster than the precision of private information over time. To enforce this condition, private information must flow at an increasing rate. We show that momentum never obtains when the flow of private information is linear, a customary assumption in the literature.

Among the possible mechanisms that cause information to flow at an increasing rate, we focus on word-of-mouth communication. In the context of a rational-expectations model, word-of-mouth communication represents an additional channel of information acquisition. We model word-of-mouth communication through the information percolation theory (Duffie and Manso, 2007), whereby agents exchange information in random, bilateral private meetings. Investors therefore trade in centralized markets, but also search for each other's private information—trading is centralized, but information exchange is partially decentralized.

When embedded into a centralized trading model, information percolation has two effects. First, the percolation mechanism dictates how average market precision evolves over time. As agents accumulate information through random meetings, the average precision of information in the economy increases at an accelerated—exponential—rate. Beyond a certain threshold of the intensity at which agents meet and talk, this exponential increase in precision generates short-term momentum and long-term reversal. We fully characterize this critical threshold of the meeting intensity.

Second, the percolation mechanism dictates how individual precisions are distributed across agents. Through the meeting process, agents acquire heterogeneous amounts of information, which causes them to implement different trading strategies. We first show that the "average agent" in our model is neutral to the market—she is not a momentum trader, nor a contrarian. It follows that, without information percolation, all investors in our model are market neutral. Information percolation, instead, allows agents' precision to differ from average market precision. In this case, the distance between agents' precision and average market precision determines agents' strategies. Agents who are better informed than the average agent are contrarians, while others are momentum traders. Although everyone (in-

cluding the econometrician) observes momentum, better informed investors trade against it, thus allowing momentum to persist in the presence of momentum traders.

We argue that word-of-mouth communication is a plausible mechanism, as it produces several predictions that are supported by empirical evidence. Our model can simultaneously generate short-term momentum and long-term reversal (Moskowitz et al., 2012) and a hump-shaped pattern of momentum, similar to that documented by Hong, Lim, and Stein (2000). Moreover, our model is consistent with the empirical finding that stock returns exhibit strong reversals at shorter horizons (Jegadeesh, 1990; Lehmann, 1990) and with empirically documented trading strategies (Grinblatt, Jostova, Petrasek, and Philipov, 2016).

We extend our model along three dimensions. A first extension is based on the idea that word-of-mouth communication is a natural propagator of rumors (Shiller, 2000). When private information contains a rumor, this rumor circulates among investors, who are aware of its existence but cannot observe it, creating a disconnect between the stock price and the fundamental. Ultimately, the rumor subsides, leading to a price reversal. Second, while we derive our results in a model with a finite horizon, we show that they carry over to a fully dynamic setup. In particular, momentum obtains whether the asset pays a single liquidating dividend or an infinite stream of dividends. Finally, in the Appendix (Section C.4), we introduce a large, unconstrained, risk-neutral arbitrageur who could conceivably eliminate momentum. We find that this is not the case—the arbitrageur must also consider that her trades move prices adversely.

Among the large theoretical literature on momentum, leading rational theories are based on growth-options models (Berk, Green, and Naik, 1999; Johnson, 2002; Sagi and Seasholes, 2007). Our theory abstracts from firm decisions and directly builds on information transmission as a driver of investors' decisions and thereby of stock returns.[3] Previous rational-

[3]We believe that private exchange of information is linked to momentum for several reasons. First, private information is an important driver of stock price variations (French and Roll, 1986) and provides an incentive for investors to implement heterogeneous trading strategies. Public news, instead, do not predict prices (Roll, 1988), nor do they explain price changes (Chan, Fong, Kho, and Stulz, 1996) or generate trading heterogeneity (Mitchell and Mulherin, 1994; Gropp and Kadareja, 2012; Tetlock, 2010; Koudijs, 2016). Second, word-of-mouth communication is an innate channel of information processing, "*a central part*

expectations models (Holden and Subrahmanyam, 2002; Cespa and Vives, 2012) suggest that an increase in information precision generates momentum. Our model offers a unified explanation for short-term momentum, long-term reversal, and the persistence of momentum, despite the presence of investors who profitably trade on it. Albuquerque and Miao (2014) show that "advance information" produces momentum in a dynamic model. Importantly, our model delivers opposite conclusions regarding trading strategies: in Albuquerque and Miao (2014) informed investors are contrarians and uninformed investors are momentum traders, a difference that could be used to distinguish both theories empirically. As in Biais, Bossaerts, and Spatt (2010), investors in our model follow different investment strategies and extract information from prices. Our focus, however, is on the role that word-of-mouth communication plays in generating momentum and reversal.[4] We adopt the definition of "price drift," as well as portfolio decompositions from Banerjee, Kaniel, and Kremer (2009), who show that stock returns can exhibit momentum when investors have higher-order differences of opinions.

Finally, among the well-established behavioral explanations of momentum, Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1999) are related to this paper. Hong and Stein (1999) show that the slow diffusion of information leads to underreaction. Our theory differs in two key respects. First, we show that learning from prices is instrumental in generating momentum in our model, whereas momentum does not obtain in Hong and Stein (1999) if investors learn from prices. Second, we do not assume that investors follow different trading strategies, but rather let investors optimally decide whether they want to be

of economic life" (Stein, 2008), and plays an important role in stock market fluctuations and in investors' decisions (Shiller, 2000; Shiller and Pound, 1989; Grinblatt and Keloharju, 2001; Hong, Kubik, and Stein, 2004; Feng and Seasholes, 2004; Ivkovic and Weisbenner, 2005; Brown, Ivkovic, Smith, and Weisbenner, 2008; Shive, 2010; Cohen, Frazzini, and Malloy, 2008). Furthermore, evidence suggests that word-of-mouth communication is related to momentum: momentum profits are decreasing in analyst coverage, supporting the notion that momentum is caused by slow information diffusion (Hong et al., 2000; Hou and Moskowitz, 2005; Verardo, 2009).

[4]Biais et al. (2010) build a dynamic model in which momentum may arise depending on the relative persistence of the fundamental and noise trading risk. Instead, in our model, persistence arises endogenously through the acceleration of information through word-of-mouth communication among investors. Other papers related to momentum, but unrelated to social interactions and information diffusion, include Vayanos and Woolley (2013), Makarov and Rytchkov (2012), and Wang (1993).

momentum traders or contrarians. Barberis et al. (1998) and Daniel et al. (1998) introduce two behavioral biases, one of which explains momentum and the other reversal. Our model abstracts from behavioral biases and simultaneously explains momentum and reversal solely based on information percolation. Our work complements existing behavioral theories, as information percolation and behavioral biases could reinforce each other.

The remainder of the paper is organized as follows. Section 2 presents and solves the model, Sections 3 and 4 contain the main results on momentum and momentum trading, Section 5 presents extensions of the model, and Section 6 concludes. All proofs are provided in the Appendix.

2 Information percolation in centralized markets

In this section, we build a model of *centralized* trading (a noisy rational-expectations equilibrium) with *decentralized* information gathering (information percolation). We start by describing the information diffusion mechanism.

2.1. Information percolation

Consider an economy with T trading dates, indexed by $t = 0, 1, ..., T-1$, and a final liquidation date, T. The economy is populated by a continuum of investors indexed by $i \in [0, 1]$. There is a risky security with payoff $\widetilde{U}$ realized at the liquidation date. The payoff of this security is unobservable and follows a normal distribution with zero mean and precision H.[5]

Immediately prior to trading session $t = 0$, each investor i obtains a private signal about the asset payoff, $\widetilde{z}^i$:

$$\widetilde{z}^i = \widetilde{U} + \widetilde{\epsilon}^i, \tag{1}$$

[5]We refer to the precision of a random variable as the inverse of its variance. The zero mean assumption is without loss of generality.

where $\widetilde{\epsilon}^i$ is distributed normally and independently of $\widetilde{U}$, has zero mean, precision S, and is independent of $\widetilde{\epsilon}^k$ if $k \neq i$. The precision of individual private signals is the same across investors.

We now introduce a mechanism that causes information to flow at an increasing rate and information precision to become heterogeneous across agents. To do so, we use the information percolation theory (Duffie, Malamud, and Manso, 2009). From date $t = 0$ onward, agents meet each other randomly and share their information. Meetings take place continuously at Poisson arrival times with intensity λ—the only parameter we add to this standard equilibrium model.

When agents meet, they exchange their initial signal and other signals that they received during previous meetings (if any). This assumption is simply a matter of convention, since incomplete exchange of information can always be incorporated by scaling the meeting intensity.[6] Moreover, agents are infinitesimally small and therefore are indifferent between telling the truth or lying—if they attempt to lie, they will not be able to move prices, and therefore will not benefit from their lies. For this reason, we assume that they tell the truth.[7] This assumption along with the normality of individual signals imply that an agent's private information is completely summarized by two statistics: her total number of signals and her posterior expectation of the fundamental. These two statistics are sufficient for the information agents actually exchange when they meet and "talk."

To illustrate how information percolation works, pick two agents—say D and J—out of the crowd. D and J start with one signal. Suppose the first time they meet someone,

[6] A stylized way of incorporating incomplete exchange of information is to assume that agents share nothing with probability p or share their entire set of signals with probability $1-p$. In this case, the meeting intensity can be reinterpreted as $\widehat{\lambda} \equiv \lambda(1-p)$. While incomplete exchange of signals dampens the effect of percolation, the increase in the cross-sectional average number of signals remains exponential (as we will show shortly in Proposition 1). This exponential increase is the crucial feature which leads to our main results.

[7] While infinitesimal agents do not have a strict incentive to tell the truth in centralized markets, a large but finite number of agents may. Possible incentives to tell the truth include, for instance, short-term investment horizons (Schmidt, 2015), reputation (Benabou and Laroque, 1992), complementarity in information sets (Stein, 2008), the presence of highly connected agents in the market (Acemoglu, Bimpikis, and Ozdaglar, 2014), or coordination motives (Abreu and Brunnermeier, 2002).

they meet each other. They exchange their signals truthfully and therefore end up with two signals after the meeting. Suppose further that D meets someone else, say M, who also has two signals (i.e., M also met someone before). Since D and M are part of an infinite crowd of agents, the person that M has met cannot be J, it must be someone else, i.e., meetings do not overlap.[8] Hence, after the meeting, D and M both part with four signals each. Signals keep on adding up randomly in the exact same way for every agent in the economy.

Random meetings introduce heterogeneity in information precisions: while agents start off holding one signal, they end up with different numbers of signals as soon as they meet and talk. This heterogeneity is captured by the cross-sectional distribution of the number of additional signals, π_t. Formally, between trading dates $t-1$ and t, each agent i collects a number $\omega_t^i \in \mathbb{N}$ of signals, *excluding* the signals she received up to and including time $t-1$.[9] An important statistic in this economy is the cross-sectional average of the number of additional signals at time t:

$$\Omega_t \equiv \sum_{n \in \mathbb{N}} \pi_t(n)\, n. \tag{2}$$

Since agents are initially endowed with a single signal, the initial distribution of signals has 100% probability mass at $n = 1$, and therefore $\omega_0^i = \Omega_0 = 1$, $\forall i \in [0,1]$. As information diffuses (at dates $t > 0$), the distribution π_t takes values over $\mathbb{N}$. For example, an agent who did not meet anyone between $t-1$ and t is of type $n = 0$; an agent who collected ten signals between $t-1$ and t is of type $n = 10$, and so on. Following Duffie et al. (2009), the

[8] In other words, there is a zero probability that the set of agents that D has met before time t overlaps with the set of agents that J has met before time t. This eliminates the concern that we are introducing *persuasion bias* in the terms of Demarzo, Vayanos, and Zwiebel (2003): an agent might share her signals to another agent who passes those signals at subsequent meetings to other agents and maybe the same signals will come back to the first agent—without her knowledge. The infinite mass of agents prevents this double accounting of signals to happen, since the probability for an agent to meet in the future precisely those agents who got her signals is zero. Thus, for every pair (i, j) of agents, their signal sets are always disjoint prior to their meeting.

[9] Notice that both the distribution over the total number of signals and the distribution over additional signals may be equivalently used; we choose to use distribution of additional signals because it helps us better separate and understand the effects of information percolation on the equilibrium price and trading strategies.

cross-sectional distribution of the number of additional signals satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}\pi_t(n) = \lambda\pi_t * \mu_t - \lambda\pi_t = \lambda\sum_{m=1}^{n-1}\pi_t(n-m)\mu_t(m) - \lambda\pi_t(n), \tag{3}$$

where "$*$" denotes the discrete convolution product and μ represents the cross-sectional distribution of the *total* number of signals, which we define in Appendix A.1. The summation term on the right-hand side in (3) represents the rate at which new agents of a given type are created, whereas the second term in (3) captures the rate at which agents leave a given type.

This setup leads to a closed-form solution for both the cross-sectional distribution π_t and the cross-sectional average of the number of additional signals Ω_t.

Proposition 1. *At time $t \in \{1, 2, ..., T-1\}$, the probability density function π_t over the additional number of signals collected by agents between $t-1$ and t is given by*

$$\pi_t(n) = \begin{cases} e^{-\lambda} & \text{if } n = 0 \\ e^{-n\lambda t}(e^{\lambda t} - 1)^{n-1}(1 - e^{-\lambda}) & \text{if } n \geq 1. \end{cases} \tag{4}$$

The cross-sectional average of the number of additional signals at time t is given by

$$\Omega_t = e^{(t-1)\lambda}(e^{\lambda} - 1). \tag{5}$$

Fig. 1 illustrates the evolution of the cross-sectional distribution of both the *total* number of signals $\mu_t(n)$ (the left-hand side) and the *additional* number of signals $\pi_t(n)$ (the right-hand side). The meeting intensity is set at $\lambda = 1$ and each distribution is depicted at times $t = 1$ and $t = 2$. The distribution of the total number of signals is defined over $\mathbb{N}^*$, whereas the distribution of the additional number of signals is defined over $\mathbb{N}$. For both distributions, the average precision and the precision heterogeneity change over time. First, the mass of the distributions gradually shifts towards larger number of signals. As a result, the average

number of signals, and therefore the average precision, increases over time. Second, while both distributions are initially concentrated at $n = 1$ (each agent starts off with one signal), they rapidly spread to reflect the growing heterogeneity in precision across the population. This heterogeneity itself varies through time.[10]

[insert Figure 1 here]

2.2. The economy

We now describe the structure of the economy. The main difference with a standard rational-expectations model (Grossman and Stiglitz, 1980) is that we allow the precision of information to increase over time and to differ across agents.

Investors have exponential utility with common coefficient of absolute risk aversion $1/\tau$, where τ denotes investors' risk tolerance. The asset payoff is realized and consumption takes place at time $t = T$, while trading takes place at times $t = 0, 1, ..., T-1$. Each investor i is endowed at time $t = 0$ with a quantity of the risky asset represented by X^i. At each trading date, investor i chooses a position in the risky asset, $\widetilde{D}_t^i$, to maximize her expected utility of terminal wealth, denoted by $\widetilde{W}_T^i$:

$$\begin{aligned} &\max_{\widetilde{D}_t^i} \mathbb{E}\left[e^{-\frac{1}{\tau}\widetilde{W}_T^i}\middle|\mathcal{F}_t^i\right], \\ \text{subject to} \quad &\widetilde{W}_T^i = X^i\widetilde{P}_0 + \sum_{t=0}^{T-2}\left[\widetilde{D}_t^i\left(\widetilde{P}_{t+1} - \widetilde{P}_t\right)\right] + \widetilde{D}_{T-1}^i\left(\widetilde{U} - \widetilde{P}_{T-1}\right). \end{aligned} \tag{6}$$

The information set of investor i at time t, $\mathcal{F}_t^i$, contains private signals collected through information percolation, and prices (endogenously determined in equilibrium and denoted by $\widetilde{P}_t$) as public signals.[11]

[10] Other aspects are worth mentioning. First, the two distributions have identical shapes at $t = 1$, although their respective support differs. Second, the cross-sectional distribution of additional number of signals assigns the same probability mass at $n = 0$ at all times, as shown in (4); these are investors who did not meet anyone during the last period and consequently end up with zero additional signals.

[11] Our model bears similarities with Brennan and Cao (1997), with the main difference that it embeds an information diffusion mechanism. To keep the setup comparable to leading momentum theories, such as

The aggregate per capita supply of the risky asset at time $t = 0$, $\widetilde{X}_0 = \int_0^1 X^i di$, is normally and independently distributed with zero mean and precision Φ. New liquidity traders enter the market in trading sessions $t = 1, ..., T-1$. The incremental net supply of liquidity traders, $\widetilde{X}_t$, is normally distributed with zero mean and precision Φ.

The noisy supply prevents asset prices from fully revealing the final payoff $\widetilde{U}$. We adopt a random walk specification for the noisy supply, i.e., the total supply at time t is $\sum_{j=0}^{t} \widetilde{X}_j$.[12] Under this specification and in the absence of additional private information at dates $t \geq 1$, prices are martingales. As a result, any pattern in the correlation of returns depends *only* on the pattern of private information arrival. That is, our setup allows us to isolate the link between the diffusion of information and the serial correlation of returns.[13]

The solution method for finding a linear, partially revealing rational-expectations equilibrium is standard and is relegated to Appendix A.2. We describe the equilibrium below.

2.3. Equilibrium

We first introduce notation and terminology for further use. At each date t, agent i receives ω_t^i new signals. From Gaussian theory, these signals are equivalent to a single signal with precision $S\omega_t^i$. We denote this signal by $\widetilde{Z}_t^i$:

$$\widetilde{Z}_t^i = \widetilde{U} + \overline{\varepsilon}_t^i, \quad \text{where } \overline{\varepsilon}_t^i \equiv \left(\frac{1}{\omega_t^i} \sum_{j=1}^{\omega_t^i} \widetilde{\epsilon}^j \right) \sim N\left(0, \frac{1}{S\omega_t^i}\right). \tag{7}$$

Daniel et al. (1998) and Hong and Stein (1999), we focus on a single asset economy, featuring several trading dates and a final liquidation date.

[12] Equivalently, we assume that increments in the noisy supply are independent and identically distributed, which is likely to happen when time between consecutive trading dates is small.

[13] Other specifications, such as an AR(1) noise trading process, give qualitatively similar results, but complicate unnecessarily the analysis.

The conditional precision of agent i about the final payoff $\widetilde{U}$, given all available information, is denoted by K_t^i,

$$K_t^i \equiv \text{Var}^{-1}\left[\widetilde{U}\middle|\mathcal{F}_t^i\right], \tag{8}$$

whereas the cross-sectional average of conditional precisions over the entire population of agents is denoted by K_t,

$$K_t \equiv \sum_{n\in\mathbb{N}} K_t^i(n)\pi_t(n). \tag{9}$$

Throughout the paper, we will often refer to the "average agent" as the agent whose precision of information equals the average precision at time t, K_t. Note that, without information percolation, the average agent represents any agent in the economy, because $K_t^i = K_t,\ \forall i,t$. Finally, we refer to the normalized price signals as

$$\widetilde{Q}_t \equiv \widetilde{U} - \frac{1}{\tau S \Omega_t}\widetilde{X}_t. \tag{10}$$

Observing the signals $\{\widetilde{Q}_j\}_{j=0}^t$ or past prices $\{\widetilde{P}_j\}_{j=0}^t$ generates equivalent information sets. These information sets represent the information available to an econometrician at time t. We denote the precision of the econometrician, conditional on any of these two *common* information sets, by K_t^c.

Theorem 2.1 describes the risky asset prices at each date in a noisy rational-expectations equilibrium with information percolation.

Theorem 2.1. ***(Equilibrium)*** *There exists a partially revealing rational-expectations equilibrium in the T trading session economy in which the price of the risky asset, $\widetilde{P}_t$, for*

$t = 0, ..., T-1$, *is given by:*

$$\widetilde{P}_t = \frac{K_t - H}{K_t}\widetilde{U} - \sum_{j=0}^{t} \frac{1 + \tau^2 S \Omega_j \Phi}{\tau K_t} \widetilde{X}_j. \tag{11}$$

The individual and average market precisions, K_t^i *and* K_t*, are given by*

$$K_t^i = H + \sum_{j=0}^{t} S\omega_j^i + \sum_{j=0}^{t} \tau^2 S^2 \Omega_j^2 \Phi, \tag{12}$$

$$K_t = H + \sum_{j=0}^{t} S\Omega_j + \sum_{j=0}^{t} \tau^2 S^2 \Omega_j^2 \Phi, \tag{13}$$

and the precision of the econometrician is given by

$$K_t^c = H + \sum_{j=0}^{t} \tau^2 S^2 \Omega_j^2 \Phi. \tag{14}$$

The individual asset demands, $\widetilde{D}_t^i$*, are given by*

$$\widetilde{D}_t^i = \tau K_t^i \left(\mathbb{E}[\widetilde{U} | \mathcal{F}_t^i] - \widetilde{P}_t \right) \tag{15}$$

$$= \tau \left(\underbrace{S \sum_{j=0}^{t} \omega_j^i \widetilde{Z}_j^i}_{\textit{Private information}} + \underbrace{\tau^2 S^2 \Phi \sum_{j=0}^{t} \Omega_j^2 \widetilde{Q}_j}_{\textit{Public information}} - K_t^i \widetilde{P}_t \right). \tag{16}$$

The asset price in Eq. (11) is a linear function of the final payoff and supply shocks, as is customary in the noisy rational-expectations literature. Following the interpretation in Wang (1993), the second term in (11) is a discount on the price that compensates informed investors for bearing noise trading risk: as noise traders sell (i.e., the supply increases), the price decreases through the discount, which generates a higher risk premium and thus induces investors to hold more stocks. The size of this risk premium is inversely related to average market precision, because higher information precision reduces the risk of holding

the asset.[14]

Setting λ equal to zero we obtain an economy in which average precision is constant ($\Omega_0 = 1,\ \Omega_t = 0\ \forall t \geq 1$) and agents have identical precisions ($K_t^i = K_t,\ \forall i, t$). This economy serves as our benchmark model. In contrast, when $\lambda > 0$, the average precision increases exponentially over time, as can be seen by replacing $\Omega_j = e^{(j-1)\lambda}(e^{\lambda}-1)$ in (13), and agents become heterogeneous with respect to their information precision. Finally, Eq. (16) shows how agents build their demands based on private and public information, a standard decomposition in the noisy rational-expectations literature (e.g., Brennan and Cao, 1997).

3 Momentum and reversal

In this section we analyze the implications of the increase in average precision caused by information percolation for the serial correlation of returns. In the benchmark model ($\lambda = 0$), the average precision is constant over time and prices are martingales. Return predictability arises only if agents gather new private information over time ($\lambda > 0$). In this case, we derive a general condition on the "shape of learning" that is necessary for momentum to arise in a rational-expectations model. We then describe how information percolation can enforce this condition beyond a certain threshold of the meeting intensity, which we fully characterize. We finally relate our theoretical predictions to existing empirical evidence.

3.1. Predictability of returns

The following proposition establishes the condition under which future returns are predictable.

[14]This negative relationship is consistent with empirical findings. In particular, there is consensus in the accounting literature that increasing the precision of information reflected in prices decreases the cost of capital (Lambert, Leuz, and Verrecchia, 2011). See also Botosan, Plumlee, and Xie (2004), Francis, LaFond, Olsson, and Schipper (2005), and Amir and Levi (2014).

Proposition 2. *Agent i's expectation regarding future returns conditional on $\mathcal{F}_t^i$ satisfies:*

$$\mathbb{E}\left[\widetilde{P}_{t+1} - \widetilde{P}_t \middle| \mathcal{F}_t^i\right] = \underbrace{\frac{K_{t+1} - K_t}{K_{t+1}}}_{\geq 0}\left(\mathbb{E}\left[\widetilde{U}\middle| \mathcal{F}_t^i\right] - \widetilde{P}_t\right). \tag{17}$$

Returns are predictable only if average market precision is strictly increasing over time.

The first term in Eq. (17) represents the relative evolution of average market precision K over time. Clearly, if average market precision is constant, prices are martingales and no agent, even perfectly informed, can predict future returns. If, instead, average market precision increases over time, Eq. (17) shows that an agent can predict future returns by comparing the current price she observes to her current expectation of the fundamental. When she perceives that the stock is overvalued, she predicts negative future returns; when she perceives that the stock is undervalued, she predicts positive future returns.

Future returns can be further decomposed into three main sources of predictability, as we show in Proposition 3.

Proposition 3. *Stock returns from time t to time $t+1$ admit the following decomposition:*

$$\widetilde{P}_{t+1} - \widetilde{P}_t = \underbrace{\left(\overline{\mathbb{E}}_{t+1}[\widetilde{U}] - \overline{\mathbb{E}}_t[\widetilde{U}]\right)}_{\textit{Evolution of market consensus}} + \frac{K_{t+1} - K_t}{\tau K_t K_{t+1}} \underbrace{\sum_{j=0}^{t} \widetilde{X}_j}_{\substack{\textit{Past and current}\\ \textit{supply shocks}}} - \frac{1}{\tau K_{t+1}} \underbrace{\widetilde{X}_{t+1}}_{\substack{\textit{Future}\\ \textit{supply shock}}} \tag{18}$$

where $\overline{\mathbb{E}}_t[\cdot] \equiv \int_i \mathbb{E}[\cdot|\mathcal{F}_t^i]di$ denotes the weighted average market expectation at time t:

$$\overline{\mathbb{E}}_t[\widetilde{U}] = \int_0^1 \frac{K_t^i \mathbb{E}[\widetilde{U}|\mathcal{F}_t^i]}{K_t} di. \tag{19}$$

Three elements drive future returns: *(i)* the evolution of the market consensus, *(ii)* current and past supply shocks, and *(iii)* the supply shock occurring in the future.[15] Notice that

[15] Alternatively, supply shocks can be interpreted as $\widetilde{X}_j = -\tau S \Omega_j (\widetilde{Q}_j - \widetilde{U})$, where $\widetilde{Q}_j - \widetilde{U}$ represents the forecast error that the market makes at time j in estimating the fundamental (a positive error means that the market overvalues the fundamental and vice versa). Market errors are independent across time and are

no agent, even perfectly informed, can predict future supply shocks. Hence, predictability must arise through the first two components of returns. In general, agents' expectations regarding these two components differ, because they have heterogeneous information sets. Agents, however, share a common information set $\mathcal{F}_t^c$, which consists of the history of normalized price signals

$$\mathcal{F}_t^c = \left\{ \widetilde{Q}_j : 0 \leq j \leq t \right\}, \tag{20}$$

and is equivalent to the information set of the econometrician, who observes all past prices. From the point of view of the econometrician, the evolution of market consensus is not predictable (see Appendix B.3 for a proof):

$$\mathbb{E}\left[\overline{\mathbb{E}}_{t+1}[\widetilde{U}] - \overline{\mathbb{E}}_t[\widetilde{U}] \middle| \mathcal{F}_t^c \right] = 0. \tag{21}$$

Therefore, observing past prices only, predictability arises exclusively through the inference of the current and past supply shocks, a result we summarize in the following proposition.

Proposition 4. *From the point of view of the econometrician, return predictability arises solely from the inference of current and past supply shocks:*

$$\mathbb{E}\left[\widetilde{P}_{t+1} - \widetilde{P}_t \middle| \mathcal{F}_t^c \right] = \frac{K_{t+1} - K_t}{\tau K_t K_{t+1}} \sum_{j=0}^{t} \mathbb{E}\left[\widetilde{X}_j \middle| \mathcal{F}_t^c \right] \tag{22}$$

$$= \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \sum_{j=0}^{t} S\Omega_j \left(\widetilde{P}_t - \widetilde{Q}_j \right). \tag{23}$$

Proposition 4 highlights the relation between public information and expected returns. However, the relation in (23) is not based on a standard definition of past returns. While observationally equivalent, the common information set $\mathcal{F}^c$ has a different economic meaning

normally distributed with mean zero and precision $\tau^2 S^2 \Omega_j^2 \Phi$. Substituting supply shocks by market errors in the decomposition in (18) yields an alternative interpretation of return predictability in terms of information, as opposed to risk. These two interpretations of return predictability—information and risk—can be viewed as two sides of the same coin.

than the information set containing all past returns:

$$\mathcal{F}_t^r = \left\{ \widetilde{P}_{t-l+1} - \widetilde{P}_{t-l} : 1 \leq l \leq t+1 \right\} \tag{24}$$

where $\widetilde{P}_{-1} = \Omega_{-1} \equiv 0$ and $K_{-1} \equiv H$. We therefore follow the convention introduced by Banerjee et al. (2009) and condition future returns on past returns, as opposed to the common information set $\mathcal{F}^c$. We obtain from Eq. (23) an expression for the serial correlation of returns at different lags.

Proposition 5. *Conditional on past returns, expected future returns satisfy*

$$\mathbb{E}\left[\widetilde{P}_{t+1} - \widetilde{P}_t \middle| \mathcal{F}_t^r \right] = \sum_{l=1}^{t+1} \underbrace{\frac{K_{t+1} - K_t}{K_{t+1} K_t^c} m_{t-l}}_{\substack{\textit{Serial correlation of returns} \\ \textit{at the } l-\textit{th lag}}} \left(\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l} \right), \tag{25}$$

where the coefficients m_{t-l}, for lags $l = 1, ..., t+1$, are defined as:

$$m_{t-l} \equiv \underbrace{\sum_{k=0}^{t-l} S\Omega_k}_{\substack{\textit{Momentum} \\ \textit{effect}}} - \underbrace{\frac{S\Omega_{t-l+1}}{\left(K_{t-l+1} - K_{t-l}\right)/K_{t-l}}}_{\substack{\textit{Reversal} \\ \textit{effect}}}. \tag{26}$$

To understand the sign of the serial correlation coefficient m in (25), consider the first lag, $l = 1$, and use Proposition 3 to write current returns as

$$\widetilde{P}_t - \widetilde{P}_{t-1} = \overline{\mathbb{E}}_t[\widetilde{U}] - \overline{\mathbb{E}}_{t-1}[\widetilde{U}] + \frac{K_t - K_{t-1}}{\tau K_{t-1} K_t} \sum_{j=0}^{t-1} \widetilde{X}_j - \frac{1}{\tau K_t} \widetilde{X}_t. \tag{27}$$

Eqs. (18) and (27) then reveal that the current supply shock, $\widetilde{X}_t$, moves current and future returns in opposite directions, generating reversal in stock returns. This result originates from inventory considerations (Grossman and Miller, 1988). Because risk-averse informed investors act as market makers and accommodate the noninformational demand of noise traders, they require a risk premium for holding the asset. As a result, current supply shocks

create a negative relation between current and future returns. For instance, a positive supply shock today (i.e., noise traders sell the stock) simultaneously decreases the stock price and increases the risk premium for holding a larger supply of the asset. Similarly, a negative supply shock today leads to high current returns and low future returns. While the current supply shock generates reversal, past supply shocks, $(\widetilde{X}_j)_{j=0}^{t-1}$, produce momentum in stock returns. In particular, Eqs. (18) and (27) show that the coefficients of past supply shocks are always nonnegative. The reason is that the market continues to "absorb" past supply shocks in future trading rounds.

Whether the *reversal effect* of the current supply shock or the *momentum effect* of past supply shocks in Eq. (26) dominates determines the sign of the serial correlation of stock returns at the l-th lag (equivalently, the sign of the coefficient m_{t-l}). First, the momentum effect strengthens as private signals "accumulate." Second, the reversal effect also strengthens with the accumulation of private signals (numerator), but *weakens* as average market precision increases (denominator). For momentum to arise, the increase in average market precision must be sufficiently large to restore the balance in favor of the momentum effect.

To determine the necessary condition on the increase in average market precision that enforces momentum, we introduce the concept of precision elasticity. This concept relies on the average precision of private information at time t, $S\overline{\Omega}_t$, where $\overline{\Omega}_t \equiv \sum_{j=0}^{t} \Omega_j$ denotes the total number of private signals that the average agent holds at time t.

Definition 1. ***(Precision Elasticity)*** *Precision elasticity at time t is the percentage change of average market precision, K_t, relative to a percentage change in the average precision of private information, $S\overline{\Omega}_t$:*

$$\epsilon_t \equiv \frac{(K_{t+1} - K_t)/K_t}{(S\overline{\Omega}_{t+1} - S\overline{\Omega}_t)/(S\overline{\Omega}_t)} = \frac{\sum_{j=0}^{t} S\Omega_j}{H + \sum_{j=0}^{t} S\Omega_j + \sum_{j=0}^{t} \gamma^2 S^2 \Omega_j^2 \Phi} \left(1 + \gamma^2 S\Omega_{t+1}\Phi\right). \tag{28}$$

Precision elasticity is a general concept that characterizes the "shape of learning" in a rational-expectations model. It measures how average market precision responds to a change

in the average precision of private information. Eq. (28) shows that this response is always positive.

Importantly, precision elasticity is lower than one if information arrives at a linear rate (i.e., if $\Omega_t \equiv \Omega$, for all t),

$$\epsilon_t|_{\Omega_t \equiv \Omega} = 1 - \frac{H}{H + S\Omega(1 + \tau^2 S\Phi\Omega)(t+1)} \leq 1, \tag{29}$$

a customary assumption in rational-expectations models. In contrast, for momentum to obtain, precision elasticity must be higher than one, a condition we establish in Theorem 3.1.

Theorem 3.1. ***(Momentum Condition)*** *Returns exhibit momentum at the $l-th$ lag if and only if precision elasticity at lag $l \leq t$ is higher than one:*

$$\epsilon_{t-l} > 1. \tag{30}$$

At lag $l = t + 1$, the serial correlation of returns is always negative or zero.

The main implication of Theorem 3.1 is that learning must have a specific "shape" for momentum to arise in a rational-expectations model—the precision elasticity needs to be greater than one so that the percentage increase in average market precision exceeds the percentage increase in the precision of private information.

Learning from prices is instrumental in generating this pattern. Theorem 2.1 shows that the price-learning channel improves investors' precision by the square of the incremental number of private signals, Ω_j, $j = 1, .., t$. This quadratic increase precisely allows learning from prices to create a larger increase in average market precision relative to the precision of private information. It follows that learning from prices is the main channel through which momentum arises in this model. To emphasize the particularity of this result, notice that in Hong and Stein (1999) momentum does not obtain if investors learn from prices.

Learning from prices is necessary, but not sufficient to produce momentum. Not only

does momentum require the price-learning channel for average market precision to increase faster than the flow of private information, it also requires the flow of private information to have the "right dynamics." For instance, Theorem 3.1 and Eq. (29) show that momentum never arises in a standard rational-expectations model when the flow of private information is linear. We now study how information percolation can generate dynamics that enforce the condition in (30) through an exponential increase in the average number of signals.

3.2. Information percolation and momentum

We formalize the effect of information percolation on the serial correlation of returns and show that the momentum condition in Eq. (30) is always satisfied beyond a certain threshold of the meeting intensity.

Theorem 3.2. *For each horizon $t - l \geq 0$,*

1. *There exists a unique threshold, $\lambda^\star(H, S, \Phi, \tau, t-l) \in \left(0, \log\left(2 + \frac{H}{\Phi\tau^2 S^2}\right)\right]$, of the meeting intensity above which stock returns always exhibit momentum. This threshold satisfies the following implicit equation:*

$$\lambda^\star(H, S, \Phi, \tau, t-l) = \ln\left(\frac{K_{t-l+1}}{K_{t-l}}\right). \tag{31}$$

2. *The threshold $\lambda^\star(H, S, \Phi, \tau, t-l)$ is increasing in H and decreasing in S, Φ, τ, and $t - l$.*
3. *Returns are martingales when $\lambda = 0$ or when $\lambda \to \infty$.*

The first part of Theorem 3.2 characterizes the threshold $\lambda^\star$ of the meeting intensity above which information percolation creates momentum in stock returns. This threshold directly follows from the momentum condition of Theorem 3.1. At the threshold $\lambda^\star$ of the meeting intensity, Eq. (31) is exactly satisfied, precision elasticity is equal to one, and returns are martingales: the (logarithmic) increase in the precision of private information

coincides with the (logarithmic) increase in average market precision. As a result, when the meeting intensity is below $\lambda^\star$, the increase in the precision of private signals dominates, precision elasticity is less than one, and returns therefore exhibit reversals; when the meeting intensity is above $\lambda^\star$, the increase in average market precision dominates, precision elasticity is higher than one, and thus returns exhibit momentum.

The second part of Theorem 3.2 shows how the threshold $\lambda^\star$ reacts to changes in the parameters of the model. An increase in fundamental precision requires stronger information percolation to generate momentum, whereas the other parameters have the opposite effect. For instance, decreasing noise trading helps information percolation generate momentum. Less noise trading allows more information to be revealed through prices, which decreases the risk premium and the reversal effect associated with it. A similar reasoning applies to risk aversion and the precision of individual signals.

The momentum threshold decreases with the horizon $t-l$ and therefore increases with the lag l. Hence, as we increase the lag, we need a higher meeting intensity to generate momentum, i.e., $\lambda^\star(\cdot, t-l-1) > \lambda^\star(\cdot, t-l)$. An immediate consequence of this result, which is empirically appealing, is that there always exists a meeting intensity $\lambda \in (\lambda^\star(\cdot, t-l), \lambda^\star(\cdot, t-l-1))$, such that we simultaneously obtain short-term momentum and long-term reversal. A second consequence is that the serial correlation of returns in Eq. (25) decays with the lag in the momentum region, thus generating a downward-sloping term structure of momentum.

To illustrate the different points of Theorem 3.2, we plot in Fig. 2 the serial correlation of returns as a function of the meeting intensity for different lags. Without information percolation, returns are unpredictable, consistent with the last point of Theorem 3.2. With information percolation, stock returns exhibit reversals when the meeting intensity is below the threshold $\lambda^\star$ defined in Theorem 3.2. As information percolation intensifies and the meeting intensity rises above the threshold $\lambda^\star$, stock returns become positively autocorrelated. Furthermore, the momentum thresholds increase with the lag (the second point of Theorem

3.2), while the magnitude of momentum decays with the lag. Finally, for large values of the meeting intensity, returns become serially uncorrelated (the last part of Theorem 3.2).

[insert Figure 2 here]

Importantly, Fig. 2 shows that momentum is hump-shaped in the meeting intensity. To see this, pick the momentum threshold as the meeting intensity, $\lambda = \lambda^{\star}(\cdot, t-l)$. At this point, the reversal effect associated with current supply shocks exactly offsets the momentum effect associated with the revision of past supply shocks. An increase in the meeting intensity beyond this threshold weakens the reversal effect and strengthens the momentum effect, thereby creating an increasing relation between momentum and the meeting intensity. As the meeting intensity becomes infinite, not only does the reversal effect die out, but the momentum effect also disappears. As a result, the relation between momentum and the meeting intensity becomes decreasing as the meeting intensity increases, resulting in a hump-shaped pattern.

3.3. Model predictions and empirical evidence

Our model predicts a hump-shaped relation between momentum and the meeting intensity, as apparent from Fig. 2. This prediction is consistent with Hong et al. (2000), who test the momentum theory of Hong and Stein (1999). Both our theory and that of Hong and Stein (1999) centrally rely on the speed at which information diffuses in the market, which Hong et al. (2000) proxy using firm size. Specifically, Hong et al. (2000) document a hump-shaped pattern between firm size and the profitability of momentum.[16] While Hong and Stein (1999) rationalize the decreasing part of this relation, our model further explains reversals for small firms through liquidity shocks (see also French and Roll, 1986).

[16]Note that the empirical study of Hong et al. (2000) is related to cross-sectional momentum, whereas both Hong and Stein (1999) and our theoretical study relate to time-series momentum. These two forms of momentum are strongly related but not identical. See Moskowitz et al. (2012) for a discussion.

Lehmann (1990) and Jegadeesh (1990) find that stock returns exhibit strong reversals at frequencies less than a month. In our model, the amount of information that agents accumulate depends on the time elapsed between trading rounds—the longer the time within trading rounds is, the more information agents accumulate through random meetings. Consequently, fixing the meeting intensity, our model predicts that the sign of serial correlation varies for different trading frequencies. While the information percolation mechanism generates momentum at lower trading frequencies, agents have little time to talk between trading rounds at high trading frequencies and short-term reversal therefore prevails.

An additional empirical finding is that the magnitude of momentum is large for short lookback periods (one to six months) and decays as the lookback period increases, with weaker evidence of reversal for periods longer than 12 months (Moskowitz et al., 2012).[17] Information percolation bears similar time-series implications. To see this, note that the specification of Proposition 5 can also be written for different lookback periods:

$$\mathbb{E}\left[\widetilde{P}_{t+1}-\widetilde{P}_t\middle|\,\mathcal{F}_t^r\right]=\sum_{l=1}^{t}\frac{K_{t+1}-K_t}{K_{t+1}K_t^c}m'_{t-l}\left(\widetilde{P}_t-\widetilde{P}_{t-l}\right)-\frac{K_{t+1}-K_t}{K_{t+1}K_t^c}\frac{H}{1+\tau^2\Phi S\Omega_0}\widetilde{P}_0, \tag{32}$$

and

$$m'_{t-l}\equiv K_{t-l}\left(\frac{1}{1+\tau^2 S\Omega_{t-l}\Phi}-\frac{1}{1+\tau^2 S\Omega_{t-l+1}\Phi}\right)-\frac{H}{1+\tau^2\Phi S\Omega_0}. \tag{33}$$

Eq. (32) indicates that momentum also arises for larger lookback windows. Since the second term in (32) is negative, Eq. (33) suggests a decaying "term-structure" of momentum, whereby returns exhibit momentum for short lookback periods and reversal over longer lookback periods. To illustrate this decaying pattern of momentum, we plot the serial correlation in Eqs. (32) – (33) as a function of the lookback window in Fig. 3. For low values of the meeting intensity, returns exhibit reversals at all horizons (the solid bars). In contrast,

[17]Moskowitz et al. (2012) find that this decaying pattern differs across asset classes. The pattern is decaying for commodities, equities, and currencies and U-shaped for other asset classes.

information percolation at a higher intensity generates short-run momentum and long-run reversals (the dashed bars), consistent with Moskowitz et al. (2012).[18]

[insert Figure 3 here]

Novy-Marx (2012) proposes an alternative measure of momentum, which includes past returns at both recent and intermediate horizons. This way of measuring momentum is similar to our momentum measure in Proposition 5, which includes all past returns. In the context of our model, ignoring lags leads to an omitted-variables bias, which can potentially result in overestimating the magnitude of time-series momentum, a matter on which we now elaborate.

3.4. Standard definition of momentum

Our measure of momentum in Proposition 5 uses all past returns to obtain a complete description of momentum. Except for Novy-Marx (2012), including more than one lag is not a standard way of measuring momentum empirically. To be consistent with a large body of empirical literature, we compute the regression coefficient in Proposition 5 when the right-hand side includes only the most recent past return.

Proposition 6. *Under the standard definition of momentum, expected future returns satisfy*

$$\mathbb{E}\left[\widetilde{P}_{t+1}-\widetilde{P}_t\middle|\,\widetilde{P}_t-\widetilde{P}_{t-1}\right]=\frac{K_{t+1}-K_t}{K_{t+1}K_t^c}\left(m_{t-1}+b_{t-1}\right)\left(\widetilde{P}_t-\widetilde{P}_{t-1}\right), \tag{34}$$

where $b_{t-1}\geq 0$ is a positive bias, which arises through the omission of past returns:

$$b_{t-1}=\frac{K_t-K_{t-1}}{K_tK_{t-1}^c}\frac{\mathrm{Var}[\sum_{l=1}^{t}m_{t-1-l}(\widetilde{P}_{t-l}-\widetilde{P}_{t-1-l})]}{\mathrm{Var}[\widetilde{P}_t-\widetilde{P}_{t-1}]}. \tag{35}$$

[18] We do not attempt to match the magnitude of momentum, nor the exact length of the decaying pattern. Our purpose is to highlight a theoretical mechanism that can generate such pattern. Empirical work is yet needed to estimate plausible values of λ and other parameters to better match the data.

In our setup, omitting past realized returns creates a positive bias in the estimate of the coefficient of serial correlation. To illustrate the magnitude of this bias, we plot in the left panel of Fig. 4 the serial correlation of returns as a function of the meeting intensity, including or excluding past realized returns. The left panel shows that the bias introduced by the standard measure is non-monotonic in the meeting intensity. For large or small values of the meeting intensity, both our measure and the standard measure coincide. For intermediate values of the meeting intensity, however, the bias can be substantial, suggesting that our measure provides a conservative estimate of momentum relative to the standard measure.

[insert Figure 4 here]

The size of the estimation bias decreases with the length of the lookback period considered. In particular, the right panel of Fig. 4 depicts momentum for lookback periods of different lengths (Moskowitz et al., 2012), while keeping the meeting intensity constant. Excluding past realized returns (dashed bars) produces a strong positive bias at short horizons (relative to including all realized returns).[19]

In general, Proposition 6 and Fig. 4 demonstrate that including all past returns produces a lower estimate of momentum relative to standard measures, a result that strengthens the conclusions of the previous section. Moreover, the relation between momentum and the meeting intensity remains robust to the inclusion or the exclusion of past returns. Finally, the bias arising from the exclusion of past returns prevails independently of the pattern of information arrival that we propose in this paper—omitting lagged returns always results in overestimating momentum in a rational-expectations model.

Empirically, however, the difference between the two measures in Propositions 5 and 6

[19]While the right panel of Fig. 4 shows that returns exhibit momentum over the range of lookback periods considered, extending the lookback period beyond 12 months shows that they eventually exhibit long-term reversal, consistent with a vast majority of empirical literature. Hence, for certain values of λ, there exists a threshold of the lookback period beyond which the sign of the serial correlation changes from positive to negative. Comparative statics about this threshold open the possibility of additional tests of the model. We thank a referee for this suggestion.

does not necessarily constitute an estimation bias. Rather, this difference is a matter of how one defines momentum empirically. In that respect, the existence of a bias in our model suggests a possible way of validating the model empirically by comparing the two definitions; it also provides an alternative approach to rationalize the term structure of momentum documented in Novy-Marx (2012).

4 Trading strategies

In this section we analyze investors' trading strategies. We first decompose investors' demand into two components, a short-term and long-term component. We show that information percolation induces investors to trade on short-term price moves, as opposed to long-term fundamentals. We then show that information percolation generates heterogeneity in precision, which induces better informed investors to front run those lesser informed. Specifically, better informed investors act as "profit takers," while lesser informed investors follow the public opinion. As a result, better informed investors systematically trade against the serial correlation of returns: when returns exhibit momentum, better informed investors are contrarians, while lesser informed investors are momentum traders. In contrast, with homogeneous precisions, all agents are market neutral in the eyes of the econometrician.

By trading at the expense of momentum traders, contrarians optimally allow momentum to persist, despite the existence of momentum traders. This result is key to our theory of momentum: while an exogenous increase in average market precision is sufficient to generate momentum (Holden and Subrahmanyam, 2002), investors do not trade on it if they have homogeneous precisions. However, what makes momentum a puzzle is that it persists, despite the presence of momentum traders. Our model offers a potential answer to this puzzle based on heterogeneity in individual precisions—momentum survives in the presence of momentum traders because better informed investors trade against it.

4.1. Predictability of trading strategies

We start by decomposing investors' trading strategies in (16) into two components.

Proposition 7. *At date t, agent i's optimal demand is given by*

$$\widetilde{D}_t^i = \frac{\tau K_t^i}{K_t}\left(\frac{K_t^2}{K_{t+1}}\underbrace{\left(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^i]-\widetilde{P}_t\right)}_{\substack{\textit{Long-term}\\ \textit{position}}}+K_t\underbrace{\left(\mathbb{E}[\widetilde{P}_{t+1}|\mathcal{F}_t^i]-\widetilde{P}_t\right)}_{\substack{\textit{Short-term}\\ \textit{position}}}\right). \tag{36}$$

Agent i's optimal demand is the product of two terms. The first term, $\tau K_t^i/K_t$, shows that each agent i compares the precision of her own information with the average market precision and trades more aggressively when her precision is higher. The second term (in brackets) has the same structure for all agents and consists of two components [see also Banerjee et al. (2009) for a similar decomposition]:

1. A *long-term position*, reflecting the agent's view about the long-term payoff.
2. A *short-term position*, reflecting the agent's view about short-term price moves.

When average market precision remains constant over time (i.e., $K_{t+1} = K_t$), prices are martingales (see Proposition 2) and agents' short-term position drops out of (36): because the price tomorrow does not contain more information than the price today, agents focus on their long-term view of the fundamental. With information percolation, in contrast, the price tomorrow incorporates increasingly precise information about the fundamental, causing investors' short-term position to dominate their demand (in the extreme case whereby agents collect an infinite amount of information between t and $t+1$, their optimal demand at time t becomes myopic and consists of their short-term position exclusively). Hence, the decomposition in (36) shows that information percolation, by generating an increase in average market precision, induces investors to optimally adopt trading strategies based on short-term views. This short-term trading activity arises as investors anticipate an increase

in average market precision next period, K_{t+1}, which they can perfectly predict.[20]

We now analyze how investors expect to trade in the future. To do so, we first establish in Proposition 8 a key relationship necessary to understand investors' trading behavior.

Proposition 8. *The portfolio of any agent i in the economy, rescaled by the inverse of her relative precision, K_t/K_t^i, is a martingale:*

$$\mathbb{E}\left[\left.\frac{K_{t+1}}{K_{t+1}^i}\widetilde{D}_{t+1}^i\right|\mathcal{F}_t^i\right] = \frac{K_t}{K_t^i}\widetilde{D}_t^i. \tag{37}$$

Proposition 8 has two important implications, which we present as corollaries. First, an agent whose current precision coincides with average market precision, K_t, cannot predict how she will trade next period.

Corollary 1. *If $K_t^i = K_t$ then*

$$\mathbb{E}\left[\left.\widetilde{D}_{t+1}^i - \widetilde{D}_t^i\right|\mathcal{F}_t^i\right] = 0. \tag{38}$$

As a consequence, the average agent is neutral to the market: she is neither a momentum trader nor a contrarian.[21] This agent therefore serves as a useful benchmark when analyzing the heterogeneity of trading strategies generated by information percolation.

Second, the expected trading strategy of any agent i conditioned on the common information set, $\mathcal{F}^c$, can be written as follows.

Corollary 2. *The trading strategy of agent i, as measured by the econometrician, satisfies*

$$\mathbb{E}\left[\left.\widetilde{D}_{t+1}^i - \widetilde{D}_t^i\right|\mathcal{F}_t^c\right] = \tau\frac{(K_t - K_t^i)(K_{t+1} - K_t)}{K_{t+1}}\left(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^c] - \widetilde{P}_t\right). \tag{39}$$

[20] The cross-sectional average of investors' precision for the next period, K_{t+1}, is known at time t, because it is just a function of time. In other words, investors know today how precise they will be *on average* next period, although they do not know what their *individual* precision will be next period.

[21] One can see this by applying the law of iterated expectations on (38) and conditioning on the last period price move, $\widetilde{P}_t - \widetilde{P}_{t-1}$. This directly implies that $\text{cov}(\widetilde{D}_{t+1}^i - \widetilde{D}_t^i, \widetilde{P}_t - \widetilde{P}_{t-1}) = 0$.

From now on we adopt the point of view of the econometrician and describe agents' trading strategies with respect to the common information set $\mathcal{F}_t^c$, under which strategies are comparable directly. The econometrician can predict how agent i trades only if average market precision improves over time *and* if agent i's precision differs from the average market precision. In particular, better informed agents ($K_t^i > K_t$) trade against the "public opinion," as measured by the term $(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^c] - \widetilde{P}_t)$, whereas less informed agents follow the public opinion. We draw two conclusions from this observation. First, a model in which all agents have the same precision does not produce predictable trading—even though an exogenous increase in precision can generate momentum (Theorem 3.1), no one would trade on it. Second, heterogeneous precisions create an additional layer of trading activity, whereby informed agents not only trade against noise traders, but also trade against less informed traders.

That better informed agents trade against lesser informed agents can be interpreted as a competitive form of "front running." To illustrate this, suppose that the public opinion today is that the stock is undervalued, $\mathbb{E}[\widetilde{U}|\mathcal{F}_t^c] > \widetilde{P}_t$. All agents then buy the stock today—the better informed they are, the more aggressively they buy.[22] Eq. (39) in turn indicates that lesser informed investors expect to further increase their position tomorrow, thus following the public opinion (their trades tomorrow are positively correlated with the public opinion today). In contrast, while better informed investors build a large position today, they expect to partly unwind it tomorrow at the expense of the lesser informed agents, thus acting as "profit takers." We emphasize, however, that this "front-running" behavior is not strategic, since investors, who belong to a continuum, do not have price impact in our model.

[22]To see this, notice that $\widetilde{D}_t^i = \tau K_t^i(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^i] - \widetilde{P}_t)$. Thus, $\mathbb{E}[\widetilde{D}_t^i|\mathcal{F}_t^c] = \tau K_t^i(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^c] - \widetilde{P}_t)$, and therefore if $\mathbb{E}[\widetilde{U}|\mathcal{F}_t^c] - \widetilde{P}_t > 0$, investors buy the stock today.

4.2. Information percolation and momentum trading

To identify trend-following and contrarian strategies, we follow the convention introduced by Brennan and Cao (1997). This approach is consistent with the convention we adopted to measure momentum: we condition future trading strategies on the information set containing past returns, $\mathcal{F}_t^r$. We provide this trading measure in Proposition 9.

Proposition 9. *Conditional on past returns, the expected trading strategy of investor i from time t to $t+1$ satisfies*

$$\mathbb{E}\left[\widetilde{D}_{t+1}^i - \widetilde{D}_t^i | \mathcal{F}_t^r\right] = \sum_{l=1}^{t+1} \underbrace{\tau(K_t - K_t^i)\frac{K_{t+1}-K_t}{K_{t+1}K_t^c} m_{t-l}}_{\textit{Momentum trading coefficient}} (\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l}), \tag{40}$$

where the coefficients m_{t-l} are defined in Proposition 5.

Investors' trading behavior is tightly related to the serial correlation of returns: Eq. (40) shows that the trading coefficient of an investor i is the serial correlation of returns multiplied by a factor $\tau(K_t - K_t^i)$, measuring how investor i's precision compares to average market precision. As a result, better informed investors trade systematically against the serial correlation of returns: when returns exhibit reversals they are momentum traders and when returns exhibit momentum they are contrarians. The opposite mechanism applies for lesser informed investors. This trading behavior is consistent with the front-running pattern we previously discussed: better informed agents speculate against the public opinion and front run the trades of the lesser informed agents, who, they expect, trade on momentum.

To illustrate these points, we plot in Fig. 5 the "trading coefficient" at lag $l = 1$, as a function of the meeting intensity and for two investor types: (i) the 5% percentile *least informed investor* (solid line) and (ii) the 95% percentile *best informed investor* (dashed line). The area between the lines therefore captures 90% of the investor population. In the absence of information percolation, all investors are neutral. Because they have information with identical precision, they are neither momentum traders nor contrarians. For positive

values of the meeting intensity, optimal trading strategies differ. Better informed investors are momentum traders in the reversal region and contrarians in the momentum region, whereas the opposite holds for the lesser informed investors. Finally, the spectrum of trading strategies expands as the magnitude of momentum increases and contracts as the magnitude of momentum decreases. At the threshold λ^*, prices are martingales and therefore investors do not trade based on past prices.[23]

[insert Figure 5 here]

These trading patterns are consistent with empirical evidence. A recent empirical study by Grinblatt et al. (2016) finds that most hedge funds are contrarians, whereas most mutual funds tend to follow momentum strategies. Furthermore, contrarian hedge funds make profits on mutual funds by buying stocks that mutual funds sell. While hedge funds outperform mutual funds on average, mutual funds consistently profit from momentum trading (Grinblatt, Titman, and Wermers, 1995). Another strand of the literature documents that investors who have a broader experience on how the market operates—specialists and commercial investors—are contrarians and liquidity providers (Hendershott and Seasholes, 2007; Moskowitz et al., 2012). In contrast, mutual fund flows chase past performance and further exacerbate market anomalies (Akbas, Armstrong, Sorescu, and Subrahmanyam, 2015; Lou, 2012). Kelley and Tetlock (2013) observe that informed retail trades predict returns, all the more so in markets with higher investor heterogeneity, consistent with our idea that heterogeneity is a key element in understanding return predictability. Finally, Baltzer, Jank, and Smajlbegovic (2015) show that foreign investors trade on momentum, while domestic investors, who presumably possess more information about domestic stocks, are contrarians. While this evidence can be used to distinguish empirically our theory from Albuquerque and

[23]Because the momentum trading coefficient is just the serial correlation of returns rescaled by an agent's relative precision and risk tolerance, the analysis of Section 3.4 also applies to trading strategies: conditioning only on the most recent lagged return (as opposed to all past returns) leads to a positively biased estimator for the momentum trading coefficient. The econometrician may therefore overestimate the magnitude of momentum trading by market participants.

Miao (2014), who predict different trading patterns, further research is needed to distinguish our theory from theirs.

Overall, our model provides an explanation to the puzzling observation that time-series momentum persists, even though investors trade on it (Moskowitz et al., 2012). In our setup, better informed individuals trade systematically against the serial correlation of returns, front running the lesser informed agents. Conversely, the main force that could eliminate momentum—the lesser informed investors—is also the weakest one. A potential caveat is that an unconstrained, risk-neutral arbitrageur could enter the market and conceivably eliminate momentum. We consider this possibility in Appendix C.4 and show that this arbitrageur must necessarily impact prices to eliminate momentum. Since her trades move prices adversely, she faces a tradeoff between trading aggressively on momentum and moderating her price impact. Hence, she optimally decides not to eliminate momentum completely.

5 Extensions

In this section, we extend our model along two dimensions. First, we extend the model to a stationary equilibrium—a setup in which the asset pays an infinite stream of dividends instead of a single liquidating dividend. In this setup, we demonstrate that information percolation generates and amplifies momentum, thus generalizing our previous results. Second, we incorporate a "rumor" in our benchmark model and show that it can generate a phase of price over-shooting followed by a phase of price correction. Under certain conditions, this convergence pattern can jointly produce short-term momentum and long-term reversal.

5.1. Dynamic setup

We present a simplified version of the stationary model and relegate all technical details to Appendix D.1. Consider an economy that goes on forever and in which the stock pays a stochastic dividend D_t per share. As in the finite-horizon version of the model, new liquidity

traders enter the market in every trading session. For simplicity, we assume that the dividend process D_t and the supply process X_t follow random walks (we solve a general version of the model with AR(1) processes in Appendix D.1):

$$D_t = D_{t-1} + \varepsilon_t^d \tag{41}$$

$$X_t = X_{t-1} + \varepsilon_t^x. \tag{42}$$

All investors observe the past and current realizations of dividends and stock prices. Each investor observes a signal about the dividend innovation 3-steps ahead:[24]

$$\widetilde{z}_t^i = \varepsilon_{t+3}^d + \widetilde{\epsilon}_t^i. \tag{43}$$

As in the baseline model, investors meet and share private information over time. A fundamental difference, however, is that investors do not only talk about a single liquidation value, but about several dividends revealed at different times in the future—they share information about the dividend 3-steps ahead, 2-steps ahead, and 1-step ahead.

Unlike the baseline model, we consider an overlapping generation of agents, as in Bacchetta and Wincoop (2006), Watanabe (2008), Banerjee (2011), and Andrei (2013). This assumption considerably simplifies the analysis by ruling out dynamic hedging demands and does not change the results qualitatively.[25] The solution method, which follows Andrei (2013), proceeds by specifying an equilibrium price that is a linear function of model

[24]Note that the model can be extended to a general case in which investors receive information about the dividend T-steps ahead at the expense of analytical complexity and without altering the main intuition presented here.

[25]In the infinite-horizon case the portfolio maximization problem is substantially more complicated. The fixed-point problem cannot be reduced to a finite dimensional one, but Bacchetta and Wincoop (2006) and Andrei (2013) show how to approximate the problem to a desired accuracy level by truncating the state space. The (numerical) results for the infinite-horizon model are very close to those obtained in the overlapping generations model. See also Albuquerque and Miao (2014).

innovations:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3\ a_2\ a_1)\epsilon_t^d + (b_3\ b_2\ b_1)\epsilon_t^x, \tag{44}$$

where $\epsilon_t^d \equiv (\varepsilon_{t+1}^d\ \varepsilon_{t+2}^d\ \varepsilon_{t+3}^d)^\top$ are the three unobservable dividend innovations occurring in the future and $\epsilon_t^x \equiv (\varepsilon_{t-2}^x\ \varepsilon_{t-1}^x\ \varepsilon_t^x)^\top$ are the last three supply innovations. In general, the coefficients a are positive, whereas the coefficients b are negative. The main difference with respect to our baseline model is that equilibrium prices are now stationary: the coefficients α, β, a, and b in (44) do not change over time, in contrast to the price coefficients in Theorem 2.1. These coefficients, however, have a term structure capturing the price effect of the next three dividend shocks and the last three supply shocks.

Information percolation significantly affects the term structure of the price coefficients. Because investors dynamically talk about the next three dividends, they spend more time talking about the dividend one step ahead, as compared to dividends occurring two or three periods ahead. As a result, they have more information regarding the dividend one step ahead. It follows that information percolation causes the coefficient a_3 to increase faster than the coefficient a_2, which itself increases faster than the coefficient a_1. The upper left panel of Fig. 6 illustrates the term structure of coefficients a for $\lambda = 0$ (solid line) and $\lambda = 1$ (dashed line). Clearly, information percolation "steepens" the term structure of coefficients and magnifies the differences between them. The upper right panel of Fig. 6 shows that a similar term structure prevails for supply coefficients. In particular, information percolation exacerbates the effect of current supply shocks (the coefficient b_1 becomes larger in absolute terms), relative to past supply shocks (as measured by the coefficients b_2 and b_3). Overall, the percolation mechanism steepens the term structure of price coefficients.

[insert Figure 6 here]

The term structure of price coefficients is the main determinant of the sign of the serial correlation of returns. To see this, consider two consecutive price changes, $P_t - P_{t-1}$ and

$P_{t+1} - P_t$. Each price change depends differently on future dividend shocks and past supply innovations. Table 1 decomposes the dependence of price changes on each shock. Computing the covariance between $P_t - P_{t-1}$ and $P_{t+1} - P_t$ in turn involves multiplying the differences between consecutive coefficients a and consecutive coefficients b, as given in each column of Table 1.[26] The term structure of price coefficients precisely dictates the magnitude of these differences, which information percolation amplifies.

[insert Table 1 here]

Importantly, the current supply shock ε_t^x is the only column of Table 1 that creates reversal, as in our benchmark model; other shocks generate momentum. While information percolation amplifies the price effect of all shocks—it steepens the term structure of price coefficients—it causes the momentum effect to dominate the reversal effect beyond a certain threshold of the meeting intensity. In the case of the random walk specification in (41) – (42), stock returns are serially uncorrelated when investors do not receive private information. Hence, any flow of private information creates momentum, which information percolation simply amplifies, as apparent from the solid line in the lower panel of Fig. 6. In contrast, when the dividend and the supply follow mean-reverting processes, returns exhibit reversals without information percolation. In this case, information percolation not only amplifies momentum but allows momentum to arise in the first place, as in our benchmark model. For instance, the dashed line in the lower panel of Fig. 6 shows that information percolation creates momentum when the dividend and supply have a reversion parameter of 0.9.

5.2. Rumors

Our baseline model can jointly generate short-term momentum, consistent with the empirical finding of Jegadeesh and Titman (1993), and long-term reversal, consistent with the

[26] To be consistent with our main model, we compute the serial correlation of returns using ex-dividend prices. Alternatively, one could assume several trading rounds in-between dividend payment dates (which would bring this extension even closer to our baseline case), with similar results. See Makarov and Rytchkov (2012) for a detailed analysis when returns are computed using cum-dividend prices.

over-reaction phenomenon of De Bondt and Thaler (1985). However, an important question is whether these effects can be amplified by rumors. It is natural to think of social interactions as propagators of rumors.[27] We introduce a rumor in our model by assuming that agents receive at time $t = 0$ signals of the form:

$$\widetilde{z}_0^i = \widetilde{U} + \widetilde{V} + \widetilde{\epsilon}_0^i \tag{45}$$

where $\widetilde{V}$ is normally distributed with zero mean and precision ν. We build a simplified version of the model in which we assume that the asset is liquidated at time $T = 4$.

The *common noise* $\widetilde{V}$ satisfies two important properties of a rumor: *(i)* it circulates from person to person and *(ii)* it is unverifiable. The first property arises as private signals now contain the variable $\widetilde{V}$, which now circulates from one agent to another through word-of-mouth communication. The second property results from the signal specification in (45): on average, private signals only reveal the sum of the fundamental value and the rumor ($\widetilde{U}+\widetilde{V}$). As a result, the rumor is unverifiable as agents cannot distinguish fundamental information from the rumor, either using prices or their private signals.

After receiving the initial signal at time $t = 0$, agents meet with each other exactly as in the baseline setup. When they meet, they exchange the information they have, which includes the rumor $\widetilde{V}$. Agents are aware of the existence of the rumor, but cannot disentangle it from fundamental information because the signals they exchange through private meetings all contain the rumor. To allow agents to eventually learn about the rumor, we assume that agents receive an additional signal at $t = 3$, as the economy approaches the final liquidation date. This information is now centered on the fundamental:

$$\widetilde{Z}_3^i = \widetilde{U} + \widetilde{\epsilon}_3^i. \tag{46}$$

[27] Peterson and Gist (1951) define a rumor as "*an unverified account or explanation of events circulating from person to person and pertaining to an object, event, or issue in public concern.*"

Although this signal does not allow an agent to perfectly back out the content of rumor, it allows prices to become more informative about the fundamental. The reason is that the private signal in (46) is centered on $\widetilde{U}$. This assumption incorporates the idea that rumors do not last forever, but eventually subside.

In the presence of a rumor, asset prices and investors' asset demands do not have a closed-form solution. Theorem 5.1 describes a system of recursive equations for the equilibrium price coefficients. We provide the proof of Theorem 5.1 and we solve this system of equations through a numerical scheme that we describe in Appendix D.2.

Theorem 5.1. *In the presence of a rumor, equilibrium prices have the following form*

$$\widetilde{P}_t = \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j + \beta_t \underbrace{\left[\widetilde{U} - \frac{1}{\tau S \Omega'_t} \left(\widetilde{X}_t - \Lambda_t \widetilde{V} \right) \right]}_{\widetilde{Q}_t}, \tag{47}$$

where $\{\widetilde{Q}_j\}_{j=0}^t$ *are the normalized price signals, which can be written as*

$$\widetilde{Q}_j \equiv \widetilde{U} - \frac{1}{\tau S \Omega'_j} \left(\widetilde{X}_j - \Lambda_j \widetilde{V} \right). \tag{48}$$

The coefficients $\{\Omega'_j\}_{j=0}^t$ *and* $\{\Lambda_j\}_{j=0}^t$ *are positive and solve a fixed-point problem given by a system of recursive equations:*

$$\begin{aligned} \Omega'_t &= \frac{1}{\tau S} \sum_{j=0}^{t} \bar{\theta}_j - \sum_{j=0}^{t-1} \Omega'_j \\ \Lambda_t &= \sum_{j=0}^{t} \bar{\theta}_j - \sum_{j=0}^{t-1} \Lambda_j, \end{aligned} \tag{49}$$

in which $\bar{\theta}$ *denotes the average coefficients of agents' private signals in their optimal demand.*

The normalized price signals in (48) contain a rumor. The price signal now reflects fundamental information $\widetilde{U}$, supply shocks $\widetilde{X}$, and the rumor $\widetilde{V}$. When signals do not contain a rumor, we recover the result of Theorem 2.1, in which the coefficients $\Omega'_j = \Omega_j$, for

$j = 0, 1, ...3$, represent the average number of incremental signals and increase exponentially over time. In contrast, when signals contain a rumor, the coefficients Ω'_j increase initially but then revert back to zero, as we show in Fig. 7. Intuitively, agents know they possess information of lower quality due to the presence of the rumor and therefore apply a discount on the actual number of signals they have—the coefficients Ω'_j now represent *discounted* averages of incremental signals. At time $t = 2$, the discounted average Ω' declines, as agents anticipate that they will get better information at time $t = 3$ and apply a stronger discount on their number of signals. At time $t = 3$, the discounted average number of signals almost reaches zero for $\lambda = 3$. When agents have collected a large number of signals, they can accurately forecast $\widetilde{U} + \widetilde{V}$. Hence, when they get the signal centered on the fundamental, they ignore their other signals. The rumor thus induces agents to interpret their information with caution.

[insert Figure 7 here]

We now investigate how this convergence pattern relates to the serial correlation of stock returns. Intuitively, the first phase of price "over-shooting" generates short-term momentum and the second phase of price correction generates long-term reversal. To show this, we plot the serial correlation of returns in Fig. 8. When the rumor is fairly precise (Panel a), returns mostly exhibit momentum: despite the presence of the rumor, agents' precision rises over time, generating momentum. As the precision of the rumor decreases (Panel b), agents discount their actual number of signals more strongly. As a result, agents progressively cut back their positions—they adjust their trades to reflect that their information is of lower quality. While these portfolio adjustments do not prevent returns to exhibit momentum in the first period, they induce reversal in the second period as the price gradually corrects. Finally, when the rumor's precision is low (Panel c), agents become cautious about their information and the improvement in their precision is not sufficient to generate momentum.

[insert Figure 8 here]

6 Conclusion

This paper suggests several interesting avenues for future research. For instance, in this paper we abstract from individual behavioral biases, but we believe that individual biases, such as in Daniel et al. (1998) or Barberis et al. (1998), would amplify the effects we analyze. Other questions are worthwhile investigating, such as extending the setup to multiple assets, where information percolation could generate rich dynamics of the conditional correlation among assets. It is also interesting to study precisely the mechanism of information transmission and find conditions under which investors find it beneficial to tell the truth (Stein, 2008).

A legitimate question is what empirical exercise would validate our model. We believe that natural experiments capturing an exogenous increase or decrease in the intensity of word-of-mouth communication could make a worthwhile empirical point. For example, Shiller (2000) relates the increase in the word-of-mouth communication intensity once the telephone became effective during the 1920s with the steady increase of volatility during the same period. Alternatively, the Regulation Fair Disclosure, promulgated by the U.S. Securities and Exchange Commission in August 2000, forbids firms and their insiders to provide information to some investors (often large institutional investors). Hence, after August 2000, there should be less information propagated through the word-of-mouth communication channel.

Appendix A

A.1. Proof of Proposition 1

To obtain the closed-form solution for the distribution π of incremental signals, we first derive the equation for its dynamics.

Lemma A.1. *The probability density function π over the additional number of signals collected by each agent satisfies*

$$\frac{d}{dt}\pi_t(n) = -\lambda\pi_t(n) + \lambda(\pi_t * \mu_t)(n), \quad \pi_0 = \delta_{n=1}. \tag{50}$$

Proof. We use an argument made in Proposition 3 of Duffie, Giroux, and Manso (2010) and Proposition 4.2 in Duffie, Malamud, and Manso (2010): the probability density function π_t solves (50) if and only if its Fourier transform $\widehat{\pi}_t$ solves

$$\frac{\mathrm{d}}{\mathrm{d}t}\widehat{\pi}_t = -\lambda\widehat{\pi}_t + \lambda\widehat{\mu}_t\widehat{\pi}_t, \quad \widehat{\pi}_0 = 1. \tag{51}$$

This differential equation has a unique solution, which is given by

$$\widehat{\pi}_t = \exp\left(-\lambda\int_0^t \widehat{\mu}_s \mathrm{d}s - \lambda t\right). \tag{52}$$

Our goal is to show that $\widehat{\pi}_t$ has the solution in (52). Denote by X_{t_i} the number of new signals gathered if a meeting occurs at time t_i and observe that it is distributed as

$$X_{t_i} \sim \mu\left(t_i, \cdot\right)$$

where the distribution $\mu\left(t, x\right)$ satisfies the Boltzmann equation (Duffie et al., 2009)

$$\frac{\mathrm{d}}{\mathrm{d}t}\mu_t(n) = -\lambda\mu_t(n) + \lambda(\mu_t * \mu_t)(n), \quad \mu_0 = \delta_{n=1}. \tag{53}$$

Furthermore, the number $N(t)$ of meetings that took place between time 0 and t is a Poisson counter with intensity λ; accordingly, the total number Y_t of new signals gathered between time 0 and t is given by $\sum_{i=1}^{N(t)} X_{t_i}$. We now characterize its distribution. First, observe that Y_t, conditional on the set of times $\{0 \leq t_1 \leq t_2 \leq ... \leq t_{N(t)} \leq t\}$ at which a meeting occurs (up to time t) and the total number of meetings $N(t)$ (that is, conditioning on the whole trajectory A_t^N of the Poisson process), is distributed as

$$\begin{aligned} Y_t | A_t^N &\sim \int_{\mathbb{R}^{N-1}} \mu\left(Y_{t_N} - Y_{t_{N-1}}, t_N\right) \mathrm{d}\mu\left(Y_{t_{N-1}} - Y_{t_{N-2}}, t_{N-1}\right) ... \mathrm{d}\mu\left(Y_{t_1} - 0, t_1\right) \\ &\equiv \int_{\mathbb{R}^{N-1}} \mu\left(X_{t_N}, t_N\right) \mathrm{d}\mu\left(X_{t_{N-1}}, t_{N-1}\right) ... \mathrm{d}\mu\left(X_{t_1}, t_1\right). \end{aligned} \tag{54}$$

The distribution in (54) may be written as

$$Y_t | A_t^N \sim \Gamma_{i=1}^{N(t)} \mu_{t_i}$$

where, for any probability measures $\alpha_1, ..., \alpha_k$, we write $\Gamma_{i=1}^k = \alpha_1 * \alpha_2 * ... * \alpha_k$.

Now, observe that each t_i in the sequence of meetings $\{0 \leq t_1 \leq t_2 \leq ... \leq t_{N(\tau)} \leq t\}$ conditional on $N(t)$ is uniformly distributed over t; accordingly, we have that

$$Y_t | N(t) \sim \Gamma_{i=1}^{N(t)} \frac{1}{t} \int_0^t \mu_{t_i} \mathrm{d}t_i = \left(\frac{1}{t^{N(t)}} \left(\int_0^t \mu_s \mathrm{d}s \right)^{*N(t)} \right)$$

where $*n$ denotes the $n-$fold convolution.

Using that $N(t)$ is a Poisson(λ) counter, we can write

$$Y_t \sim \sum_{k=0}^{\infty} e^{-\lambda t} \frac{(\lambda t)^k}{k!} \frac{1}{t^k} \left(\int_0^t \mu_s \mathrm{d}s \right)^{*k},$$

and thus

$$\pi_t \equiv e^{-\lambda t} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \left(\int_0^t \mu_s \mathrm{d}s \right)^{*k}. \tag{55}$$

Furthermore, computing the Fourier transform of (55), using that the transform of a convolution is the product of the transforms (e.g., Duffie and Manso, 2007), we obtain

$$\widehat{\pi}_t \equiv e^{-\lambda t} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \left(\int_0^t \widehat{\mu}_s \mathrm{d}s \right)^k. \tag{56}$$

Finally, using the Taylor expansion of $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$, we can write

$$\widehat{\pi}_t = \exp\left(-\lambda \int_0^t \widehat{\mu}_s \mathrm{d}s - \lambda t \right), \tag{57}$$

which proves (52) and thus (50), corresponding to Eq. (3) in the paper. □

To prove Proposition 1, write the Boltzmann equation in (53) of the cross-sectional distribution μ_t of the number of *total* signals at time t as

$$\frac{d}{dt}\mu_t(n) = \lambda \sum_{k=1}^{n-1} \mu_t(n-k)\mu_t(k) - \lambda \mu_t(n). \tag{58}$$

Since agents are assumed to be initially endowed with a single signal, the initial distribution of signals is a Dirac mass at 1, i.e., $\mu_0(1) = 1$. This initial distribution has the advantage of leading to a closed-form solution for the cross-sectional distribution of the total number of signals at time $t \geq 1$ and the average number of total signals at time t, denoted by $\overline{\Omega}_t$:

$$\mu_t(n) = e^{-n\lambda t} \left(e^{\lambda t} - 1 \right)^{n-1} \tag{59}$$

$$\overline{\Omega}_t = e^{\lambda t}. \tag{60}$$

To obtain the distribution of incremental number of signals between time $t-1$ and t, notice that the probability of getting n new signals over $[t-1, t]$ is independent of an agent's current type and is thus given by the cross-sectional distribution:

$$\mathbb{P}\left[\text{n new signals over } [t-1,t] | \text{meeting someone}\right] = \mu_t(n). \tag{61}$$

Since the probability of meeting no one between time $t-1$ and t is

$$\mathbb{P}\left[\text{meeting no one over } [t-1,t]\right] = e^{-\lambda}, \tag{62}$$

it follows that the distribution π of incremental signals satisfies

$$\begin{aligned}\pi_t(n) =& \mathbb{P}\left[\text{n new signals over } [t-1,t] | \text{meet someone}\right] \\ & \times \left(1 - \mathbb{P}\left[\text{meet no one over } [t-1,t]\right]\right),\end{aligned} \tag{63}$$

and thus

$$\pi_t(n) = \mu_t(n)\left(1-e^{-\lambda}\right) = e^{-n\lambda t}\left(e^{\lambda t}-1\right)^{n-1}\left(1-e^{-\lambda}\right), \quad n \geq 1. \tag{64}$$

The average number of incremental signals is then given by

$$\Omega_t = \overline{\Omega}_t - \overline{\Omega}_{t-1} = e^{\lambda(t-1)}(e^{\lambda}-1). \tag{65}$$

A.2. Proof of Theorem 2.1 (Equilibrium)

We provide the proof for a two-trading session economy. Once the equilibrium quantities are written in a recursive form, as in Brennan and Cao (1997), or in He and Wang (1995), it is straightforward to derive the full recursive equilibrium solution. The model is solved backwards, starting from date 1 and then going back to date 0. First, we conjecture that prices in period 0 and period 1 have the following form

$$\widetilde{P}_0 = \beta_0 \widetilde{U} - \alpha_{0,0}\widetilde{X}_0 \tag{66}$$

$$\widetilde{P}_1 = \beta_1 \widetilde{U} - \alpha_{1,0}\widetilde{X}_0 - \alpha_{1,1}\widetilde{X}_1. \tag{67}$$

Consider the normalized price signal in period zero, informationally equivalent to $\widetilde{P}_0$:

$$\widetilde{Q}_0 \equiv \frac{1}{\beta_0}\widetilde{P}_0 = \widetilde{U} - \frac{\alpha_{0,0}}{\beta_0}\widetilde{X}_0. \tag{68}$$

Replace $\widetilde{X}_0$ from (68) into (67) to obtain

$$\widetilde{P}_1 = \varphi_1\widetilde{U} + \xi_1\widetilde{Q}_0 - \alpha_{1,1}\widetilde{X}_1, \tag{69}$$

where $\varphi_1 \equiv \beta_1 - \alpha_{1,0}\frac{\beta_0}{\alpha_{0,0}}$ and $\xi_1 \equiv \alpha_{1,0}\frac{\beta_0}{\alpha_{0,0}}$. These coefficients are to be determined in equilibrium. We normalize the price signal in period $t = 1$ and obtain $\widetilde{Q}_1$:

$$\widetilde{Q}_1 \equiv \frac{1}{\varphi_1}\left(\widetilde{P}_1 - \xi_1\widetilde{Q}_0\right) = \widetilde{U} - \frac{\alpha_{1,1}}{\varphi_1}\widetilde{X}_1. \tag{70}$$

Thus, observing $\{\widetilde{Q}_0, \widetilde{Q}_1\}$ is equivalent to observing $\{\widetilde{P}_0, \widetilde{P}_1\}$. We conjecture the following relationships (see Admati, 1985), which are to be verified once the solution is obtained:

$$\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\tau S\Omega_0} \tag{71}$$

$$\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\tau S\Omega_1}. \tag{72}$$

In our setup, $\Omega_0 = 1\ \forall\lambda$, $\Omega_1 = 0$ if $\lambda = 0$ (in this case the price $\widetilde{P}_1$ is not informative), and $\Omega_1 > 0$ if $\lambda > 0$. Relationships (71) and (72) make the calculations that follow straightforward. The normalized price signals become:

$$\widetilde{Q}_0 = \widetilde{U} - \frac{1}{\tau S\Omega_0}\widetilde{X}_0 \tag{73}$$

$$\widetilde{Q}_1 = \widetilde{U} - \frac{1}{\tau S\Omega_1}\widetilde{X}_1. \tag{74}$$

Period 1 Consider an investor i who, at date $t = 1$, collects $\omega_1^i \geq 1$ additional signals. At date $t = 1$, investor i chooses $\widetilde{D}_1^i$ to maximize expected utility of final wealth:

$$\max_{\widetilde{D}_1^i} \mathbb{E}\left[-e^{-\frac{1}{\tau}\widetilde{W}_2^i}\middle|\mathcal{F}_1^i\right], \tag{75}$$

where the final wealth at date $t = 2$ (at liquidation) is

$$\widetilde{W}_2^i = X^i\widetilde{P}_0 + \widetilde{D}_0^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) + \widetilde{D}_1^i\left(\widetilde{U} - \widetilde{P}_1\right), \tag{76}$$

and $\mathcal{F}_1^i$ represents the total information available at date $t = 1$. This information is given by $\widetilde{Z}_1^i$, $\widetilde{Z}_0^i$ (private signals) and $\widetilde{Q}_1$, $\widetilde{Q}_0$ (public signals, informationally equivalent to prices). Note that $\widetilde{Z}_0^i$ represents only one signal of precision S, but $\widetilde{Z}_1^i$ represents the average of the ω_1^i additional signals collected by the investor at date $t = 1$ (ω_1^i signals of equal precision S are informationally equivalent to their average, a single signal with precision $\omega_1^i S$). Based on this information (i.e., $\widetilde{Z}_0^i$, $\widetilde{Z}_1^i$, $\widetilde{Q}_0$, and $\widetilde{Q}_1$), investor i updates her expectations regarding $\widetilde{U}$ and her posterior variance

$$K_1^i = Var^{-1}\left[\widetilde{U}\middle|\widetilde{Z}_1^i, \widetilde{Z}_0^i, \widetilde{Q}_1, \widetilde{Q}_0\right] \tag{77}$$

$$\widetilde{\mu}_1^i = \mathbb{E}\left[\widetilde{U}\middle|\widetilde{Z}_1^i, \widetilde{Z}_0^i, \widetilde{Q}_1, \widetilde{Q}_0\right], \tag{78}$$

using the Projection Theorem (see, e.g., DeGroot, 2005):

(**Projection Theorem).** *Consider the n-dimensional normal random variable*

$$(\theta, s) \sim \mathcal{N}\left(\begin{bmatrix} \mu_\theta \\ \mu_s \end{bmatrix}, \begin{bmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,s} \\ \Sigma_{s,\theta} & \Sigma_{s,s} \end{bmatrix}\right). \tag{79}$$

Provided $\Sigma_{s,s}$ is non-singular, the conditional density of θ given s is normal with conditional

mean and conditional variance-covariance matrix:

$$\mathbb{E}[\theta|s] = \mu_\theta + \Sigma_{\theta,s}\Sigma_{s,s}^{-1}\left(s - \mu_s\right) \tag{80}$$

$$\text{Var}[\theta|s] = \Sigma_{\theta,\theta} - \Sigma_{\theta,s}\Sigma_{s,s}^{-1}\Sigma_{s,\theta}. \tag{81}$$

From this theorem, we obtain

$$K_1^i = H + S + S\omega_1^i + \tau^2 S^2 \Phi\left(\Omega_0^2 + \Omega_1^2\right) \tag{82}$$

$$\widetilde{\mu}_1^i = \frac{1}{K_1^i}\left[S\widetilde{Z}_0^i + S\omega_1^i\widetilde{Z}_1^i + \tau^2 S^2\Phi\left(\Omega_0^2\widetilde{Q}_0 + \Omega_1^2\widetilde{Q}_1\right)\right]. \tag{83}$$

The normality assumption along with the exponential utility function then imply that the optimal demand of trader i in period 1 has the standard form:

$$\widetilde{D}_1^i = \tau K_1^i\left(\widetilde{\mu}_1^i - \widetilde{P}_1\right). \tag{84}$$

Replacing (83) in (84) we obtain

$$\widetilde{D}_1^i = \tau\left[S\widetilde{Z}_0^i + S\omega_1^i\widetilde{Z}_1^i + \tau^2 S^2\Phi\left(\Omega_0^2\widetilde{Q}_0 + \Omega_1^2\widetilde{Q}_1\right) - K_1^i\widetilde{P}_1\right]. \tag{85}$$

We can now integrate the optimal demands to obtain the total demand. We follow the convention used by Admati (1985) that $\int_0^1 \widetilde{Z}_j^i = \widetilde{U}$, almost surely. Importantly, we have to keep track of the heterogeneity in information endowments when aggregating all individual demands. In particular, at time $t = 1$ there is an infinity of types of investors with respect to their number of signals, and within each type there is a continuum of investors. Consequently, the total demand at time $t = 1$ satisfies

$$\widetilde{D}_1 = \int_0^1 \widetilde{D}_1^i = \sum_{n\in\mathbb{N}}\left[\pi_1(n)\int_{\{i:\omega_1^i=n\}}\widetilde{D}_1^i\right], \tag{86}$$

which yields

$$\widetilde{D}_1 = \tau \left[S\left(\Omega_0 + \Omega_1\right) \widetilde{U} + \tau^2 S^2 \Phi \left(\Omega_0^2 \widetilde{Q}_0 + \Omega_1^2 \widetilde{Q}_1 \right) - K_1 \widetilde{P}_1 \right], \tag{87}$$

where K_1 is the average precision across the entire population of agents:

$$K_1 \equiv \int_0^1 K_1^i = \sum_{\omega_1^i=0}^{\infty} K_1(\omega_1^i)\pi_1(\omega_1^i) = H + S\left(\Omega_0 + \Omega_1\right) + \tau^2 S^2 \Phi \left(\Omega_0^2 + \Omega_1^2\right). \tag{88}$$

Replacing (74) in (87) we obtain

$$\widetilde{D}_1 = \tau \left[\left(S\Omega_0 + S\Omega_1 + \tau^2 S^2 \Phi \Omega_1^2 \right) \widetilde{U} + \tau^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0 - \tau \Phi S \Omega_1 \widetilde{X}_1 - K_1 \widetilde{P}_1 \right]. \tag{89}$$

Once we impose market clearing, $\widetilde{D}_1 = \widetilde{X}_0 + \widetilde{X}_1$, we can use the conjecture for the price $\widetilde{P}_1$ in Eq. (69) to get the undetermined coefficients φ_1, ξ_1, and $\alpha_{1,1}$:

$$\varphi_1 = \frac{S\Omega_1 \left(1 + \tau^2 S \Phi \Omega_1\right)}{K_1}, \tag{90}$$

$$\xi_1 = \frac{S\Omega_0 \left(1 + \tau^2 S \Phi \Omega_0\right)}{K_1}, \tag{91}$$

$$\alpha_{1,1} = \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1}. \tag{92}$$

We can now verify that, indeed, $\frac{\alpha_{1,1}}{\varphi_1} = \frac{1}{\tau S \Omega_1}$, hence validating the conjecture in (72). Furthermore, the undetermined coefficients of $\widetilde{P}_1$ of the conjectured form in (67) are

$$\beta_1 = \frac{K_1 - H}{K_1} \tag{93}$$

$$\alpha_{1,0} = \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_1} \tag{94}$$

$$\alpha_{1,1} = \frac{1 + \tau^2 S \Phi \Omega_1}{\tau K_1}, \tag{95}$$

and thus $\widetilde{P}_1$ can be written

$$\widetilde{P}_1 = \frac{K_1 - H}{K_1}\widetilde{U} - \frac{1+\tau^2 S\Phi\Omega_0}{\tau K_1}\widetilde{X}_0 - \frac{1+\tau^2 S\Phi\Omega_1}{\tau K_1}\widetilde{X}_1, \tag{96}$$

which, after using (73) and (74), becomes:

$$\widetilde{P}_1 = \frac{S\Omega_0 + \tau^2 S^2\Phi\Omega_0^2}{\tau K_1}\widetilde{Q}_0 + \frac{S\Omega_1 + \tau^2 S^2\Phi\Omega_1^2}{\tau K_1}\widetilde{Q}_1. \tag{97}$$

Period 0 The problem of investor i at time $t = 0$ is

$$\max_{\widetilde{D}_0^i} \mathbb{E}\left[-e^{-\frac{1}{\tau}\left[X^i\widetilde{P}_0 + \widetilde{D}_0^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) + \widetilde{D}_1^i\left(\widetilde{U} - \widetilde{P}_1\right)\right]} \middle| \widetilde{Z}_0^i, \widetilde{Q}_0\right]. \tag{98}$$

Observe that, at time $t = 0$, investor i needs to estimate $\widetilde{U}$, $\widetilde{D}_1^i$, and $\widetilde{P}_1$, after observing $\widetilde{Z}_0^i$ and $\widetilde{Q}_0$. $\widetilde{D}_1^i$ and $\widetilde{P}_1$ are given by (84) and (96), respectively. Note also that $\widetilde{D}_1^i$ depends on the future number of additional signals at time $t = 1$, which is unknown to the investor at time $t = 0$. The following lemma shows that this uncertainty about ω_1^i is irrelevant for portfolio choice.

Lemma A.2. *When agent i builds her portfolio, her future number of signals is irrelevant.*

Proof. The value function V^i of agent i at time $t = 0$ is given by

$$V^i(W_0) = e^{-\frac{1}{\tau}W_0} \max_{\widetilde{D}_0^i} \mathbb{E}\left[-e^{-\frac{1}{\tau}\left[\widetilde{D}_0^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) + \widetilde{D}_1^i(\omega_1^i)\left(\widetilde{U} - \widetilde{P}_1\right)\right]} \middle| \widetilde{Z}_0^i, \widetilde{Q}_0\right] \tag{99}$$

$$= e^{-\frac{1}{\tau}W_0} \max_{\widetilde{D}_0^i} \sum_{k\in\mathbb{N}} \pi_1(k) \underbrace{\mathbb{E}\left[-e^{-\frac{1}{\tau}\left[\widetilde{D}_0^i\left(\widetilde{P}_1 - \widetilde{P}_0\right) + \widetilde{D}_1^i(k)\left(\widetilde{U} - \widetilde{P}_1\right)\right]} \middle| \widetilde{Z}_0^i, \widetilde{Q}_0; \omega_1^i = k\right]}_{g\left(k, \widetilde{D}_0^i\right)}, \tag{100}$$

where $\pi_1(k)$ represents the probability of receiving k additional signals at time 1 and g represents an expectation of an exponential affine quadratic normal variable. To derive its explicit form, we use the following standard result from multivariate normal calculus:

Lemma A.3. *Consider a random vector $z \sim N(0, \Sigma)$. Then,*

$$E\left[e^{z'Fz+G'z+H}\right] = |I - 2\Sigma F|^{-\frac{1}{2}}\, e^{\frac{1}{2}G'(I-2\Sigma F)^{-1}\Sigma G+H}.$$

In our particular case, the vector z of random variables is given by $z = [\widetilde{U}\ \ \widetilde{D}_1^i(k)\ \ \widetilde{P}_1]'$. Tedious computations then show that

$$g\left(k, \widetilde{D}_0^i\right) = -\,|I - 2\Sigma(k)F|^{-\frac{1}{2}}\, e^{\frac{1}{2}G\left(k,\widetilde{D}_0^i\right)'(I-2\Sigma(k)F)^{-1}\Sigma(k)G\left(k,\widetilde{D}_0^i\right)+H\left(k,\widetilde{D}_0^i\right)}, \tag{101}$$

where

$$\Sigma(k) \equiv \begin{bmatrix} \frac{1}{K_0^i} & \frac{\tau(K_1^i-K_1)}{K_1} & \frac{K_1-K_0}{K_0^iK_1} \\ \frac{\tau(K_1^i-K_1)}{K_1} & \frac{\tau^2K_1^i\left[K_0S^2\Omega_1(\Omega_1-k)+kSK_1(K_1-K_0)\right]}{K_1^2(K_1-K_0-S\Omega_1)} & -\frac{\tau S\Omega_1K_1^i(K_1-K_0)}{K_1^2(K_1-K_0-S\Omega_1)} \\ \frac{K_1-K_0}{K_0^iK_1} & -\frac{\tau S\Omega_1K_1^i(K_1-K_0)}{K_1^2(K_1-K_0-S\Omega_1)} & \frac{(K_1-K_0)^2(K_1-S\Omega_1)}{K_0^iK_1^2(K_1-K_0-S\Omega_1)} \end{bmatrix} \tag{102}$$

$$F \equiv \begin{bmatrix} 0 & -\frac{1}{2\tau} & 0 \\ -\frac{1}{2\tau} & 0 & \frac{1}{2\tau} \\ 0 & \frac{1}{2\tau} & 0 \end{bmatrix} \tag{103}$$

$$G(k, \widetilde{D}_0^i) \equiv \begin{bmatrix} \frac{SK_1^i(\widetilde{Q}_0-\widetilde{Z}_0^i)}{K_1} \\ \frac{S(\widetilde{Q}_0-\widetilde{Z}_0^i)}{\tau K_1} \\ \frac{\tau SK_1^i(\widetilde{Z}_0^i-\widetilde{Q}_0)-K_1\widetilde{D}_0^i}{\tau K_1} \end{bmatrix} \tag{104}$$

$$H(k, \widetilde{D}_0^i) \equiv \frac{\widetilde{D}_0^i\left[K_0^i(K_1\widetilde{P}_0 - S(\widetilde{Q}_0 - \widetilde{Z}_0^i)) - SK_1(\widetilde{Z}_0^i + \tau^2S\Phi\widetilde{Q}_0)\right]}{\tau K_0^iK_1} - \frac{S^2K_1^i(\widetilde{Q}_0 - \widetilde{Z}_0^i)^2}{K_1^2}. \tag{105}$$

Further computations show that

$$h(k) \equiv |I - 2\Sigma(k)F|^{-\frac{1}{2}} = \sqrt{\frac{\tau^2\Phi K_1^2}{K_1^i(1 + \tau^2S\Phi\Omega_1 + \tau^2\Phi K_1)}}, \tag{106}$$

and

$$q(\widetilde{D}_0^i) \equiv \frac{1}{2} G\left(k, \widetilde{D}_0^i\right)' \left(I - 2\Sigma(k)F\right)^{-1} \Sigma(k) G\left(k, \widetilde{D}_0^i\right) + H\left(k, \widetilde{D}_0^i\right) \tag{107}$$

$$= \frac{\widetilde{P}_0 \widetilde{D}_0^i}{\tau} - \frac{\Phi\left[\widetilde{D}_0^i + \tau S(\widetilde{Q}_0 - \widetilde{Z}_0^i)\right]^2}{2\left[1 + \tau^2 \Phi(K_1 + S\Omega_1)\right]} + \frac{\widetilde{D}_0^i\left[2\tau S(\widetilde{Z}_0^i + \tau^2 S\Phi\widetilde{Q}_0) - \widetilde{D}_0^i\right]}{2\tau^2\left[S\Omega_1(1 + \tau^2 S\Phi\Omega_1) - K_1\right]}, \tag{108}$$

which does not depend on k. Plugging these expressions into (101), agent i solves

$$V^i(W_0) = e^{-\frac{1}{\tau}W_0} \underbrace{\left(\sum_{k\in\mathbb{N}} \pi_1(k) h(k)\right)}_{\text{Anticipation of future signals}} \max_{\widetilde{D}_0^i} -e^{q\left(\widetilde{D}_0^i\right)}, \tag{109}$$

and it follows that her portfolio decision is independent of her expectation regarding her future number of signals. □

To obtain agent i's optimal demand, we solve the problem in (109) and impose optimality

$$\frac{\partial}{\partial \widetilde{D}_0^i} q\left(n_0, \widetilde{D}_0^i\right) = 0.$$

We integrate the resulting optimal demand and impose market clearing in order to solve for the undetermined coefficients of $\widetilde{P}_0$, i.e., β_0 and $\alpha_{0,0}$. The solutions for these coefficients are:

$$\beta_0 = \frac{K_0 - H}{K_0}, \ \alpha_{0,0} = \frac{1 + \tau^2 S\Phi\Omega_0}{\tau K_0}, \tag{110}$$

where

$$K_0 = K_0^i = H + S + \tau^2 S^2 \Phi \Omega_0^2 \tag{111}$$

$$\widetilde{\mu}_0^i = \frac{1}{K_0}\left(S\widetilde{Z}_0^i + \tau^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0\right). \tag{112}$$

Note that $K_0 = K_0^i$ because all investors start with one signal at time 0. The optimal

demand of investor i at time $t = 0$ is

$$\widetilde{D}_0^i = \tau S\left(\widetilde{Z}_0^i - \widetilde{Q}_0\right) = \tau\left(S\widetilde{Z}_0^i + \tau^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0 - \tau^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0 - S\widetilde{Q}_0\right) \tag{113}$$

$$= \tau\left[S\widetilde{Z}_0^i + \tau^2 S^2 \Phi \Omega_0^2 \widetilde{Q}_0 - \left(S + \tau^2 S^2 \Phi \Omega_0^2\right)\frac{1}{\beta_0}\widetilde{P}_0\right] \tag{114}$$

$$= \tau K_0^i\left(\mathbb{E}_0^i[\widetilde{U}|\mathcal{F}_0^i] - \widetilde{P}_0\right). \tag{115}$$

At this point, we can use (110) and (111) to verify that, indeed, $\frac{\alpha_{0,0}}{\beta_0} = \frac{1}{\tau S \Omega_0}$, which validates the conjecture in (71). Then

$$\widetilde{P}_0 = \beta_0 \widetilde{Q}_0 = \frac{K_0 - H}{K_0}\widetilde{U} - \frac{1 + \tau^2 S \Phi \Omega_0}{\tau K_0}\widetilde{X}_0. \tag{116}$$

The solution can then be written in a recursive form and extended to more than two trading periods, as done in Theorem 2.1. The recursive form for prices follows from (116) and (96); the recursive form for individual precisions follows from (111) and (82); the recursive form for individual demands follows from (113) – (115) and (84) – (85).

Appendix B

B.1. Proof of Proposition 2

Define $\widetilde{\mu}_t^i \equiv \mathbb{E}_t[\widetilde{U}|\mathcal{F}_t^i]$ and start with the following lemma.

Lemma B.1. *$Y_t^i \equiv K_t(\widetilde{P}_t - \widetilde{\mu}_t^i)$ is a martingale under agent i's information set:*

$$\mathbb{E}\left[Y_{t+1}^i|\mathcal{F}_t^i\right] = Y_t^i. \tag{117}$$

Proof. Compute first the expected stock price tomorrow as

$$\mathbb{E}\left[P_{t+1}|\mathcal{F}_t^i\right] = \frac{K_{t+1}-H}{K_{t+1}}\widetilde{\mu}_t^i - \sum_{j=0}^{t}\frac{1+\tau^2 S\Omega_j\Phi}{\tau K_{t+1}}\mathbb{E}\left[\widetilde{X}_j\middle|\mathcal{F}_t^i\right] \tag{118}$$

$$= \frac{K_{t+1}-H}{K_{t+1}}\widetilde{\mu}_t^i - \sum_{j=0}^{t}\frac{S\Omega_j+\tau^2 S^2\Omega_j^2\Phi}{K_{t+1}}(\widetilde{\mu}_t^i - \widetilde{Q}_j) \tag{119}$$

$$= \frac{K_{t+1}-K_t}{K_{t+1}}\widetilde{\mu}_t^i + \sum_{j=0}^{t}\frac{S\Omega_j+\tau^2 S^2\Omega_j^2\Phi}{K_{t+1}}\widetilde{Q}_j. \tag{120}$$

Moreover, observe that since $\widetilde{P}_t \in \mathcal{F}_t^i$, we also have that

$$\widetilde{P}_t = \mathbb{E}\left[\widetilde{P}_t\middle|\mathcal{F}_t^i\right] = \sum_{j=0}^{t}\frac{S\Omega_j+\tau^2 S^2\Omega_j^2\Phi}{K_t}\widetilde{Q}_j. \tag{121}$$

Multiply (120) by K_{t+1} and subtract $K_{t+1}\widetilde{\mu}_t^i$ to obtain

$$\mathbb{E}\left[K_{t+1}(P_{t+1}-\widetilde{\mu}_{t+1}^i)|\mathcal{F}_t^i\right] = -K_t\widetilde{\mu}_t^i + \sum_{j=0}^{t}(S\Omega_j+\tau^2 S^2\Omega_j^2\Phi)\widetilde{Q}_j, \tag{122}$$

which follows from the fact that, by the law of iterated expectations

$$\mathbb{E}[\widetilde{\mu}_{t+1}^i|\mathcal{F}_t^i] = \widetilde{\mu}_t^i. \tag{123}$$

Similarly, multiply (121) by K_t and subtract $K_t\widetilde{\mu}_t^i$ to obtain

$$K_t(\widetilde{P}_t - \widetilde{\mu}_t^i) = -K_t\widetilde{\mu}_t^i + \sum_{j=0}^{t}(S\Omega_j+\tau^2 S^2\Omega_j^2\Phi)\widetilde{Q}_j. \tag{124}$$

Clearly, comparing (122) and (124), $K_t(\widetilde{P}_t - \widetilde{\mu}_t^i)$ is a martingale under $\mathcal{F}_t^i$. □

Rearranging the martingale relation of Lemma B.1, we obtain

$$\mathbb{E}\left[P_{t+1}|\mathcal{F}_t^i\right] = \frac{K_t}{K_{t+1}}\widetilde{P}_t + \frac{K_{t+1}-K_t}{K_{t+1}}\widetilde{\mu}_t^i \tag{125}$$

and Proposition 2 follows.

B.2. Proof of Proposition 3

The weighted average $\overline{\mathbb{E}}_t[\widetilde{U}]$ obtains from the market clearing condition

$$\sum_{j=0}^{t} \widetilde{X}_j = \int_0^1 \widetilde{D}_t^i di \tag{126}$$

$$= \int_0^1 \tau K_t^i \left(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^i] - \widetilde{P}_t \right) di \tag{127}$$

$$= \int_0^1 \tau K_t^i \mathbb{E}[\widetilde{U}|\mathcal{F}_t^i] di - \tau K_t \widetilde{P}_t \tag{128}$$

$$= \tau K_t \left(\underbrace{\int_0^1 \frac{K_t^i \mathbb{E}[\widetilde{U}|\mathcal{F}_t^i]}{K_t} di}_{\overline{\mathbb{E}}_t[\widetilde{U}]} - \widetilde{P}_t \right), \tag{129}$$

and thus every price is of the form

$$\widetilde{P}_t = \overline{\mathbb{E}}_t[\widetilde{U}] - \frac{1}{\tau K_t} \sum_{j=0}^{t} \widetilde{X}_j, \tag{130}$$

and thus

$$\widetilde{P}_{t+1} - \widetilde{P}_t = \left(\overline{\mathbb{E}}_{t+1}[\widetilde{U}] - \overline{\mathbb{E}}_t[\widetilde{U}] \right) + \frac{K_{t+1} - K_t}{\tau K_t K_{t+1}} \sum_{j=0}^{t} \widetilde{X}_j - \frac{1}{\tau K_{t+1}} \widetilde{X}_{t+1}. \tag{131}$$

B.3. Proof of Proposition 4

First, notice that

$$\mathbb{E}\left[\widetilde{U}|\mathcal{F}_t^c \right] = \frac{1}{K_t^c} \sum_{j=0}^{t} \tau^2 S^2 \Omega_j^2 \Phi \widetilde{Q}_j, \tag{132}$$

and thus the price in (121) satisfies

$$\widetilde{P}_t = \sum_{j=0}^{t} \frac{S\Omega_j}{K_t} \widetilde{Q}_j + \frac{K_t^c}{K_t} \mathbb{E}\left[\widetilde{U} | \mathcal{F}_t^c\right]. \tag{133}$$

Further, we know that

$$\widetilde{P}_t = \overline{\mathbb{E}}_t[\widetilde{U}] - \frac{1}{\tau K_t} \sum_{j=0}^{t} \widetilde{X}_j, \tag{134}$$

and thus the average market expectations at time t can be written as

$$\overline{\mathbb{E}}_t[\widetilde{U}] = \frac{\sum_{j=0}^{t} S\Omega_j}{K_t} \widetilde{U} + \frac{K_t - \sum_{j=0}^{t} S\Omega_j}{K_t} \mathbb{E}\left[\widetilde{U} | \mathcal{F}_t^c\right] \tag{135}$$

$$\equiv \alpha_t \widetilde{U} + (1 - \alpha_t) \mathbb{E}\left[\widetilde{U} | \mathcal{F}_t^c\right]. \tag{136}$$

Taking this expression one step forward and applying the law of iterated expectations with respect to the common information set at time t, $\mathcal{F}_t^c$, we obtain

$$\mathbb{E}\left[\bar{\mathbb{E}}_{t+1}[\widetilde{U}] \middle| \mathcal{F}_t^c\right] = \mathbb{E}\left[\alpha_{t+1} \widetilde{U} + (1 - \alpha_{t+1}) \mathbb{E}\left[\widetilde{U} | F_{t+1}^c\right] \middle| \mathcal{F}_t^c\right] \tag{137}$$

$$= \alpha_{t+1} \mathbb{E}\left[\widetilde{U} | F_t^c\right] + (1 - \alpha_{t+1}) \mathbb{E}\left[\widetilde{U} | F_t^c\right] \tag{138}$$

$$= \mathbb{E}\left[\widetilde{U} | F_t^c\right]. \tag{139}$$

Furthermore, notice that

$$\mathbb{E}\left[\overline{\mathbb{E}}_t[\widetilde{U}] | \mathcal{F}_t^c\right] = \mathbb{E}\left[\alpha_t \widetilde{U} | \mathcal{F}_t^c\right] + (1 - \alpha_t) \mathbb{E}\left[\widetilde{U} | F_t^c\right] \tag{140}$$

$$= \alpha_t \mathbb{E}\left[\widetilde{U} | F_t^c\right] + (1 - \alpha_t) \mathbb{E}\left[\widetilde{U} | F_t^c\right] \tag{141}$$

$$= \mathbb{E}\left[\widetilde{U} | F_t^c\right] \tag{142}$$

and thus

$$\mathbb{E}\left[\overline{\mathbb{E}}_{t+1}[\widetilde{U}] - \overline{\mathbb{E}}_t[\widetilde{U}]\middle|\,\mathcal{F}_t^c\right] = 0. \tag{143}$$

Using (143) to compute the common expectation of (18), we finally get

$$\mathbb{E}\left[\widetilde{P}_{t+1} - \widetilde{P}_t\middle|\,\mathcal{F}_t^c\right] = \frac{K_{t+1} - K_t}{\tau K_t K_{t+1}} \sum_{j=0}^{t} \mathbb{E}\left[\widetilde{X}_j\middle|\,\mathcal{F}_t^c\right] \tag{144}$$

which yields the first part of Proposition 4. To obtain the second part of the proposition, apply the law of iterated expectations to (125) with respect to $\mathcal{F}^c$ to obtain

$$\mathbb{E}\left[P_{t+1}|\mathcal{F}_t^c\right] = \frac{K_t}{K_{t+1}}\widetilde{P}_t + \frac{K_{t+1} - K_t}{K_{t+1}}\mathbb{E}\left[\widetilde{U}\middle|\,\mathcal{F}_t^c\right] \tag{145}$$

and thus

$$\mathbb{E}\left[P_{t+1} - \widetilde{P}_t|\mathcal{F}_t^c\right] = \frac{K_{t+1} - K_t}{K_{t+1}}\left(\mathbb{E}\left[\widetilde{U}\middle|\,\mathcal{F}_t^c\right] - \widetilde{P}_t\right) \tag{146}$$

$$= \frac{K_{t+1} - K_t}{K_{t+1}K_t^c}\left(K_t^c\mathbb{E}\left[\widetilde{U}\middle|\,\mathcal{F}_t^c\right] - K_t^c\widetilde{P}_t\right). \tag{147}$$

Using (121) and (132), we know that

$$K_t\widetilde{P}_t = \sum_{j=0}^{t} S\Omega_j\widetilde{Q}_j + \sum_{j=0}^{t} \tau^2 S^2\Omega_j^2\Phi\widetilde{Q}_j. \tag{148}$$

$$K_t^c\mathbb{E}\left[\widetilde{U}\middle|\,\mathcal{F}_t^c\right] = \sum_{j=0}^{t} \tau^2 S^2\Omega_j^2\Phi\widetilde{Q}_j = K_t\widetilde{P}_t - \sum_{j=0}^{t} S\Omega_j\widetilde{Q}_j, \tag{149}$$

where the second equality in (149) results from (148). Plugging this into (147), we get

$$\mathbb{E}\left[P_{t+1} - \widetilde{P}_t|\mathcal{F}_t^c\right] = \frac{K_{t+1} - K_t}{K_{t+1}K_t^c}\left((K_t - K_t^c)\widetilde{P}_t - \sum_{j=0}^{t} S\Omega_j\widetilde{Q}_j\right). \tag{150}$$

Finally, observing that $K_t - K_t^c = \sum_{j=0}^{t} S\Omega_j$, we obtain the second part of Proposition 4:

$$\mathbb{E}\left[P_{t+1} - \widetilde{P}_t | \mathcal{F}_t^c\right] = \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} \sum_{j=0}^{t} S\Omega_j \left(\widetilde{P}_t - \widetilde{Q}_j\right). \tag{151}$$

B.4. Proof of Proposition 5

Conditioning (151) on $\mathcal{F}^r$ requires computing the following expectation:

$$\mathbb{E}\left[\widetilde{P}_t - \widetilde{Q}_j | \mathcal{F}_t^r\right], \quad \forall j = 0, ..., t, \tag{152}$$

which amounts to derive the recursive relation between the information sets $\mathcal{F}^c$ and $\mathcal{F}^r$. To do so, we first use (148) and obtain

$$\widetilde{P}_t = \frac{K_{t-1}}{K_t} \widetilde{P}_{t-1} + \frac{K_t - K_{t-1}}{K_t} \widetilde{Q}_t, \tag{153}$$

from which we derive

$$\widetilde{P}_t - \widetilde{Q}_t = \frac{K_{t-1}}{K_t} (\widetilde{P}_{t-1} - \widetilde{Q}_t) \tag{154}$$

$$\widetilde{P}_t - \widetilde{P}_{t-1} = -\frac{K_t - K_{t-1}}{K_t} (\widetilde{P}_{t-1} - \widetilde{Q}_t). \tag{155}$$

Replacing (155) in (154), we further get

$$\widetilde{P}_t - \widetilde{Q}_t = -\frac{K_{t-1}}{K_t} \frac{K_t}{K_t - K_{t-1}} (\widetilde{P}_t - \widetilde{P}_{t-1}) = -\frac{K_{t-1}}{K_t - K_{t-1}} (\widetilde{P}_t - \widetilde{P}_{t-1}). \tag{156}$$

Accordingly, the expectation for $j = t$ in (152) writes

$$\mathbb{E}\left[\widetilde{P}_t - \widetilde{Q}_t | \mathcal{F}_t^r\right] = -\frac{K_{t-1}}{K_t - K_{t-1}} (\widetilde{P}_t - \widetilde{P}_{t-1}). \tag{157}$$

Proceeding similarly for $j = t-1$, we obtain the following recursive relation:

$$\mathbb{E}\left[\widetilde{P}_t - \widetilde{Q}_{t-1}|\mathcal{F}_t^r\right] = \mathbb{E}\left[\widetilde{P}_t - \widetilde{P}_{t-1} + \widetilde{P}_{t-1} - \widetilde{Q}_{t-1}|\mathcal{F}_t^r\right] \tag{158}$$

$$= (\widetilde{P}_t - \widetilde{P}_{t-1}) + \mathbb{E}\left[\widetilde{P}_{t-1} - \widetilde{Q}_{t-1}|\mathcal{F}_t^r\right]. \tag{159}$$

Iterating over this recursive relation, the sum in (151) can be written as

$$\begin{aligned}
\sum_{j=0}^{t} S\Omega_j(\widetilde{P}_t - \widetilde{Q}_j) = &\left(\sum_{k=0}^{t-1} S\Omega_k - S\Omega_t \frac{K_{t-1}}{K_t - K_{t-1}}\right)(\widetilde{P}_t - \widetilde{P}_{t-1}) \\
&+ \left(\sum_{k=0}^{t-2} S\Omega_k - S\Omega_{t-1} \frac{K_{t-2}}{K_{t-1} - K_{t-2}}\right)(\widetilde{P}_{t-1} - \widetilde{P}_{t-2}) \\
&+ ... \\
&+ \left(\sum_{k=0}^{j-1} S\Omega_k - S\Omega_j \frac{K_{j-1}}{K_j - K_{j-1}}\right)(\widetilde{P}_j - \widetilde{P}_{j-1}) \\
&+ ... \\
&+ \left(S\Omega_0 - S\Omega_1 \frac{K_0}{K_1 - K_0}\right)(\widetilde{P}_1 - \widetilde{P}_0) \\
&+ \left(-S\Omega_0 \frac{H}{K_0 - H}\right)(\widetilde{P}_0 - 0),
\end{aligned} \tag{160}$$

which pins down the recursive equivalence between $\mathcal{F}^c$ and $\mathcal{F}^r$. Inspecting (160) shows that the coefficient of $(\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l})$ is:

$$\sum_{k=0}^{t-l} S\Omega_k - \frac{S\Omega_{t-l+1}}{(K_{t-l+1} - K_{t-l})/K_{t-l}} \tag{161}$$

and thus the sum in (151) can be written recursively as

$$\sum_{j=0}^{t} S\Omega_j(\widetilde{P}_t - \widetilde{Q}_j) = \sum_{l=1}^{t+1}\left(\sum_{k=0}^{t-l} S\Omega_k - \frac{S\Omega_{t-l+1}}{(K_{t-l+1} - K_{t-l})/K_{t-l}}\right)(\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l}) \tag{162}$$

$$\equiv \sum_{l=1}^{t+1} m_{t-l}(\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l}) \tag{163}$$

and the relation in (25) follows.

B.5. Proof of Theorem 3.1 (Momentum Condition)

Using the relation in (25), if $K_{t+1} > K_t$, a sufficient condition for momentum to obtain at lag l is that $m_{t-l} > 0$. We can therefore express the momentum condition at lag l as

$$\sum_{j=0}^{t-l} S\Omega_j > \frac{K_{t-l}}{K_{t-l+1} - K_{t-l}} S\Omega_{t-l+1}, \tag{164}$$

which implies

$$\frac{K_{t-l+1} - K_{t-l}}{K_{t-l}} > \frac{S\Omega_{t-l+1}}{\sum_{j=0}^{t-l} S\Omega_j}. \tag{165}$$

This gives the momentum condition (30) in Theorem 3.1:

$$\frac{(K_{t-l+1} - K_{t-l})/K_{t-l}}{S\Omega_{t-l+1}/\sum_{j=0}^{t-l} S\Omega_j} = \epsilon_{t-l} > 1. \tag{166}$$

The last part of the claim follows directly from inspecting the last lag $l = t + 1$, which satisfies

$$m_{-1}(\widetilde{P}_0 - \widetilde{P}_{-1}) \equiv -\frac{H}{1 + \tau^2 S\Phi\Omega_0} \widetilde{P}_0. \tag{167}$$

For the limit when $t \to \infty$, we need to compute:

$$\lim_{t\to\infty} \frac{K_{t+1} - K_t}{K_{t+1} K_t^c} = \lim_{t\to\infty} \frac{1}{K_t^c} - \lim_{t\to\infty} \frac{K_t}{K_{t+1} K_t^c}. \tag{168}$$

The first limit is zero. For the second limit, notice that:

$$\frac{1}{K_{t+1}} < \frac{K_t}{K_{t+1} K_t^c} \leq \frac{1}{K_t^c}, \tag{169}$$

and both bounds go to zero as $t \to \infty$. Thus, the second limit is also zero, and we obtain

$$\lim_{t\to\infty} \frac{K_{t+1} - K_t}{K_{t+1}K_t^c} m_{-1} = 0. \tag{170}$$

B.6. Proof of Theorem 3.2

The proof is organized in three parts. We first prove that there exists a unique threshold such that the momentum condition in (30) is satisfied. Second, we show how this threshold is related to the parameters of the model. Third, we prove that prices are martingales for $\lambda = 0$ and $\lambda \to \infty$.

Note first that

$$\sum_{k=0}^{t} \Omega_k = e^{\lambda t} \tag{171}$$

$$\sum_{k=0}^{t} \Omega_k^2 = \frac{e^{2\lambda t}(e^\lambda - 1) + 2}{e^\lambda + 1}, \tag{172}$$

and thus

$$K_{t-l} = H + Se^{\lambda(t-l)} + \tau^2 S^2 \Phi \frac{e^{2\lambda(t-l)}(e^\lambda - 1) + 2}{e^\lambda + 1} \tag{173}$$

$$K_{t-l+1} = H + Se^{\lambda(t-l+1)} + \tau^2 S^2 \Phi \frac{e^{2\lambda(t-l+1)}(e^\lambda - 1) + 2}{e^\lambda + 1}. \tag{174}$$

The momentum threshold Eq. (30) can be written

$$(e^\lambda - 1)K_{t-l} = K_{t-l+1} - K_{t-l}, \tag{175}$$

from which we obtain Eq. (31):

$$\lambda^\star(H, S, \Phi, \tau, t-l) = \ln\left(\frac{K_{t-l+1}}{K_{t-l}}\right). \tag{176}$$

Furthermore, plugging (173) – (174) in the right-hand side of (175) we can write

$$(e^{\lambda}-1)K_{t-l} = Se^{\lambda(t-l)}(e^{\lambda}-1) + \tau^2 S^2 \Phi \frac{e^{2\lambda(t-l)}(e^{\lambda}-1)}{e^{\lambda}+1}(e^{2\lambda}-1), \tag{177}$$

and thus

$$K_{t-l} = Se^{\lambda(t-l)} + \tau^2 S^2 \Phi e^{2\lambda(t-l)}(e^{\lambda}-1), \tag{178}$$

from which we derive the following equation in λ:

$$\frac{e^{\lambda[2(t-l)+1]}\left(e^{\lambda}-1\right)-2}{e^{\lambda}+1} = \frac{H}{\tau^2 S^2 \Phi}. \tag{179}$$

Define the function on the left-hand side as $g(\lambda)$. It takes values on $[-1,\infty)$, with $g(0)=-1$ and $\lim_{\lambda\to\infty} g(\lambda)=\infty$, and is increasing in λ:

$$g'(\lambda) = \frac{e^{\lambda[2(t-l)+1]}\left[2(t-l)\sinh(\lambda)+\sinh(\lambda)+1\right]+1}{(e^{\lambda}+1)^2} 2e^{\lambda} > 0, \tag{180}$$

and thus Eq. (179) has a unique solution.

Applying the Implicit Function Theorem to Eq. (179) allows us to prove the second part of Theorem 3.2: If H is larger, then the threshold is harder to reach. If τ, S, and Φ are larger, the threshold is easier to reach. If $t-l$ is large, the threshold is easier to reach. Therefore, the threshold $\lambda^{\star}(H,S,\Phi,\tau,t-l)$ is increasing in H and decreasing in S, Φ, τ and, $t-l$.

Using the fact that the threshold is decreasing in $t-l$, we obtain an upper bound for $\lambda^{\star}$. It takes its largest value with $t-l=0$. In this special case, the threshold $\lambda^{\star}$ is given by

$$e^{\lambda} = \frac{H+2\tau^2 S^2 \Phi}{\tau^2 S^2 \Phi}, \tag{181}$$

and thus

$$\lambda^{\star}(H,S,\Phi,\tau,t-l)\in\left(0,\log\left(2+\frac{H}{\Phi\tau^{2}S^{2}}\right)\right]. \tag{182}$$

For the third part of the theorem, we know from Proposition 2 that when $\lambda=0$, returns are not predictable. When $\lambda\to\infty$, the coefficients of supply shocks in Eqs. (18) and (27) become zero. Since return predictability arises solely from the inference of current and past supply shocks (Proposition 4), prices become martingales in this case.

B.7. Proof of Proposition 6

Using Proposition 5, we can write

$$\mathbb{E}\left[\widetilde{P}_{t+1}-\widetilde{P}_{t}|\mathcal{F}_{t}^{r}\right]=\frac{K_{t+1}-K_{t}}{K_{t+1}K_{t}^{c}}\left[m_{t-1}(\widetilde{P}_{t}-\widetilde{P}_{t-1})+\widetilde{X}_{t-1}\right], \tag{183}$$

where $\widetilde{X}_{t-1}\equiv\sum_{l=1}^{t}m_{t-1-l}(\widetilde{P}_{t-l}-\widetilde{P}_{t-1-l})$. Observing that $\{\widetilde{P}_{t}-\widetilde{P}_{t-1}\}\subset\mathcal{F}_{t}^{r}$ and applying the law of iterated expectations, it follows that

$$\mathbb{E}\left[\widetilde{P}_{t+1}-\widetilde{P}_{t}|\widetilde{P}_{t}-\widetilde{P}_{t-1}\right]=\frac{K_{t+1}-K_{t}}{K_{t+1}K_{t}^{c}}\left\{m_{t-1}(\widetilde{P}_{t}-\widetilde{P}_{t-1})+\mathbb{E}\left[\widetilde{X}_{t-1}|\widetilde{P}_{t}-\widetilde{P}_{t-1}\right]\right\}. \tag{184}$$

To compute the conditional expectation in the inner bracket, further observe that Proposition 5 implies that

$$\widetilde{P}_{t}-\widetilde{P}_{t-1}=\frac{K_{t}-K_{t-1}}{K_{t}K_{t-1}^{c}}\widetilde{X}_{t-1}+\varepsilon_{t-1}, \tag{185}$$

where the noise ε_{t-1} is independent of $(\widetilde{P}_{t-l}-\widetilde{P}_{t-1-l})$, for $l=1,...,t$, and thus of $\widetilde{X}_{t-1}$. Hence,

$$\mathrm{Cov}\left[\widetilde{P}_{t}-\widetilde{P}_{t-1},\widetilde{X}_{t-1}\right]=\frac{K_{t}-K_{t-1}}{K_{t}K_{t-1}^{c}}\mathrm{Var}\left[\widetilde{X}_{t-1}\right], \tag{186}$$

which implies

$$\mathbb{E}\left[\widetilde{X}_{t-1}|\widetilde{P}_t-\widetilde{P}_{t-1}\right]=\frac{\text{Cov}\left[\widetilde{P}_t-\widetilde{P}_{t-1},\widetilde{X}_{t-1}\right]}{\text{Var}\left[\widetilde{P}_t-\widetilde{P}_{t-1}\right]}\left(\widetilde{P}_t-\widetilde{P}_{t-1}\right) \tag{187}$$

$$=\frac{K_t-K_{t-1}}{K_tK_{t-1}^c}\frac{\text{Var}\left[\widetilde{X}_{t-1}\right]}{\text{Var}\left[\widetilde{P}_t-\widetilde{P}_{t-1}\right]}\left(\widetilde{P}_t-\widetilde{P}_{t-1}\right). \tag{188}$$

Substituting back into (184), we obtain

$$\mathbb{E}\left[\widetilde{P}_{t+1}-\widetilde{P}_t|\widetilde{P}_t-\widetilde{P}_{t-1}\right]=\frac{K_{t+1}-K_t}{K_{t+1}K_t^c}\left[m_{t-1}+\frac{K_t-K_{t-1}}{K_tK_{t-1}^c}\frac{\text{Var}\left[\widetilde{X}_{t-1}\right]}{\text{Var}\left[\widetilde{P}_t-\widetilde{P}_{t-1}\right]}\right](\widetilde{P}_t-\widetilde{P}_{t-1}). \tag{189}$$

Appendix C

C.1. Proof of Proposition 7

An application of Lemma B.1 yields agent i's expectation regarding the future price:

$$\mathbb{E}\left[\widetilde{P}_{t+1}|\mathcal{F}_t^i\right]=\frac{K_t}{K_{t+1}}\widetilde{P}_t+\frac{K_{t+1}-K_t}{K_{t+1}}\widetilde{\mu}_t^i. \tag{190}$$

Reorganize the relation in (190) as

$$\widetilde{\mu}_t^i=\mathbb{E}\left[\widetilde{P}_{t+1}|\mathcal{F}_t^i\right]-\frac{K_t}{K_{t+1}}\widetilde{P}_t+\frac{K_t}{K_{t+1}}\widetilde{\mu}_t^i, \tag{191}$$

and substitute it in individual portfolio demands $\widetilde{D}_t^i=\tau K_t^i(\widetilde{\mu}_t^i-\widetilde{P}_t)$ to obtain

$$\widetilde{D}_t^i=\tau K_t^i\left(\mathbb{E}\left[\widetilde{P}_{t+1}|\mathcal{F}_t^i\right]-\frac{K_t}{K_{t+1}}\widetilde{P}_t+\frac{K_t}{K_{t+1}}\widetilde{\mu}_t^i-\widetilde{P}_t\right). \tag{192}$$

Reorganizing yields the following decomposition

$$\widetilde{D}_t^i = \tau K_t^i \frac{K_t}{K_{t+1}} \left(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^i] - \widetilde{P}_t \right) + \tau K_t^i \left(\mathbb{E}[\widetilde{P}_{t+1}|\mathcal{F}_t^i] - \widetilde{P}_t \right), \tag{193}$$

which can also be written as in Proposition 7:

$$\widetilde{D}_t^i = \frac{\tau K_t^i}{K_t} \left(\frac{K_t^2}{K_{t+1}} \left(\mathbb{E}[\widetilde{U}|\mathcal{F}_t^i] - \widetilde{P}_t \right) + K_t \left(\mathbb{E}[\widetilde{P}_{t+1}|\mathcal{F}_t^i] - \widetilde{P}_t \right) \right). \tag{194}$$

C.2. Proof of Proposition 8, Corollary 1, and Corollary 2

Recall that individual demands are given by

$$\widetilde{D}_t^i = K_t^i(\widetilde{\mu}_t^i - \widetilde{P}_t), \tag{195}$$

and the martingale condition of Lemma B.1 is

$$\mathbb{E}\left[K_{t+1}(\widetilde{P}_{t+1} - \widetilde{\mu}_{t+1}^i)|\mathcal{F}_t^i \right] = K_t(\widetilde{P}_t - \widetilde{\mu}_t^i), \tag{196}$$

which can thus be interpreted as a condition on rescaled portfolios. In particular, we can write:

$$\mathbb{E}\left[\frac{K_{t+1}}{K_{t+1}^i} \widetilde{D}_{t+1}^i |\mathcal{F}_t^i \right] = \frac{K_t}{K_t^i} \widetilde{D}_t^i, \tag{197}$$

which is the result of Proposition 8.

Proof of Corollary 1. From Lemma B.1, we know that:

$$\mathbb{E}\left[K_{t+1}(\widetilde{\mu}_{t+1}^i - \widetilde{P}_{t+1})|\mathcal{F}_t^i \right] = K_t(\widetilde{\mu}_t^i - \widetilde{P}_t). \tag{198}$$

Further, if agent i's precision, K_t^i, coincides with the average market precision, K_t, then

$$\mathbb{E}\left[K_{t+1}^i|\mathcal{F}_t^i\right] = K_t^i + K_{t+1} - K_t = K_{t+1}, \tag{199}$$

that is, the agent expects to remain the "average agent" next period. Thus,

$$\begin{aligned}
\mathbb{E}\left[\widetilde{D}_{t+1}^i - \widetilde{D}_t^i|F_t^i\right] &= \mathbb{E}\left[\tau K_{t+1}^i(\widetilde{\mu}_{t+1}^i - \widetilde{P}_{t+1}) - \tau K_t^i(\widetilde{\mu}_t^i - \widetilde{P}_t)|F_t^i\right] &(200)\\
&= \tau\left\{\mathbb{E}\left[K_{t+1}(\widetilde{\mu}_{t+1}^i - \widetilde{P}_{t+1})|\mathcal{F}_t^i\right] - K_t(\widetilde{\mu}_t^i - \widetilde{P}_t)\right\} &(201)\\
&= 0. &(202)
\end{aligned}$$

Proof of Corollary 2. We can write

$$\begin{aligned}
\mathbb{E}\left[\widetilde{D}_{t+1}^i|\mathcal{F}_t^i\right] &= \tau\mathbb{E}\left[K_{t+1}^i\widetilde{\mu}_{t+1}^i - K_{t+1}^i\widetilde{P}_{t+1}|\mathcal{F}_t^i\right] &(203)\\
&= \tau\mathbb{E}\left[K_{t+1}^i\widetilde{\mu}_{t+1}^i|\mathcal{F}_t^i\right] - \tau\mathbb{E}\left[K_{t+1}^i|\mathcal{F}_t^i\right]\mathbb{E}\left[\widetilde{P}_{t+1}|\mathcal{F}_t^i\right] &(204)
\end{aligned}$$

where the second line follows from the fact that K_{t+1}^i and $\widetilde{P}_{t+1}$ are independent conditional on the information set $\mathcal{F}_t^i$. Using

$$\widetilde{\mu}_t^i = \frac{1}{K_t^i}\left(\sum_{j=0}^{t} S\omega_j^i\widetilde{Z}_j^i + \sum_{j=0}^{t}\Phi\tau^2S^2\Omega_j^2\widetilde{Q}_j\right), \tag{205}$$

we can express conditional expectations recursively as

$$K_{t+1}^i\widetilde{\mu}_{t+1}^i = K_t^i\widetilde{\mu}_t^i + S\omega_{t+1}^i\widetilde{Z}_{t+1}^i + \Phi\tau^2S^2\Omega_{t+1}\widetilde{Q}_{t+1}. \tag{206}$$

Observe that, since meetings are independent, an agent i expects to collect the average incremental number of signals next period

$$\mathbb{E}\left[\omega_{t+1}^i|\mathcal{F}_t^i\right] = \Omega_{t+1}. \tag{207}$$

As a result, we have

$$\mathbb{E}\left[K_{t+1}^i \widetilde{\mu}_{t+1}^i | \mathcal{F}_t^i\right] = K_t^i \widetilde{\mu}_t^i + S\Omega_{t+1}\mathbb{E}\left[\widetilde{Z}_{t+1}^i | \mathcal{F}_t^i\right] + \Phi\tau^2 S^2 \Omega_{t+1}\mathbb{E}\left[\widetilde{Q}_{t+1} | \mathcal{F}_t^i\right] \tag{208}$$

$$= K_t^i \widetilde{\mu}_t^i + S\Omega_{t+1}\widetilde{\mu}_t^i + \Phi\tau^2 S^2 \Omega_{t+1}\widetilde{\mu}_t^i \tag{209}$$

$$= (K_t^i + K_{t+1} - K_t)\widetilde{\mu}_t^i \tag{210}$$

and

$$\mathbb{E}\left[K_{t+1}^i | \mathcal{F}_t^i\right] = K_t^i + K_{t+1} - K_t. \tag{211}$$

Finally, using the relation (190), we can write

$$\mathbb{E}\left[\widetilde{D}_{t+1}^i | \mathcal{F}_t^i\right] = \tau\left(K_t^i + K_{t+1} - K_t\right)\left(\widetilde{\mu}_t^i - \mathbb{E}\left[\widetilde{P}_{t+1} | \mathcal{F}_t^i\right]\right) \tag{212}$$

$$= \tau\left(K_t^i + K_{t+1} - K_t\right)\left(\widetilde{\mu}_t^i - \frac{K_t}{K_{t+1}}\widetilde{P}_t - \frac{K_{t+1} - K_t}{K_{t+1}}\widetilde{\mu}_t^i\right) \tag{213}$$

$$= \tau\left(K_t^i + K_{t+1} - K_t\right)\frac{K_t}{K_{t+1}}\left(\widetilde{\mu}_t^i - \widetilde{P}_t\right) \tag{214}$$

$$= \left(K_t^i + K_{t+1} - K_t\right)\frac{K_t}{K_{t+1}}\frac{1}{K_t^i}\widetilde{D}_t^i. \tag{215}$$

We can therefore compute

$$\mathbb{E}\left[\widetilde{D}_{t+1}^i - \widetilde{D}_t^i | \mathcal{F}_t^c\right] = \left[\left(K_t^i + K_{t+1} - K_t\right)\frac{K_t}{K_{t+1}}\frac{1}{K_t^i} - 1\right]\mathbb{E}\left[\widetilde{D}_t^i | \mathcal{F}_t^c\right] \tag{216}$$

$$= \left[\left(K_t^i + K_{t+1} - K_t\right)\frac{K_t}{K_{t+1}} - K_t^i\right]\tau\left(\mathbb{E}[\widetilde{U} | \mathcal{F}_t^c] - \widetilde{P}_t\right) \tag{217}$$

$$= \frac{(K_t - K_t^i)(K_{t+1} - K_t)}{K_{t+1}}\tau\left(\mathbb{E}[\widetilde{U} | \mathcal{F}_t^c] - \widetilde{P}_t\right), \tag{218}$$

where the second line follows by the law of iterated expectations.

C.3. Proof of Proposition 9

We know from Proposition 8, Corollary 2 that

$$\mathbb{E}\left[\widetilde{D}^i_{t+1} - \widetilde{D}^i_t | \mathcal{F}^c_t\right] = \frac{(K_t - K^i_t)(K_{t+1} - K_t)}{K_{t+1}} \tau \left(\mathbb{E}[\widetilde{U}|\mathcal{F}^c_t] - \widetilde{P}_t\right). \tag{219}$$

Then, we restate here Eq. (146):

$$\mathbb{E}[\widetilde{U}|\mathcal{F}^c_t] - \widetilde{P}_t = \frac{K_{t+1}}{K_{t+1} - K_t} \mathbb{E}[\widetilde{P}_{t+1} - \widetilde{P}_t | \mathcal{F}^c_t]. \tag{220}$$

Replacing (220) in (219) gives

$$\mathbb{E}\left[\widetilde{D}^i_{t+1} - \widetilde{D}^i_t | \mathcal{F}^c_t\right] = \tau(K_t - K^i_t)\mathbb{E}[\widetilde{P}_{t+1} - \widetilde{P}_t | \mathcal{F}^c_t], \tag{221}$$

and then using Proposition 5 and the fact that $\mathcal{F}^c_t$ and $\mathcal{F}^r_t$ are equivalent information sets, we obtain

$$\mathbb{E}\left[\widetilde{D}^i_{t+1} - \widetilde{D}^i_t | \mathcal{F}^r_t\right] = \tau(K_t - K^i_t) \sum_{l=1}^{t+1} \frac{K_{t+1} - K_t}{K_{t+1} K^c_t} m_{t-l} (\widetilde{P}_{t-l+1} - \widetilde{P}_{t-l}). \tag{222}$$

C.4. Risk-neutral arbitrageur

In this appendix, we derive equilibrium solutions for prices and optimal demands in the presence of an unconstrained, uninformed, risk-neutral arbitrageur, which we summarize in Theorem C.1 below.

Theorem C.1. *There exists a partially revealing rational-expectations equilibrium in the four-trading session economy in which the price signal, $\widetilde{Q}_t$, for $t = 0, .., 3$, satisfies*

$$\widetilde{Q}_t = \widetilde{U} - \frac{1}{\tau S \Omega_t} \widetilde{X}_t \tag{223}$$

and in which the arbitrageur's demand $\widetilde{x}_t$ satisfies

$$\widetilde{x}_t = \frac{1}{2\lambda_t}\left(E\left[\widetilde{P}_{t+1}|\{\widetilde{Q}_j\}_{j=0}^{t}\right] - \varphi_t\widetilde{Q}_t - \sum_{j=0}^{t-1}\xi_{j,t}\widetilde{Q}_j\right). \tag{224}$$

The price coefficients satisfy

$$\varphi_t = \tfrac{A_t - \tau S\sum_{j=0}^{t-1}\Omega_j}{D_t}, \quad \xi_{j,t} = \tfrac{B_{j,t}+\tau S\Omega_j}{D_t}, \quad \gamma_t = \tfrac{C_t+1}{D_t}, \quad \lambda_t = \tfrac{1}{D_t} \tag{225}$$

where A, B_j, C, and D correspond to the coefficients of the aggregate demand of informed traders

$$\int_0^1 \widetilde{D}_t^i = A_t\widetilde{U} + \sum_{j=0}^{t-1} B_{j,t}\widetilde{Q}_j - C_t\widetilde{X}_t - D_t\widetilde{P}_t. \tag{226}$$

Proof. We provide the proof for a two-trading session economy. The model is solved backwards, starting from date 1 and then going back to date 0. First, conjecture that prices in period 0 and period 1 are

$$\widetilde{P}_0 = \beta_0\widetilde{U} - \gamma_{0,0}\widetilde{X}_0 + \lambda_0\widetilde{x}_0 \tag{227}$$

$$\widetilde{P}_1 = \beta_1\widetilde{U} - \gamma_{1,0}\widetilde{X}_0 - \gamma_{1,1}\widetilde{X}_1 + \lambda_1\widetilde{x}_1, \tag{228}$$

where $\widetilde{x}$ represents the demand of the risk-neutral arbitrageur, on which we elaborate below. Consider the normalized price signal in period zero (which is informationally equivalent to $\widetilde{P}_0$):

$$\widetilde{Q}_0 = \frac{1}{\beta_0}(\widetilde{P}_0 - \lambda_0\widetilde{x}_0) = \widetilde{U} - \frac{\gamma_{0,0}}{\beta_0}\widetilde{X}_0 \tag{229}$$

where the demand $\widetilde{x}$ of the risk-neutral trader is observable because she only trades on public

information, i.e., prices. Replace $\widetilde{X}_0$ from (229) into (227) to obtain

$$\widetilde{P}_1 = \varphi_1 \widetilde{U} + \xi_1 \widetilde{Q}_0 - \gamma_{1,1} \widetilde{X}_1 + \lambda_1 \widetilde{x}_1 \tag{230}$$

where $\varphi_1 = \beta_1 - \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$ and $\xi_1 = \gamma_{1,0} \frac{\beta_0}{\gamma_{0,0}}$. We normalize the price signal in period $t = 1$ and obtain $\widetilde{Q}_1$:

$$\widetilde{Q}_1 = \frac{1}{\varphi_1} \left(\widetilde{P}_1 - \xi_1 \widetilde{Q}_0 - \lambda_1 \widetilde{x}_1 \right) = \widetilde{U} - \frac{\gamma_{1,1}}{\varphi_1} \widetilde{X}_1. \tag{231}$$

Observing $\{\widetilde{Q}_0, \widetilde{Q}_1\}$ is equivalent to observing $\{\widetilde{P}_0, \widetilde{P}_1\}$. As in the setup of Section 2.2, we conjecture the following relationships:

$$\widetilde{Q}_0 = \widetilde{U} - \frac{1}{\tau S \Omega_0} \widetilde{X}_0 \tag{232}$$

$$\widetilde{Q}_1 = \widetilde{U} - \frac{1}{\tau S \Omega_1} \widetilde{X}_1. \tag{233}$$

Period 1 At time $t = 1$, both the precision and the posterior mean of an investor i remain identical to those of Section 2.2 in (83) along with her demand in (84). Integrating informed agents' demand again yields (87). The risk-neutral agent solves

$$\max_{\widetilde{x}_1} \widetilde{x}_1 E[\widetilde{U} - \widetilde{P}_t | \widetilde{Q}_0, \widetilde{Q}_1] = \max_{\widetilde{x}_1} \widetilde{x}_1 E[\widetilde{U} - \varphi_1 \widetilde{Q}_1 - \xi_1 \widetilde{Q_0} - \lambda_1 \widetilde{x}_1 | \widetilde{Q}_0, \widetilde{Q}_1], \tag{234}$$

and her optimal demand satisfies

$$\widetilde{x}_1 = \frac{1}{2\lambda_1} \left(E[\widetilde{U} | \widetilde{Q}_0, \widetilde{Q}_1] - \varphi_1 \widetilde{Q}_1 - \xi_1 \widetilde{Q_0} \right)$$

where

$$E[\widetilde{U} | \widetilde{Q}_0, \widetilde{Q}_1] = \frac{\tau^2 S^2 \Phi \left(\Omega_0^2 \widetilde{Q}_0 + \Omega_1^2 \widetilde{Q}_1 \right)}{H + \tau^2 S^2 \Phi (\Omega_0^2 + \Omega_1^2)}. \tag{235}$$

The market clearing condition is $\widetilde{D}_1 + \widetilde{x}_1 = \widetilde{X}_0 + \widetilde{X}_1$. Once we impose market clearing, we can use the conjectured Eq. (230) to get the undetermined coefficients φ_1, ξ_1, $\gamma_{1,1}$, and λ_1:

$$\begin{aligned}
\varphi_1 &= \frac{S\Omega_1\left(1+\tau^2 S\Phi\Omega_1\right)}{K_1}, \\
\xi_1 &= \frac{S\Omega_0\left(1+\tau^2 S\Phi\Omega_0\right)}{K_1} \\
\gamma_{1,1} &= \frac{1+\tau^2 S\Phi\Omega_1}{\tau K_1} \\
\lambda_1 &= \frac{1}{\tau K_1}.
\end{aligned} \tag{236}$$

From these solutions, we can verify that, indeed, $\frac{\gamma_{1,1}}{\varphi_1} = \frac{1}{\tau S\Omega_1}$. Hence, (72) is also verified in the presence of the risk-neutral agent.

Period 0 The problem of investor i at time $t = 0$ is, as in Section 2.2,

$$\max_{\widetilde{D}_0^i} \mathbb{E}\left[-e^{-\frac{1}{\tau}\widetilde{W}_2^i}\middle|\widetilde{Z}_0^i, \widetilde{Q}_0\right] \tag{237}$$

where the expectation now takes into account the new price function in (230). Importantly, Lemma A.2 still holds and informed agents' portfolio remains independent of the expected number of signals they will get in the future. The risk-neutral agent solves

$$\max_{\widetilde{x}_0} \widetilde{x}_0 E[\widetilde{P}_1 - \widetilde{P}_0|\widetilde{Q}_0] = \max_{\widetilde{x}_0} \widetilde{x}_0 E[\widetilde{P}_1 - \beta_0\widetilde{Q}_0 - \lambda_0\widetilde{x}_0|\widetilde{Q}_0]. \tag{238}$$

The optimization problem in (238) only involves the profits of period 0 because we assume that the risk-neutral agent does not take into account that a deviation from her strategy will affect current and future price signals for informed agents who cannot detect a deviation in her strategy; in that sense, the risk-neutral agent is myopic. As a result, her optimal demand satisfies

$$\widetilde{x}_0 = \frac{1}{\lambda_0}\left(E[\widetilde{P}_1|\widetilde{Q}_0] - \beta_0\widetilde{Q}_0\right) \tag{239}$$

where

$$E[\widetilde{P}_1|\widetilde{Q}_0] = \frac{S\left(2\tau^2 S\Phi + \frac{H}{H+S(\Omega_0+\Omega_1+\tau^2 S\Phi(\Omega_0^2+\Omega_1^2))}\right)}{2(H+\tau^2 S^2\Phi\Omega_0^2)}. \tag{240}$$

Integrating informed investors' optimal demand and imposing market clearing $D_0+\widetilde{x}_0 = \widetilde{X}_0$, we obtain β_0, $\gamma_{0,0}$, and λ_0. We can then verify that, indeed, $\frac{\gamma_{0,0}}{\beta_0} = \frac{1}{\tau S\Omega_0}$. By induction, the solution of the equilibrium for the trading dates takes the form in Theorem C.1. □

Arbitrageur's profits In this section, we show that the arbitrageur can only make profits if she allows momentum to persist. She therefore optimally forgoes profits. To illustrate this, we compute the unconditional profits Π she expects to make between time t and $t+1$. In particular, simple computations show that

$$\Pi = E\left[\widetilde{x}_t\left(\widetilde{P}_{t+1}-\widetilde{P}_t\right)\right] = \frac{1}{\lambda_t}E\left[\left(E\left[\widetilde{P}_{t+1}-\widetilde{P}_t|\{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right] = \lambda_t E\left[\widetilde{x}_t^2\right].$$

We then compare these profits to those of an econometrician, who is not strategic and ignores price impact ($\lambda_t \equiv 1$). The profits that the arbitrageur optimally forgoes to keep momentum in the model are therefore given by

$$\Pi' = E\left[\left(E\left[\widetilde{P}_{t+1}-\widetilde{P}_t|\{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right] - \Pi = \left(1-\frac{1}{\lambda_t}\right)E\left[\left(E\left[\widetilde{P}_{t+1}-\widetilde{P}_t|\{\widetilde{Q}_j\}_{j=0}^t\right]\right)^2\right]. \tag{241}$$

We plot the profits she makes and the profits she forgoes at time $t=1$ in Fig. 9. For low meeting intensities, Fig. 9 shows that the arbitrageur extracts half of the momentum rents, consistent with the behavior of a monopolist. As the meeting intensity increases, however, the momentum profits she forgoes significantly increase. The reason is that she now trades against agents who are better informed on average; accordingly, she has a larger price impact and therefore trades less aggressively on momentum. We conclude that it is difficult to

arbitrage away momentum in a market characterized by fast diffusion of information among investors.

[insert Figure 9 here]

Appendix D

D.1. Dynamic setup (Section 5.1)

This appendix mainly follows Andrei (2013). Consider the following processes for dividends and noisy supply:

$$D_t = \kappa_d D_{t-1} + \varepsilon_t^d \tag{242}$$

$$X_t = \kappa_x X_{t-1} + \varepsilon_t^x \tag{243}$$

where $0 \leq \kappa_d \leq 1$ and $0 \leq \kappa_x \leq 1$. The dividend and supply innovations are *i.i.d.* with normal distributions: $\varepsilon_t^d \sim \mathcal{N}(0, 1/H)$ and $\varepsilon_t^x \sim \mathcal{N}(0, 1/\Phi)$. There is one riskless bond assumed to have an infinitely elastic supply at positive constant gross interest rate R.

The economy is populated by a continuum of rational agents, indexed by i, with constant absolute risk aversion utilities and common risk aversion $1/\tau$. Each agent lives for two periods, while the economy goes on forever (overlapping generations). All investors observe the past and current realizations of dividends and of the stock prices. Additionally, each investor observes an information signal about the dividend innovation 3-steps ahead:

$$\widetilde{z}_t^i = \varepsilon_{t+3}^d + \widetilde{\epsilon}_t^i. \tag{244}$$

As time goes by, investors share their private information at random meetings. The information structure and the probability density function over the number of private signals is described in Andrei (2013). As usual in noisy rational expectations, we conjecture a linear

function of model innovations for the equilibrium price:

$$P_t = \alpha D_t + \beta X_{t-3} + (a_3\ a_2\ a_1)\epsilon_t^d + (b_3\ b_2\ b_1)\epsilon_t^x. \tag{245}$$

Proposition 1 in Andrei (2013) describes the rational-expectations equilibrium, which is found by solving a fixed-point problem provided by the market clearing condition. Infinite-horizon models with overlapping generations have multiple equilibria (there are 2^N equilibria for a model with N assets). The model studied here has two equilibria, one low volatility equilibrium and one high volatility equilibrium. We focus on the low volatility equilibrium, which is the limit of the unique equilibrium in the finite version of the model.

To understand how the two equilibria arise, let's assume that there is no private information. In this case, the equilibrium price has a closed-form solution:

$$P_t = \frac{\kappa_d}{R-\kappa_d} D_t - \frac{\Sigma}{\tau}\frac{\kappa_x^3}{R-\kappa_x} X_{t-3} - \frac{\Sigma}{\tau}\frac{\kappa_x^2}{R-\kappa_x}\varepsilon_{t-2}^x - \frac{\Sigma}{\tau}\frac{\kappa_x}{R-\kappa_x}\varepsilon_{t-1}^x - \frac{\Sigma}{\tau}\frac{1}{R-\kappa_x}\varepsilon_t^x, \tag{246}$$

where $\Sigma \equiv (\alpha+1)^2\sigma_d^2 + b_1^2\sigma_x^2$. Thus, the coefficient b_1 has to solve a quadratic equation:

$$b_1 = -\frac{\tau}{R-\kappa_x}\left[\left(\frac{R}{R-\kappa_d}\right)^2 \sigma_d^2 + b_1^2\sigma_x^2\right]. \tag{247}$$

For different parameter values, the above quadratic equation can have two solutions, one solution, or none. In this particular example (no private information), the autocovariance of stock returns, $\mathrm{Cov}\left(P_{t+1}-P_t, P_{t+2}-P_{t+1}\right)$, is

$$\begin{aligned}\mathrm{Cov}\left(P_{t+1}-P_t, P_{t+2}-P_{t+1}\right) = &-\alpha^2\sigma_d^2\frac{1-\kappa_d}{1+\kappa_d} + \beta^2(\kappa_x-1)^2\kappa_x\frac{\sigma_x^2}{1-\kappa_x^2} + \\ &\begin{pmatrix} \beta-b_3 & b_3-b_2 & b_2-b_1 & b_1 \end{pmatrix}\begin{pmatrix} -\beta(1-\kappa_x) \\ \beta-b_3 \\ b_3-b_2 \\ b_2-b_1 \end{pmatrix}.\end{aligned} \tag{248}$$

It can be shown numerically that this covariance is generally negative when $\kappa_d < 1$ and $\kappa_x < 1$. In the random walk specification (242) – (243), the covariance is zero.

If agents receive private information, the model has to be solved numerically using the methodology described in Andrei (2013). More precisely, α, β, a, and b solve the following equations:

$$(\alpha + 1)\kappa_d - R\alpha = 0 \tag{249}$$

$$\bar{K}_t \beta \kappa_x - \bar{K}_t R\beta - \frac{1}{\tau}\kappa_x^3 = 0 \tag{250}$$

$$\bar{K}_t b^* \mathbb{B}^{-1}\mathbb{A} + \bar{L}_t \mathbb{H} - \bar{K}_t R a = \mathbf{0}_{1\times 3} \tag{251}$$

$$\bar{K}_t b^* + \bar{L}_t \mathbb{B}^* - \bar{K}_t R b - \frac{1}{\tau}\left(\kappa_x^2 \ \kappa_x \ 1\right) = \mathbf{0}_{1\times 3}, \tag{252}$$

where $\bar{K}_t$, b^*, $\mathbb{B}$, $\mathbb{A}$, $\bar{L}_t$, $\mathbb{H}$, and $\mathbb{B}^*$ are defined in Appendix A.3 of Andrei (2013).

D.2. Proof of Theorem 5.1

To prove Theorem 5.1, we adapt the expression for the price $\widetilde{P}$ in Brennan and Cao (1997) and write

$$\widetilde{P}_t = \beta_t \widetilde{U} + \alpha_t \widetilde{V} + \sum_{j=0}^{t-1} \xi_{j,t}\widetilde{Q}_j - \gamma_t \widetilde{X}_t. \tag{253}$$

The price is informationally equivalent to

$$\widetilde{Q}_t = \frac{1}{\beta_t}\left(\widetilde{P}_t - \sum_{j=0}^{t-1}\xi_{j,t}\widetilde{Q}_j\right) = \widetilde{U} + \frac{\alpha_t}{\beta_t}\widetilde{V} - \frac{\gamma_t}{\beta_t}\widetilde{X}_t. \tag{254}$$

Furthermore, we can write agent i's individual demand as

$$\widetilde{D}_t^i = \omega_t^i \widetilde{P}_t + \sum_{k=0}^{t}\lambda_k^i \widetilde{Q}_k + \sum_{k=0}^{t}\theta_k^i \widetilde{Z}_k^i. \tag{255}$$

By the law of large numbers, we have that $\int_0^1 \widetilde{Z}_k^i = \widetilde{U} + \widetilde{V}$. As a result, when we aggregate individual demands, we obtain

$$\int_0^1 \widetilde{D}_t^i = \bar{\omega}_t \widetilde{P}_t + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k + \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} + \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V}, \tag{256}$$

where $\bar{\omega}_t = \sum_{k\in\mathbb{N}} \pi_t(k)\omega_t^i(k)$, $\bar{\lambda}_t = \sum_{k\in\mathbb{N}} \pi_t(k)\lambda_t^i(k)$, and $\bar{\theta}_t = \sum_{k\in\mathbb{N}} \pi_t(k)\theta_t^i(k)$ are average demand coefficients across the population of agents. Each average involves the distribution of types.

Imposing market clearing, we have

$$\sum_{k=0}^{t} \widetilde{X}_k - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = \bar{\omega}_t \widetilde{P}_t + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k. \tag{257}$$

Substituting

$$\widetilde{P}_t = \beta_t \widetilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j \tag{258}$$

into the above equation, we obtain

$$\sum_{k=0}^{t} \widetilde{X}_k - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = \bar{\omega}_t(\beta_t \widetilde{Q}_t + \sum_{j=0}^{t-1} \xi_{j,t} \widetilde{Q}_j) + \sum_{k=0}^{t} \bar{\lambda}_k \widetilde{Q}_k. \tag{259}$$

Furthermore, notice that

$$\widetilde{X}_k = \frac{\beta_k}{\gamma_k} \left(\widetilde{U} + \frac{\alpha_k}{\beta_k} \widetilde{V} - \widetilde{Q}_k \right). \tag{260}$$

Substituting and regrouping, we obtain

$$\widetilde{X}_t + \sum_{k=0}^{t-1} \frac{\beta_k}{\gamma_k} \left(\widetilde{U} + \frac{\alpha_k}{\beta_k} \widetilde{V} - \widetilde{Q}_k \right) - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{U} - \sum_{k=0}^{t} \bar{\theta}_k \widetilde{V} = (\bar{\omega}_t \beta_t + \bar{\lambda}_t) \widetilde{Q}_t + \sum_{j=0}^{t-1} (\xi_{j,t} \bar{\omega}_t + \bar{\lambda}_j) \widetilde{Q}_j, \tag{261}$$

or, equivalently,

$$\widetilde{X}_t + \left(\sum_{k=0}^{t-1}\frac{\beta_k}{\gamma_k} - \sum_{k=0}^{t}\bar{\theta}_k\right)\widetilde{U} + \left(\sum_{k=0}^{t-1}\frac{\beta_k}{\gamma_k}\frac{\alpha_k}{\beta_k} - \sum_{k=0}^{t}\bar{\theta}_k\right)\widetilde{V} - \sum_{k=0}^{t-1}\frac{\beta_k}{\gamma_k}\widetilde{Q}_k = \\ (\bar{\omega}_t\beta_t + \bar{\lambda}_t)\left(\widetilde{U} + \frac{\alpha_t}{\beta_t}\widetilde{V} - \frac{\gamma_t}{\beta_t}\widetilde{X}_t\right) + \sum_{j=0}^{t-1}(\xi_{j,t}\bar{\omega}_t + \bar{\lambda}_j)\widetilde{Q}_j. \tag{262}$$

By separation of variables, we get

$$-\frac{\beta_t}{\gamma_t} = \bar{\omega}_t\beta_t + \bar{\lambda}_t \tag{263}$$

$$\sum_{k=0}^{t-1}\frac{\beta_k}{\gamma_k} - \sum_{k=0}^{t}\bar{\theta}_k = \bar{\omega}_t\beta_t + \bar{\lambda}_t \tag{264}$$

$$\sum_{k=0}^{t-1}\frac{\beta_k}{\gamma_k}\frac{\alpha_k}{\beta_k} - \sum_{k=0}^{t}\bar{\theta}_k = \frac{\alpha_t}{\beta_t}\left(\bar{\omega}_t\beta_t + \bar{\lambda}_t\right) \tag{265}$$

$$-\frac{\beta_k}{\gamma_k} = \xi_{k,t}\bar{\omega}_t + \bar{\lambda}_k, \quad \text{for } k = 0, 1, ..., t-1. \tag{266}$$

Without loss of generality, we set

$$\frac{\gamma_t}{\beta_t} = \frac{1}{rS\Omega'_t} \tag{267}$$

so that

$$\sum_{k=0}^{t}\bar{\theta}_k = \sum_{k=0}^{t}\frac{\beta_k}{\gamma_k} = rS\sum_{k=0}^{t}\Omega_k, \tag{268}$$

and

$$\frac{\alpha_t}{\beta_t} = \frac{\Lambda_t}{rS\Omega_t} \tag{269}$$

so that:

$$\sum_{k=0}^{t} \bar{\theta}_k = \sum_{k=0}^{t} \frac{\beta_k}{\gamma_k} \frac{\alpha_k}{\beta_k} = \sum_{k=0}^{t} \Lambda_k. \tag{270}$$

The system of equations in (49) follows. This system of equations is a fixed point: to solve it, we solve the problem recursively (as in Appendix A.2, except accounting for the rumor) over four periods. We then start with guess values for $\{\Omega_j\}_{j=0}^{3}$ and $\{\Lambda_j\}_{j=0}^{3}$ and get, through the fixed point in (49), new values for these coefficients. Iterating and invoking the Contraction-Mapping Theorem, we obtain the equilibrium coefficients.

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Table 1
Two consecutive price differences in the dynamic model.

Price	Dividend innovations					Supply innovations				
differences	ε_t^d	ε_{t+1}^d	ε_{t+2}^d	ε_{t+3}^d	ε_{t+4}^d	ε_{t-3}^x	ε_{t-2}^x	ε_{t-1}^x	ε_t^x	ε_{t+1}^x
$P_t - P_{t-1}$	$\alpha - a_3$	$a_3 - a_2$	$a_2 - a_1$	a_1	0	$\beta - b_3$	$b_3 - b_2$	$b_2 - b_1$	b_1	0
$P_{t+1} - P_t$	0	$\alpha - a_3$	$a_3 - a_2$	$a_2 - a_1$	a_1	0	$\beta - b_3$	$b_3 - b_2$	$\underbrace{b_2 - b_1}_{\text{Reversal}}$	b_1

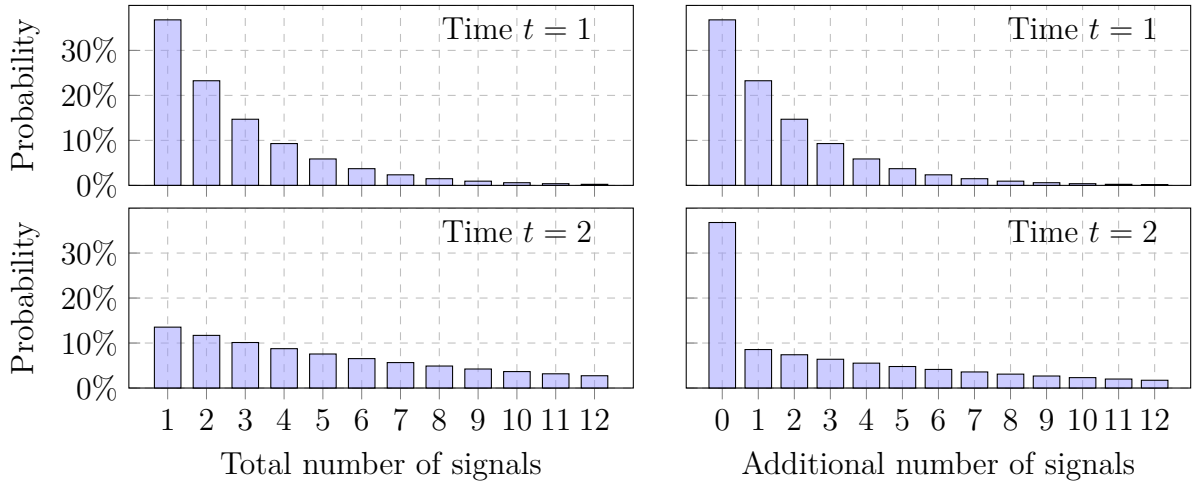

Fig. 1. Evolution of cross-sectional densities. The left-hand side panels depict the evolution of the probability density function over the *total* number of signals $\mu(\cdot)$ through time. The right-hand side panels depict the evolution of the probability density function over the *additional* number of signals $\pi(\cdot)$ through time. We fix $\lambda = 1$ for this illustration.

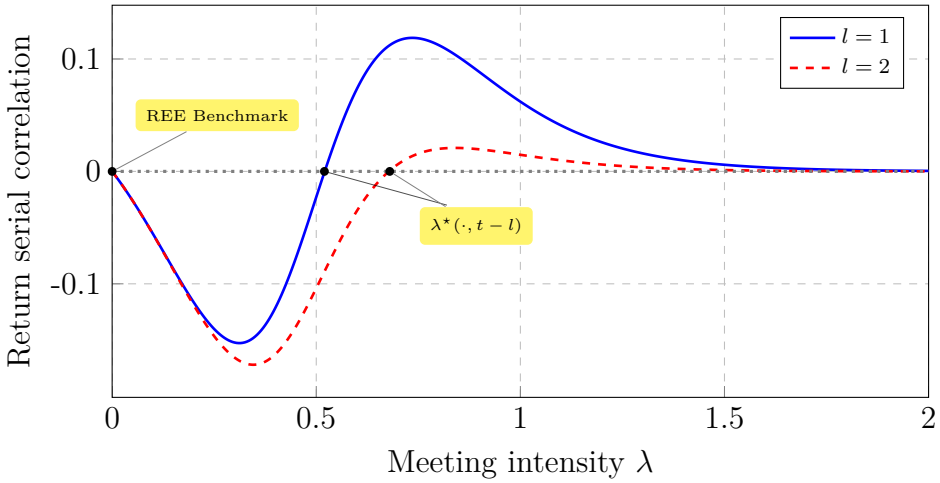

Fig. 2. Information percolation and serial correlation in Returns. Serial correlation of returns as a function of the meeting intensity λ. Serial correlation is computed at $t = 4$ for two different lags: the solid line corresponds to lag $l = 1$, and the dashed line to lag $l = 2$. The calibration used is $H = S = \Phi = 1$ and $\tau = 1/3$.

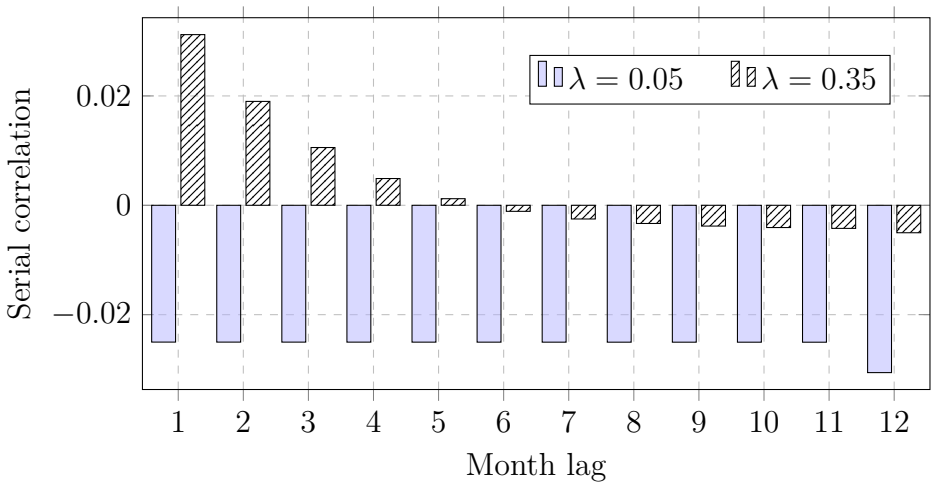

Fig. 3. Term structure of momentum. The figure depicts the serial correlation of returns when the lookback period varies from one to 12 months, as in (32) – (33). There are two sets of bars, one corresponding to $\lambda = 0.05$ and the second to $\lambda = 0.35$. The calibration used is $H = S = \Phi = 1$, and $\tau = 1/3$.

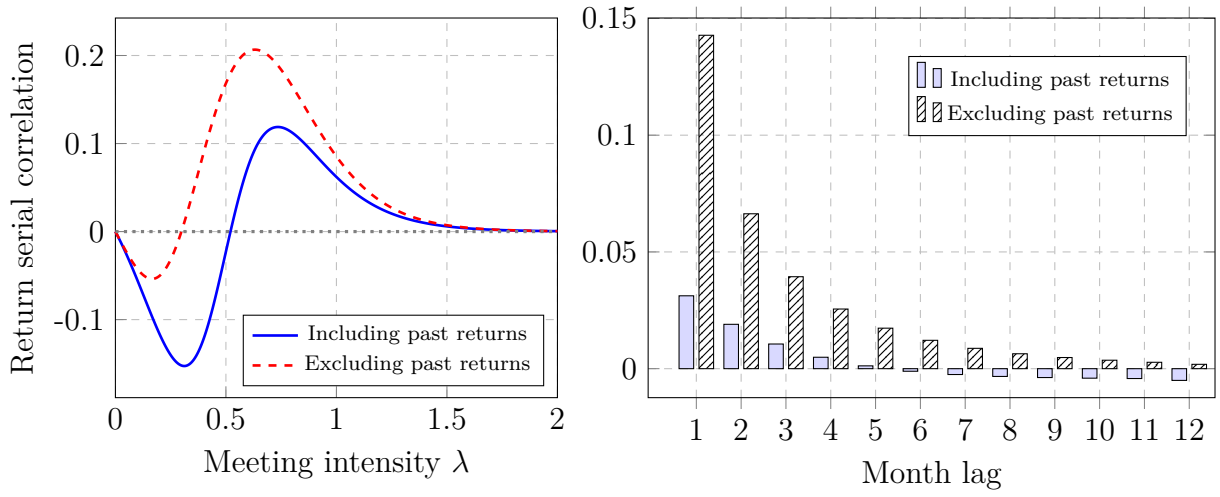

Fig. 4. Two ways of measuring momentum. The left panel depicts the serial correlation of returns at time $t = 4$ as a function of the meeting intensity λ. The solid line corresponds to our definition of momentum (Proposition 5), and the dashed line to the standard definition of momentum without additional lags (Proposition 6). The right panel depicts the serial correlation of returns for different lookback periods, as in Moskowitz et al. (2012), when $\lambda = 0.35$. The solid bars include all lags, whereas the dashed bars exclude any additional lags. The calibration used is $H = S = \Phi = 1$ and $\tau = 1/3$.

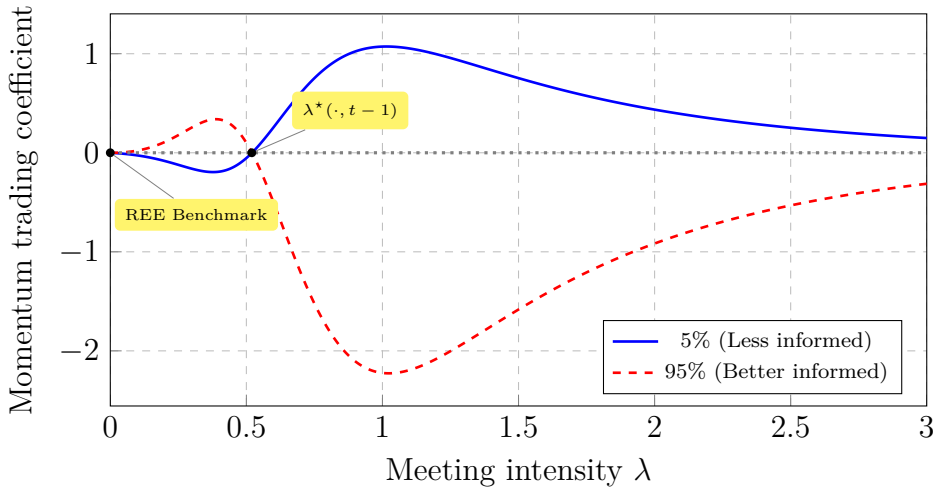

Fig. 5. Information percolation and momentum trading. Momentum trading coefficient from Eq. (40) as a function of the meeting intensity λ. A positive coefficient means momentum trading, whereas a negative coefficient means contrarian trading. The solid line corresponds to the 5% percentile less informed investor, and the dashed line to the 95% percentile better informed investor. Thus, the area between the two lines represents 90% of the investor population. The coefficient is represented at time $t = 4$ with one lag, $l = 1$. The calibration used is $H = S = \Phi = 1$ and $\tau = 1/3$.

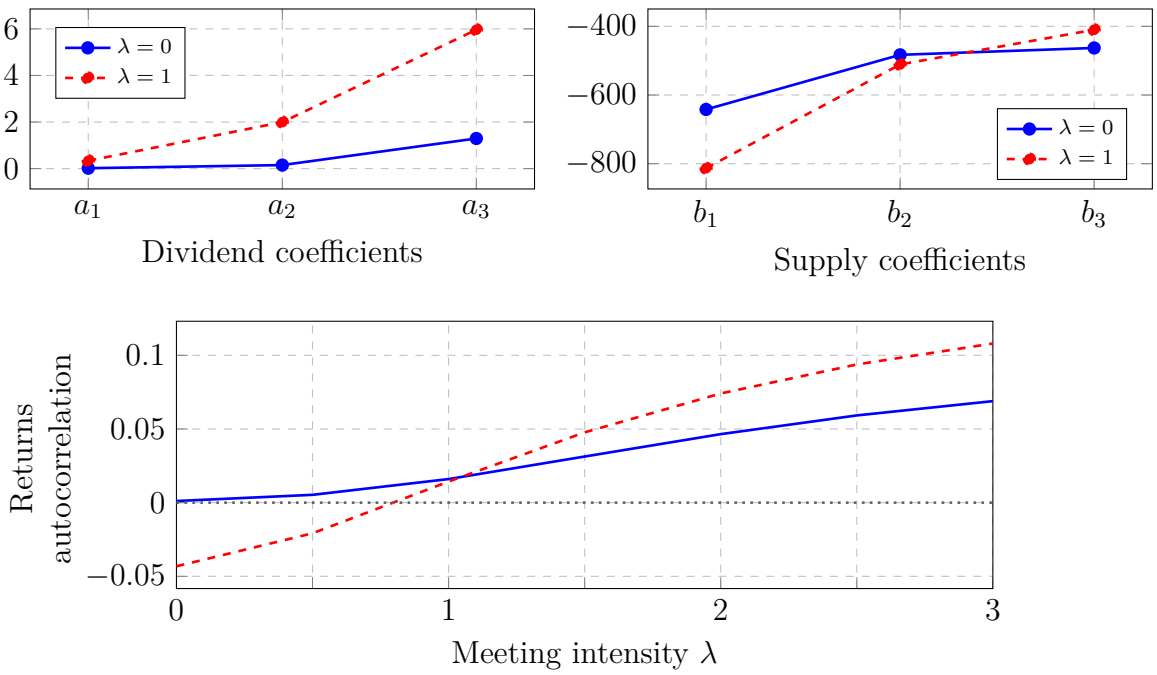

Fig. 6. Dynamic model: Price coefficients and serial correlation of returns. The upper panels plot the term structure of the coefficients a and b of the equilibrium price (44), without information percolation ($\lambda = 0$, solid lines) or with information percolation ($\lambda = 1$, dashed lines). The lower panel depicts the serial correlation of returns, $\text{corr}(P_{t+1} - P_t, P_{t+2} - P_{t+1})$, for different levels of the meeting intensity λ. There are two cases: (i) the dividend and supply processes are random walks (solid line) and (ii) the dividend and supply processes are mean-reverting with AR(1) parameter 0.9 (dashed line). The calibration for the rest of the parameters ensures the existence of an equilibrium in the stationary model: $R = 1.1$, $H = 1$, $S = 10$, $\Phi = 1/100$, and $\tau = 1/3$, although most of the calibrations we have tried yield the same qualitative results.

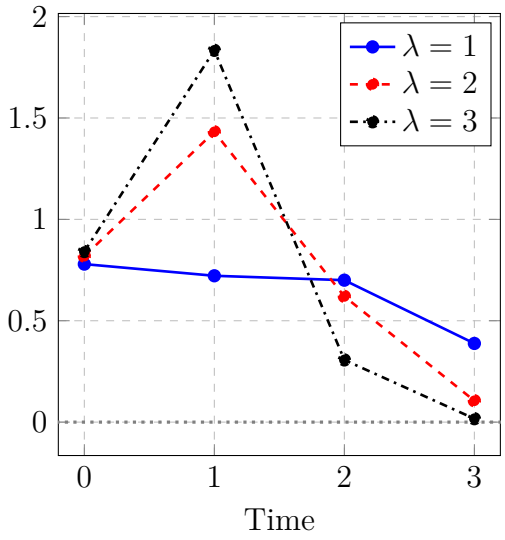

Fig. 7. Coefficients $\{\Omega'_j\}_{j=0}^t$ from Theorem 5.1 with a rumor. The figure depicts the coefficients $\{\Omega'_j\}_{j=0}^t$ over time in the presence of a rumor with precision $\nu = 3$. Each line corresponds to a meeting intensity of $\lambda = 1$, $\lambda = 2$, and $\lambda = 3$. The calibration used is $H = S = \Phi = 1$ and $\tau = 1/3$.

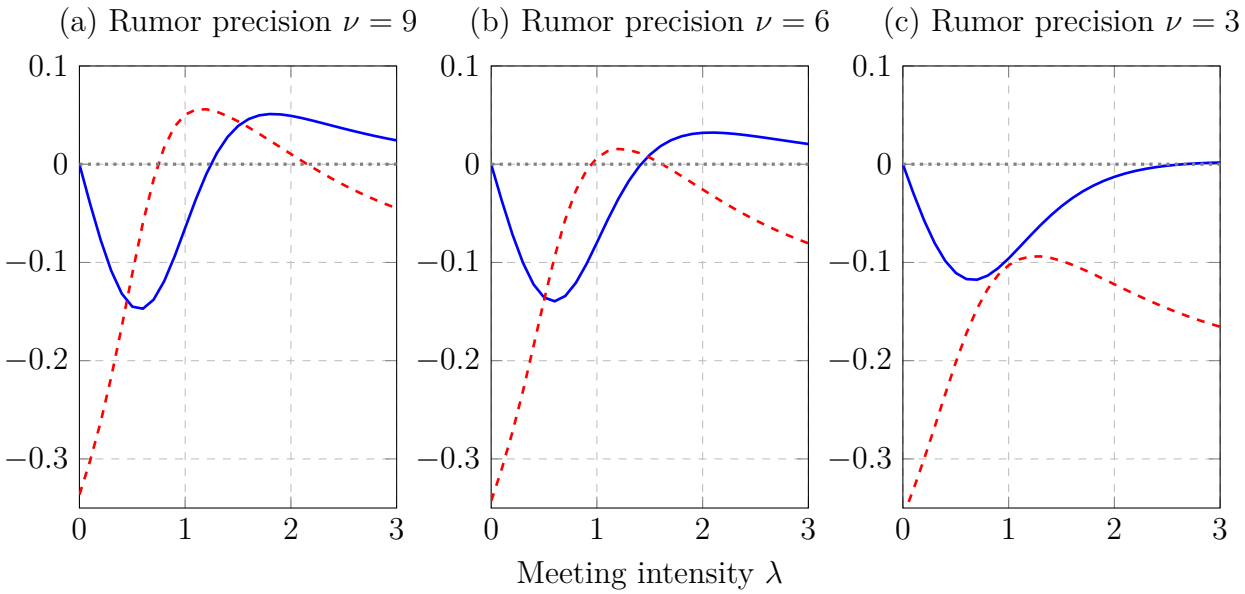

Fig. 8. Serial correlation of return and rumors. Serial correlation of stock returns over the first period (the solid line) and the second period (the dashed line) as a function of the meeting intensity. The first period serial correlation is defined as the regression coefficient between $\widetilde{P}_1 - \widetilde{P}_0$ and $\widetilde{P}_2 - \widetilde{P}_1$, whereas the second period serial correlation is defined as $\widetilde{P}_2 - \widetilde{P}_1$ and $\widetilde{P}_3 - \widetilde{P}_2$. Each panel corresponds to a different rumor precision ν. The calibration used is $H = S = \Phi = 1$ and $\tau = 1/3$.

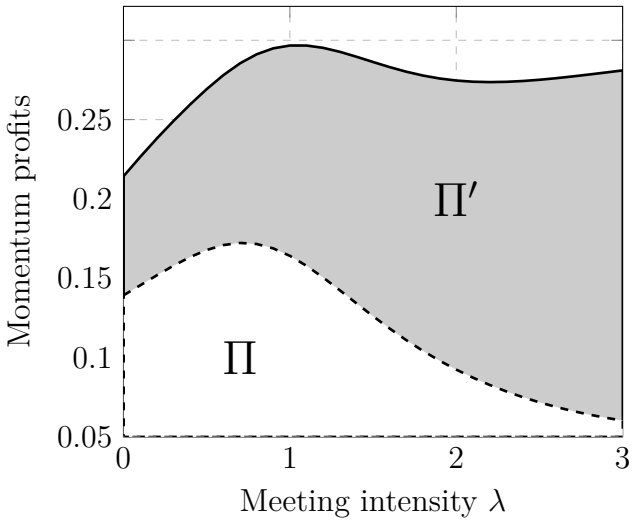

Fig. 9. Profits of the arbitrageur as a function of λ. The dashed line represents the profits made by the arbitrageur. The shaded area represents the profits that arbitrageur forgoes. The calibration is $H = S = \Phi = 1$ and $\tau = 1/3$.